Critical planes in the round section of an element taking the stress gradient into account

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ABSTRACT: *At present more and more criteria of multiaxial fatigue are based on the critical plane. The best results in the critical plane determination are obtained with use of the damage accumulation method. In this paper this method was applied for calculations of the critical plane position. The calculations were done for all the specimen section. The specimens were subjected to combined tension-compression with torsion, swinging bending with torsion. It has been shown that in different points of the section there are various local critical planes. The global critical plane is the plane in which the life calculated according to the chosen criterion is minimum.*

INTRODUCTION

The known algorithms for estimation of fatigue life of machine elements or structures have not been well verified in experiments. Many laboratory tests are necessary in order to determine a range of the algorithms application during design calculations. Some of the algorithms were verified for the selected materials (Łagoda and Macha [1], Łagoda [6]) but we do not know if they can be applied for other materials. It is very important to choose a suitable criterion of multiaxial fatigue. Most of the proposed multiaxial fatigue criteria use the critical plane. In such a case we meet problems connected with determination of this plane.

In the algorithm of the fatigue life estimation it is very important to determine the expected critical plane position in the point where the maximum effort of the material occurs. The critical plane position is strongly influenced by the stress or strain state occurring in the material. The position is determined by the direction cosines l_n, m_n, n_n ($n = \eta, s$) of the unit vectors $\bar{\eta}, \bar{s}$ occurring in the fatigue criteria, where $\bar{\eta}$ is perpendicular and $\bar{s}$ is tangent to the critical plane, so there is the relation $\bar{\eta} \cdot \bar{s} = 0$. The three methods of determination of the expected position of the critical fatigue fracture plane position ([1], Macha [3, 4]) are proposed: the weight function method, the maximum variance method and the damage accumulation method. In the last method damages are accumulated on all planes and the plane of the maximum damage is selected. As for the damage accumulation method and the variance method, the success depends on selection of a suitable criterion of fatigue effort and a stage of discretization of angle changes.

The aim of the paper is to acquaint with the damage accumulation method for determination of the critical plane position and fatigue life calculations under different loading states namely proportional cyclic tension with torsion and reversed bending with torsion. However, we should distinguish the ideas of the critical plane and the fracture plane.

THE ALGORITHM FOR FATIGUE LIFE DETERMINATION

Fig.1 shows the algorithm for fatigue life determination under multiaxial random loading. As a result of calculations we obtain the critical plane and the calculation life if we assume the same criterion for determination of the critical plane and the fatigue life.

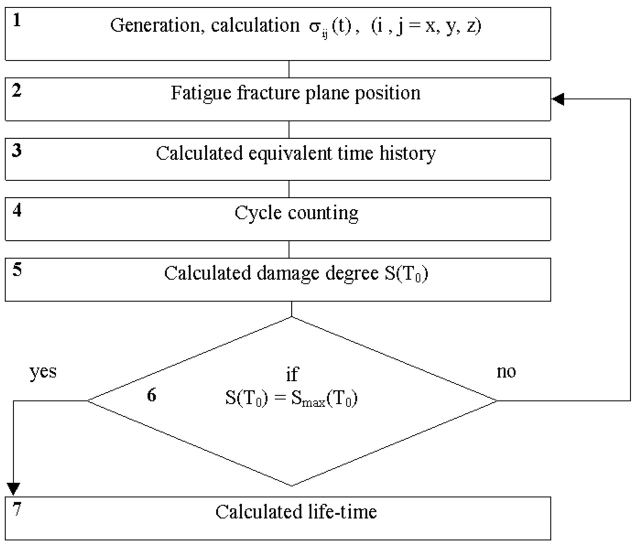

Figure 1: Algorithm for fatigue life calculations

In the first stage histories of the stress tensor components $\sigma_{ij}(t)$ are generated. In the second stage we assume the critical plane direction – it is optional at the beginning and next it is changed at the successive iterative steps. In the case of the damage accumulation method we start from the third stage, i.e. from determination of history of the equivalent (uniaxial) stress [1]. Under multiaxial loading we can use, for example, the generalized criterion of maximum shear and normal stresses in the critical plane [3]

$$\max_t \left\{ B\tau_{\eta s}(t) + K\sigma_\eta(t) \right\} = F \tag{1}$$

where B, K, F – the constants for selection of the particular criterion, $\tau_{\eta s}(t)$, $\sigma_\eta(t)$ – time courses of the stresses connected with the plane with normal $\overline{\eta}$ ([1], Downing, and Socie [2], [3-6]).

Assuming that B = 0 and K = 1, we obtain the criterion of maximum normal stress in the critical plane

$$\sigma_{eq}(t) = \sigma_\eta(t) = l_\eta^2 \sigma_{xx}(t) + m_\eta^2 \sigma_{yy}(t) + n_\eta^2 \sigma_{zz}(t) + 2 l_\eta m_\eta \sigma_{xy}(t) + 2 l_\eta n_\eta \sigma_{xz}(t) + 2 m_\eta n_\eta \sigma_{yz}(t) \tag{2}$$

For the same histories of the stress state components $\sigma_{xx}(t)$ and $\sigma_{xy}(t)$, the equivalent stress $\sigma_{eq}(t)$ according to (3) depends on a position of vector $\overline{\eta}$, i.e. on its components l_n, m_n, n_n. In the plane stress state a position of vector $\overline{\eta}$ normal to the critical plane can be described with one angle φ in relation to 0x axis. Thus, the direction cosines of the unit vectors are:

$$l_\eta = \cos\varphi, \qquad m_\eta = \sin\varphi, \qquad n_\eta = 0 \tag{3}$$

In the fourth stage we schematise a course of the random equivalent stress with the rain flow method [2].

For damage accumulation (the fifth stage) we apply the selected hypothesis. It is often the linear Palmgren-Miner hypothesis (PM):

$$S_{PM}(T_0) = \begin{cases} \sum_{i=1}^{j} \dfrac{n_i}{N_0 \left(\dfrac{\sigma_{af}}{\sigma_{ai}} \right)^m} & \text{for } \sigma_{\text{ai}} \ge a \cdot \sigma_{af} \\ 0 & \text{for } \sigma_{\text{ai}} < a \cdot \sigma_{af} \end{cases} \tag{4}$$

where n_i – a number of cycles with amplitudes σ_{ai} in T_0, m – coefficient of the Wöhler curve slope, N_0 – a number of cycles corresponding to the fatigue limit σ_{af}, a – coefficient allowing to consider influence of the amplitudes less than the fatigue limit σ_{af}.

After determination of the damage degree $S(T_0)$ at the observation time T_0 according to (3) we calculate the fatigue life

$$T_{cal} = \frac{T_0}{S(T_0)} \tag{5}$$

THE CRITICAL PLANE IN THE SPECIMEN SECTION

In the general case of the complex structures we discretize such a structure and calculate the strain and stress tensors in particular points. For calculations we can apply the finite element method. Next, we can determine the critical plane in those points. Generally speaking, the fatigue life can be different in each considered point of the analysed structure. The life in the point of the maximum damage degree is assumed as the fatigue life of all the structure. In this paper we consider only the round smooth specimen. Axis of specimen is denoted as X axis and bending takes place along Y axis. For this type of specimen searching the point of the maximum damage is not difficult and it does not require the finite element method because the stress gradient caused by bending and torsion is well known. However, it is interesting to analyse distribution of the critical planes in the specimen section under the occurring of stress gradients. Let us consider two cases of proportional cyclic tension with torsion and reversed bending with torsion. Under such combinations we have only the normal stress σ_{xx} and the shear stress σ_{xy}. Thus, the criterion of maximum normal stress (2) is reduced to the following form

$$\sigma_{eq}(t) = l_\eta^2 \sigma_{xx}(t) + 2 l_\eta m_\eta \sigma_{xy}(t) \tag{6}$$

Under proportional cyclic loading for amplitudes we have

$$\sigma_{eqa} = l_\eta^2 \sigma_{xxa} + 2 l_\eta m_\eta \sigma_{xya} \tag{7}$$

Substituting Eqs. (3) to (7) we obtain

$$\sigma_{eqa} = \left(\cos^2 \varphi\right)\sigma_{xxa} + \left(2 \sin\varphi \cos\varphi\right)\sigma_{xya} \tag{8}$$

After differentiation and equation to zero of Eq.(8) and taking into account changes of the torsional stresses in the specimen section we obtain the analytical condition for the critical plane position determined by the angle φ

$$\varphi = 0.5 arctg \frac{2k\sqrt{y^2 + z^2}}{R} \tag{9}$$

where: $k = \sigma_{xya} / \sigma_{xxa}$ - ratio of the shear stress amplitudes to the normal ones, R - specimen radius, y, z - coordinates in the specimen section.
The critical plane position varies along the radius according to

$$\varphi = 0.5 arctg \frac{2k\rho}{R} \tag{10}$$

where the actual distance from the centre is $\rho = \sqrt{y^2 + z^2}$.
Assuming that $k = 1$, this angle varies from 31.7^o on the surface to 0^o inside the specimen. The critical plane in the section of a round specimen subjected to proportional tension with torsion is shown in Fig.2.

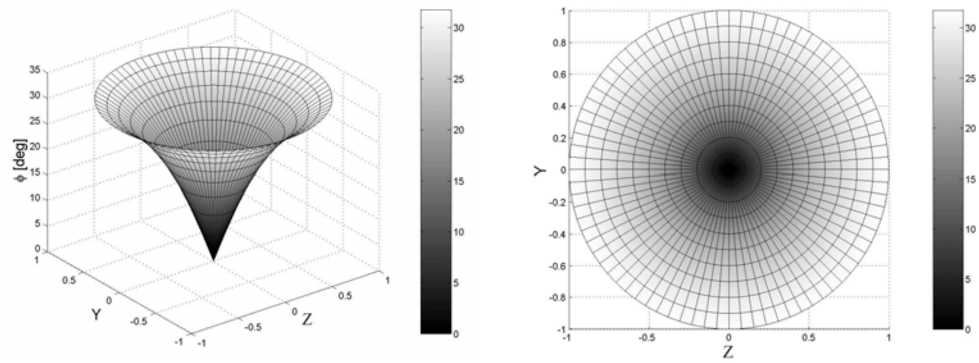

Figure 2: The critical plane positions in the section of a round specimen subjected to proportional cyclic tension with torsion

Let us consider a combination of proportional bending with torsion. The analytical condition for the critical plane position determined by the angle φ is similar to Eq.(10)

$$\varphi = 0.5 arctg \frac{2k\sqrt{y^2 + z^2}}{y} \tag{11}$$

The critical plane positions for such a case are shown in Fig. 3. From Fig.2 it appears that under tension with torsion in the same distance from the specimen axis we can observe the same equivalent amplitude σ_{eqa} and the same calculated critical plane. Under combined bending with torsion (Fig.3) we observe only torsion in the bending plane and the critical plane is inclined at 45^{o}; in the fibre which is the most distant from the bending plane the critical plane is inclined at 31.7^{o} as under the combined tension with torsion. The same critical plane directions are located radially from the specimen centre. If the critical plane is identified with the fracture plane, we should obtain outlines of cracks in the round specimens, see Fig.4. From the experiments it results that the fracture plane i.e. the experimental crack outline does not take forms shown in Fig.4. It means that the fracture plane should not be identified with the critical plane. The critical plane should be understood as the theoretical plane occurring in a fatigue criterion, allowing to determine the minimum life of the element or structure or the crack initiation plane.

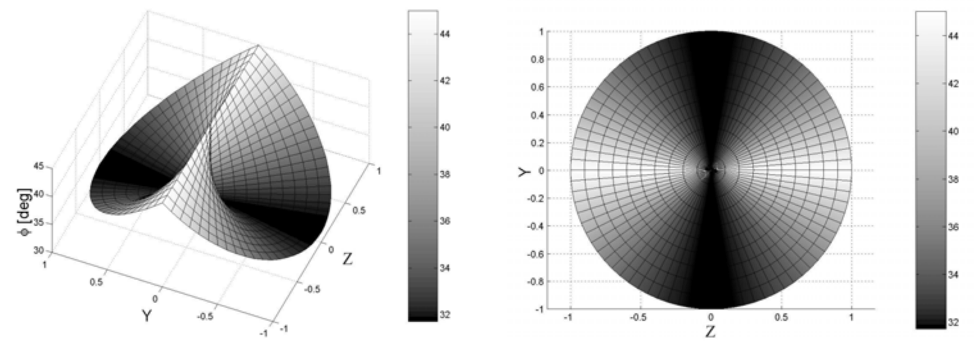

Figure 3: The critical planes in the section of a round specimen subjected to proportional cyclic bending with torsion

(a) (b)

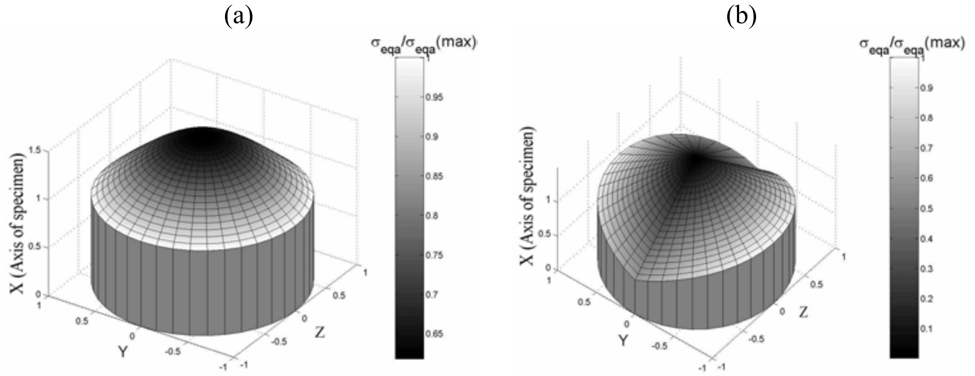

Figure 4: Theoretical crack outline in a round specimen with a map of normalized stresses amplitudes $\sigma_{eqa}/\sigma_{eqa}(max)$ subjected to proportional cyclic: (a) tension with torsion, (b) bending with torsion

A SIMULATION EXAMPLE OF THE CRITICAL PLANE DETERMINATION

Computer simulation was done for the full spatial stress state by generation of 6 stress state components $\sigma_{ij}(t)$ for $i, j = x, y, z$. All the generated stress histories were pseudorandom and had the same variances, probability distributions and power spectral density distributions. The stress tensor components were independently generated and their intercorrelation coefficients were close to zero. The extreme frequency had a normal distribution from the interval 3-6 Hz and the extreme distributions were of the Rayleigh type. The fatigue life calculations were done for 10HNAP steel [6]. The exponent of the Wöhler curve under uniaxial loading was m = 9.82, the fatigue limit σ_{af} = 252 MPa, and the corresponding theoretical number of cycles was $N_0 = 1.28\ 10^6$ cycles. For calculations the criterion of maximum normal stress (2) was used and damages were accumulated according to the PM (4). Fig. 5 shows the sphere located in the spatial coordinate system. The suitable axes are the direction cosines l_η, m_η, n_η determining the critical plane position. The calculated fatigue life, i.e. the material damage degree is grey in the picture. For such generation and the fatigue life calculations the following minimum lives were obtained for the direction cosines $l_\eta = 0.631$, $m_\eta = 0.695$, $n_\eta = -0.342$, i.e. for the corresponding angles 129^o, 46^o and 110^o. The same calculation lives were obtained for the angles turned about 180^o at the opposite side of the presented sphere Fig. 5.

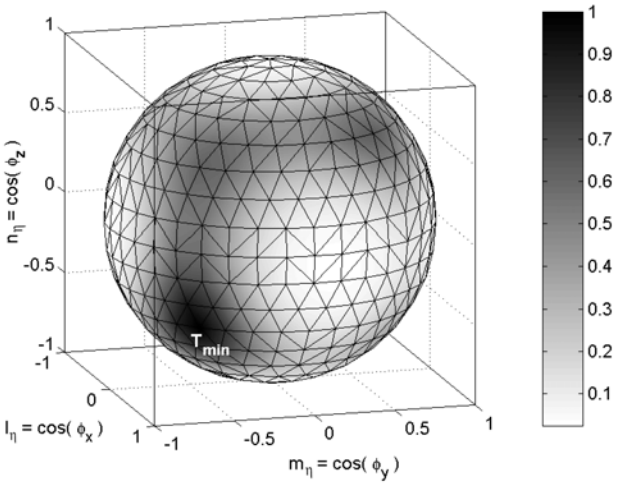

Figure 5: The calculated fatigue lives versus directional cosines (l_η, m_η, n_η)

CONCLUSIONS

1. Simulation tests of the critical plane positions under the stress gradients show that there are different planes in particular points of the section, assuming the criterion of maximum normal stress in the critical plane.
1.1 Under combined proportional tension with torsion the critical planes change along the radius and at the perimeter they are the same.
1.2 Under combined proportional bending with torsion the critical planes change radially in relation to the centre. In the bending plane it is 45^{o} and this angle decreases in the fibre which is maximally distant from the bending plane where combined bending with torsion is dominating.
2. Simulation of the spatial stress state shows that there are different calculated fatigue lives dependent on the assumed direction cosines. As a result of such simulation we obtain two sets of three angles, different at 180^{o}.

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