

# Fatigue of materials under cyclic loading with the contribution of mean stress

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***ABSTRACT:** The paper contains a mathematical model of the material's behaviour under cyclic loading taking into account the dynamics of the fatigue process, including the number of cycles to failure, induced by the mean stress value. The coefficients in the proposed equation were obtained from experimental tests under symmetrical and nonsymmetrical loading (with the stress ratio  $R=0$ ). The proposed model was used in order to modify the energetic criterion with the aim of influence of the mean load on the fatigue life. The fatigue tests were done for structural steels 10HNAP and 18G2A subjected to cyclic bending and torsion with different values of the mean stress. A good agreement between the calculated and experimental results was obtained.*

## INTRODUCTION

The energy models are the most popular in analyse of fatigue of materials. They are based on the analyses of the hysteresis loops formed as a consequence of the plastic strains induced in the material. Fatigue life of the material is connected with the plastic strain energy dissipated in the material. Calculations usually include energy of the plastic and elastic strains. The criterion based on the sum of elastic and plastic strain energies is considered in many papers and it was successfully verified during the tests under complex states of loading [1-4]. In many structures additional static loading occurs. In literature influence of the mean stress value on the material fatigue life is usually taken into account according to the Morrow's model [5]. This model includes only influence of the mean stress on the elastic part of strains. In [6] Manson and Halford developed the Morrow's proposal, taking into account influence of the mean stress on both elastic and plastic strains. Bergman and Gołoś [3, 4] proposed to include the influence of the mean stress value by the coefficient of the material sensitivity on the cycle asymmetry, describing the material behaviour in presence of the mean stress. In [7,8] Gasiak and Pawliczek analysed the specimens of 18G2A steel subjected to bending and torsion and they proved

that the coefficient changed together with a number of cycles up to failure. In their energy model Park and Nelson [9] included the stress mean value by determination of the static strain energy associated with the hydrostatic mean stress and by energy of the variable volumetric strains.

## INFLUENCE OF THE MEAN STRESS

In [7, 8] the influence of the mean stress value on the fatigue life is described as a function which the coefficient of the material sensitivity on the cycle asymmetry  $\psi(N)$  depends on the number of cycles  $N$  to failure. This function can be written as

$$\psi(N) = \eta \cdot N^\lambda, \quad (1)$$

where:  $N$  - number of cycles to failure,  $\eta$ ,  $\lambda$  - parameters determined during fatigue tests under the reversed load ( $R = -1$ ) and obtained from the pulsating load ( $R = 0$ ). The results of fatigue tests for the stress ratio  $R = -1$  were expressed by the equation  $\log N = B_w + A_w \cdot \log \sigma_{a(-1)}$ , and for  $R = 0$  by the equation  $\log N = B_j + A_j \cdot \log \sigma_{a(0)}$ . For the structural steels the coefficients  $\eta$  and  $\lambda$ , in the case of lifetime  $N=5 \cdot 10^4 \div 2.5 \cdot 10^6$ , can be determined from the following experimental equation [10]

$$\lambda = -0,588 \cdot \log \left( \frac{10^{4,7a+b} - 1}{10^{6,4a+b} - 1} \right) ; \quad \eta = 10^{4,7(a-\lambda)+b} - 10^{-4,7\lambda}, \quad (2)$$

$$\text{where: } a = \frac{1}{A_w} - \frac{1}{A_j}, \quad b = \frac{B_j}{A_j} - \frac{B_w}{A_w}.$$

In order to obtain a course with the zero mean stress, which is equivalent to the course with the non-zero mean stress value, we should increase the stress amplitude, using a transformation relation including influence of the mean stress value. For a linear transformation the following relationship is valid [8]

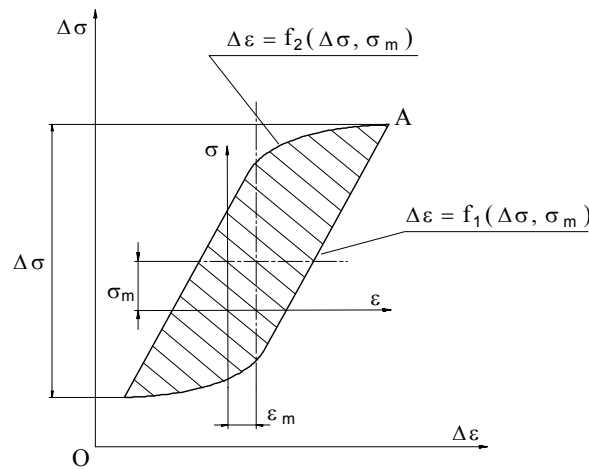
$$\sigma_{a_{-1}}(N) = \sigma_a + \psi(N) \cdot \sigma_m = \sigma_a + \gamma \cdot N^\lambda \cdot \sigma_m, \quad (3)$$

where :  $\sigma_{a-1}(N)$  - equivalent stress amplitude at the lifetime level  $N$  cycles in the case  $R = -1$  (symmetric cycles).

### MODEL OF THE STRAIN CURVE

In the paper the hysteresis loop model is used to describe the relation between the cyclic stress and cyclic strain. The additional mean loading should influence the position and shape of the stress and strain curve in the cycle. It has been assumed that the curves presenting the hysteresis loop are of the Ramberg-Osgood type and they are described by the functions  $\Delta\varepsilon = f_1(\Delta\sigma, \sigma_m)$  and  $\Delta\varepsilon = f_2(\Delta\sigma, \sigma_m)$  (Fig. 1). The area included in the loop is

$$W = \int_0^{\Delta\sigma} f_1 d(\Delta\sigma) - \int_0^{\Delta\sigma} f_2 d(\Delta\sigma). \quad (4)$$



**Figure 1:** Influence of the mean stress value on position of the hysteresis loop

Let us include influence of the mean stress value on the elastic and plastic parts of the strain according to the Eq.(3) and apply Eq.(1) for description of the change of material sensitivity on the mean stresses. Then, the functions  $f_1$  and  $f_2$  from Eq. (4) are

$$\begin{aligned}
f_2(\Delta\sigma, \sigma_m) &= \frac{\Delta\sigma}{2E} + \frac{\psi(N) \cdot \sigma_m}{E} + \left( \frac{\Delta\sigma + 2\psi(N) \cdot \sigma_m}{2K'} \right)^{\frac{1}{n'}} \\
f_1(\Delta\sigma, \sigma_m) &= \frac{\Delta\sigma}{2E} + \frac{\psi(N) \cdot \sigma_m}{E} + \\
&+ \left( \frac{1}{K'} \right)^{\frac{1}{n'}} \left[ \left( \frac{\Delta\sigma_A}{2} + \psi(N) \cdot \sigma_m \right)^{\frac{1}{n'}} - \left( \frac{\Delta\sigma_A}{2} - \frac{\Delta\sigma}{2} \right)^{\frac{1}{n'}} \right].
\end{aligned} \tag{5}$$

where:  $K'$  - coefficient of strain cyclic hardening,  $n'$  - exponent of strain cyclic hardening,  $E$  - Young's modulus,  $\Delta\sigma_A$  - stress in the point A (Fig. 1). Assuming the strain energy dissipated in the material while loading  $W^*$  as the fatigue damage parameter we obtain

$$W^* = W^e + W^m + W^p, \tag{6}$$

where:

$W^e$  - specific energy of the elastic strain coming from the variable loading

$$W^e = \frac{1}{2E} \left( \frac{\Delta\sigma}{2} \right)^2, \tag{7}$$

$W^m$  - energy of the elastic strain referred to the mean loading,

$$W^m = \frac{1}{2E} \psi(N) \cdot \sigma_m^2, \tag{8}$$

$W^p$  - specific energy of the plastic strain. It was determined by substituting relations (5) to Eq. (4):

$$\begin{aligned}
W^p &= K_1 \Delta\sigma (\Delta\sigma + 2\psi(N)\sigma_m)^{\frac{1}{n'}} - K_1 \frac{n'}{1+n'} (\Delta\sigma)^{\frac{1+n'}{n'}} - \\
&- K_1 \frac{n'}{1+n'} (\Delta\sigma + 2\psi(N)\sigma_m)^{\frac{1+n'}{n'}} + K_1 \frac{n'}{1+n'} (2\psi(N)\sigma_m)^{\frac{1+n'}{n'}},
\end{aligned} \tag{9}$$

where:  $K_1 = \frac{2}{(2K')^{\frac{1}{n'}}$ .

In the case of the complex loading, stresses should be understood as the equivalent ones, determined on the base of the assumed effort hypothesis.

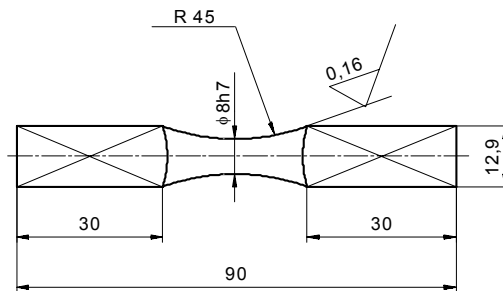
## THE FATIGUE TESTS

The fatigue tests [7, 8, 10] were done on the fatigue stand MZGS-100. The fatigue tests were carried out under cyclic bending and cyclic torsion with participation of the mean loading value. The tests were done for two steels. Their composition and properties are given in Table 1.

TABLE 1: Characteristics of 18G2A and 10HNAP steels

Steel	Chemical composition [%]	Strength properties
18G2A	0.21C 1.46Mn 0.42Si 0.019P 0.046S 0.09Cr 0.04Ni 0.17Cu	$\sigma_y=357$ [MPa], $\sigma_u=535$ [MPa], $E=2.10 \cdot 10^5$ [MPa], $\nu=0.30$ , $n' = 0.287$ , $K' = 869$ [MPa]
10HNAP	0.11C 0.52Mn 0.26Si 0.098P 0.016S 0.65Cr 0.35Ni 0.26Cu	$\sigma_y=418$ [MPa], $\sigma_u=566$ [MPa], $E=2.15 \cdot 10^5$ [MPa], $\nu=0.29$ , $n' = 0.133$ , $K' = 832$ [MPa]

Figure 2 shows a shape and dimensions of the specimens tested.



**Figure 2:** A specimen subjected to tests

The tests were done at given stress ratios,  $R = -1, -0.5, 0$ . Coefficients of the standard fatigue curve were determined for the limited fatigue life of the material. During the tests the amplitude and the mean value of the loading moment were controlled. Using the test results and Eq. (2) we formulated Eq. (1) for both steels. Mathematical forms of the obtained functions are described as follow:

for 18G2A steel

$$\text{bending} - \psi_{\sigma}(N) = 3.124 \cdot N^{-0.162}; \quad \text{torsion} - \psi_{\tau}(N) = 2.890 \cdot N^{-0.148},$$

for 10HNAP steel:

$$\text{bending} - \psi_{\sigma}(N) = 1.006 \cdot N^{-0.072}; \quad \text{torsion} - \psi_{\tau}(N) = 0.818 \cdot N^{-0.113}.$$

## ANALYSIS OF THE CALCULATION RESULTS

The fatigue test results based on Eq. (6) were applied for determination of the parameter  $W^*$  for particular cases of loading.

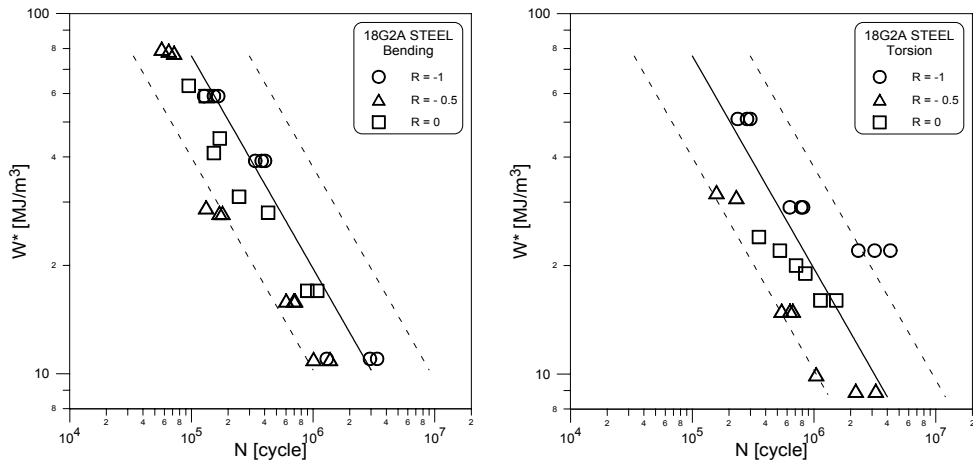
Figures 3 and 4 show values of the parameter  $W^*$  for a linear regression determined for the experimental results for loading, where the mean stress  $\sigma_m = 0$  (a solid line in Figs. 3 and 4). The broken lines mean the interval where the ratio of the experimental fatigue life  $N_{exp}$  to the calculated life  $N_{obl}$  is 3 and 1/3.

From the graphs it results that in the case of 18G2A steel and the stress ratio  $R = -1$  and  $R = 0$  (Fig. 3) we obtained a very good agreement between the calculation results (points  $\circ$ ,  $\Delta$ ,  $\square$  in the graphs) and the experimental results for bending and torsion. For the stress ratio  $R = -0.5$  the results are characterised by a greater deviation from a simple regression (a solid line), but most of the results are included in the scatter band of the coefficient  $N_{exp}/N_{obl} = 1/3$ . In the case of the specimens made of 10HNAP steel and for the bending loading  $\sigma_m = 150$  MPa and torsional loading  $\tau_m = 150$  MPa, the proposed model allows to relate the test results under asymmetric loading to those under varying loading ( $\sigma_m = 0$ ) in a sufficient way. As for loading with the mean value  $\sigma_m = 75$  MPa for bending and  $\tau_m = 75$  MPa for torsion, an error of the calculation results is greater, especially for the fatigue life  $N > 10^5$  cycles. Thus, in such cases even a low mean stress can strongly influence the damage accumulation process.

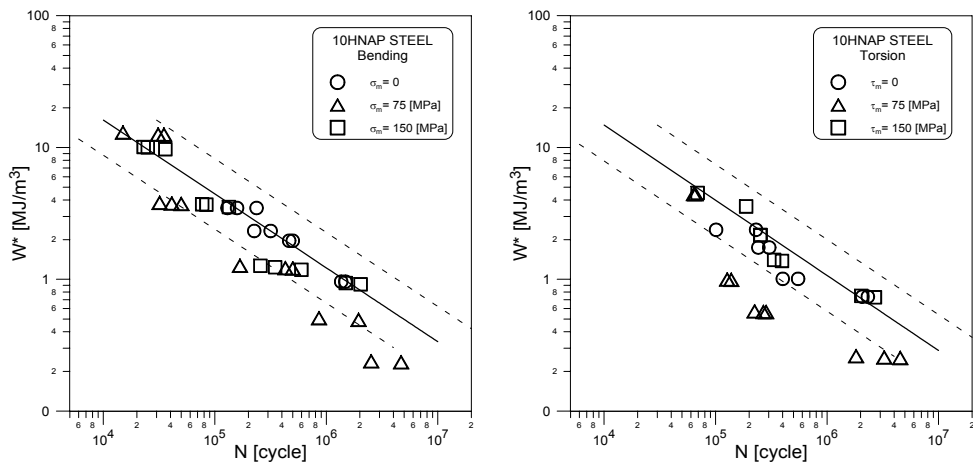
## CONCLUSIONS

From the theoretical considerations and the fatigue tests results obtained for the specimens made of two steels, 18G2A and 10HNAP, subjected to cyclic bending and torsion and with contribution of the mean stress value, the following conclusions can be drawn:

1. The hysteresis loop model derives the relationship, which determined the energy parameter  $W^*$ . This last one allows us to calculate the fatigue life under bending and torsion.
2. The tests result that the calculated energy parameter  $W^*$  for  $R = -1$ ,  $-0.5$  and  $0$  give the fatigue lives included in the scatter bands  $N_{exp}/N_{obl} = 3$  and  $1/3$ . Only the calculation results under bending for  $\sigma_m = 75$  MPa and torsion  $\tau_m = 75$  MPa are outside the scatter band (Fig.4). Thus, probably a low value of the mean stress can strongly influence the damage accumulation process.



**Figure 3:** Parameter  $W^*$  calculated for the specimens made of 18G2A steel



**Figure 4:** Parameter  $W^*$  calculated for the specimens made of 10NHAP steel

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