

# Novel Test Criteria for Determining Fracture Toughness under Pure Mode-II and Mixed-Mode Loading

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***Abstract:** Based on experimental and numerical investigations minimum size specimen requirements are proposed which allow for the determination of valid shear mode-II fracture toughnesses  $K_{IIc}$ . Following, in principle, the methodology for determining the mode-I fracture toughness  $K_{Ic}$ , the established requirements ensure a dominating plane strain state of stress at the crack tip and plastic zones that are sufficiently small with respect to the specimen dimensions to allow for applying a linear-elastic approach. The requirements have been established for specimens used in Arcan-Richard type loading fixtures which allow testing under different ratios of mode mixity. A comparison of characteristics of the crack tip stress field obtained under mode-I and mode-II conditions of loading with A-R-specimens is made with results obtained with SEB- and CT-specimens for measuring mode-I fracture toughnesses  $K_{Ic}$  according to ASTM E 399. The study demonstrates that the minimum specimen thickness for  $K_{IIc}$ -tests can be smaller, but that the minimum dimensions of the specimens in in-plane directions should be larger than for a  $K_{Ic}$ -test.*

## 1. INTRODUCTION

There is a general believe that mode-I loading is the only one worth to be considered in the design and the possible failure of mechanical and structural elements. As a consequence, mixed mode loading conditions are usually not considered or they are simplified to an unacceptable manner with respect to real practical design situations. Furthermore, the lack of

standards and criteria as regards geometry and dimensions of specimens for determining the mode-II fracture toughness  $K_{IIc}$  or, in the more general case, its equivalent in mixed mode cases, contributes to this non-satisfactory situation. Besides of this, there is a lack of consensus in the acceptance of a correct failure model under mixed mode loading conditions. Due to the complexity of the problem, it is difficult to formulate a sufficiently simple failure concept for mechanical components or real structures for which the type of load or the geometry result in stress states from which the potential of mixed mode failure arises.

A quite general model for brittle failure behaviour has recently been suggested by Podleschny and Kalthoff [1]. The model makes a clear distinction between the loading situation, the failure event and the fracture criterion. Thus, practical formulations of fracture predictions under general loading cases, i.e. under mixed mode loading, becomes possible.

## **2. SPECIMEN TYPES FOR MEASURING FRACTURE TOUGHNESSES UNDER DIFFERENT LOADING MODES**

The concept of Linear-Elastic Fracture Mechanics is based on a linear-elastic response of the material until failure, and, as a consequence, implies the fulfillment of some requirements for determining the mode-I fracture toughness  $K_{Ic}$ , which are considered in the current standards [2,3].

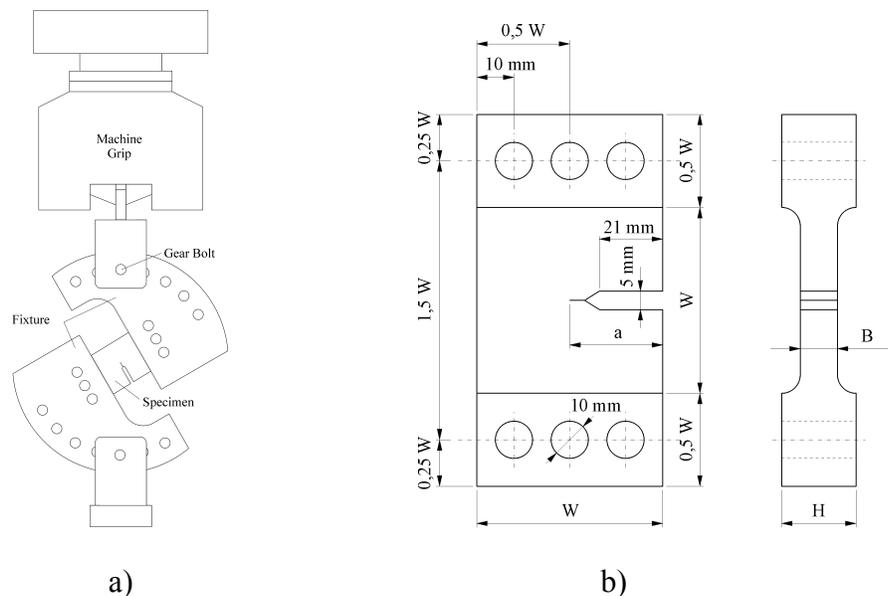
For determining the fracture toughness in mode-I, various different types of officially approved specimens can be used. Each type of these specimens has certain advantages with respect to the others, such as the need of less volume of material, better fitting of the specimen geometry to the geometry of the half-finished product, etc. Due to space limitations, only results referring to the CT- and SEB-specimens will be presented in this context.

From the fracture load determined with one of these specimens, the fracture toughness value  $K_{Ic}$  for the specific material considered is determined according to the corresponding stress intensity factor relationship with allowable crack lengths  $a/W$  in the range of 0.45 to 0.55. For determining valid fracture toughness values the conditions:

$$\begin{aligned}
 & - H/2, (W-a), a \geq 2.5 (K_{Ic}/R_{p0.2})^2 \\
 & - B \geq 2.5 (K_{Ic}/R_{p0.2})^2
 \end{aligned} \tag{1}$$

have to be fulfilled. These conditions assure, first, that the mean size of the crack tip plastic zone remains limited with respect to the specimen height, the ligament length and the crack length, and, secondly, it guarantees that plane strain conditions govern the stress condition along the crack front. In that case it can be assumed that the determined quantity represents the plane strain fracture toughness  $K_{IC}$ .

In comparison to the standardized specimens discussed above the Arcan-Richard-(A-R)-specimen represents an interesting alternative when mode-II loading conditions become of interest. The A-R-specimen allows to test specimens under different mode mixities, and, particularly, it allows testing under pure mode-I and pure mode-II conditions. The specimen is hold in a special loading fixture with different connection points to the grips (see Fig. 1a).



**Figure 1.** Arcan-Richard fixture system and A-R-specimen. From [4].

The use of the A-R-specimen, shown in Fig. 1b, allows for the determination of the mode-I fracture toughness  $K_{IC}$ , but, additionally, for the determination of the complete fracture resistance curve of the material, i.e.  $K_{IIIC}$  and critical values for mixed mode conditions, by only using the loading fixture in different orientations. Due to the unique geometry of the

specimen and the simplicity of the test, only the circumferential position of the fixture has to be varied for the different mode mixities with respect to the pull grips. This paper considers the validity and suitability of this kind of specimen for measuring fracture toughnesses under both, mode-I and mode-II, as well as under mixed mode loading.

### 3. NUMERICAL SIMULATIONS

Based on numerical calculations a comparative check is performed between characteristics obtained for the A-R-specimen and the standard SEB- and CT-specimens. The simulations apply for specimens made of the aluminum alloy Al 7075 with the following mechanical and fracture properties:  $R_{p0.2} = 488$  MPa,  $K_{IC} = 30.0$  MPa m<sup>1/2</sup> and  $K_{IIC} = 44.4$  MPa m<sup>1/2</sup>, determined in a previous research program [4]. The mode-I fracture toughness  $K_{IC}$  was measured in accordance with the requirements of ASTM E 399, the mode-II fracture toughness  $K_{IIC}$  was determined by transferring these requirements in an appropriate way from mode-I to mode-II.

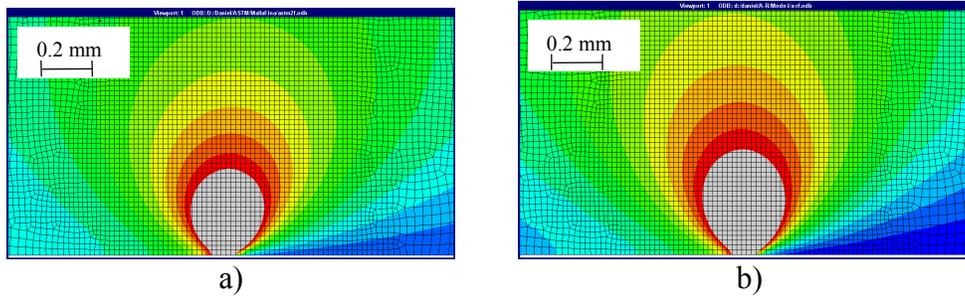
For the specimen types under consideration, the fracture loads  $P_{fract}$  were calculated; the mode-I fracture loads for all three specimen types, the mode-II fracture load for the A-R-specimen only. The results are given in Table 1.

Specimen type	$P_{fract} / B$ [N/mm]		
	SEB	CT	A-R
Mode-I	645.0	694.4	1.932.1
Mode-II	-	-	5.957.4

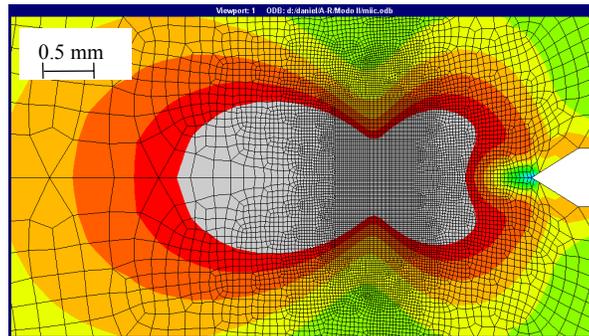
**Table 1.** Failure loads per unit thickness for mode-I and mode-II loading and different specimen types ( $W=50$  mm,  $a/W=0.50$ ).

For these fracture loads, FEM calculations of the stress distribution in the specimens were carried out with the ABAQUS code. Meshing was done automatically with use of the IDEAS program; the zone near the crack tip being defined with a finer mesh of 0.02 mm sided rectangular elements; the mesh size was increased up to 5 mm in zones of less significance. The load system was modeled through rigid linear elements, to which the constraints acting in the real system were imposed.

In the computations an ideal elastic behavior of the material was assumed with the stress-strain curve being defined by means of an uni-axial tensile test. Fig. 2 shows the stress distributions under mode-I in the neighborhood of the crack front for the CT- and A-R-specimens, the corresponding results under mode-II for the A-R-specimen are given in Fig. 3 (Note the different scales in Fig. 2 and Fig. 3).



**Figure 2.** Von Mises equivalent stresses for mode-I and strain plane conditions at  $P_{\text{fract}}$  for a) CT- and b) A-R-specimen.



**Figure 3.** Von Mises equivalent stresses for mode-II and strain plane conditions at  $P_{\text{fract}}$  for the A-R-specimen.

Results on the size (total diameter) of the crack tip plastic zones (stresses over yield stress) in x-direction ( $\varphi = 0, \pi$ ) and y-direction ( $\varphi = \pm \pi/2$ ) are summarized in Table 2 for both, plane stress and plane strain conditions. As can be seen, the plastic zones under mode-I practically coincide in shape and size for the two specimens, whereas the plastic zone in mode-II adopts the typical characteristic shape for this kind of loading and, most importantly, reaches much larger dimensions than in mode-I. Furthermore,

the size of the mode-II plastic zone size shows less dependence with respect to the lateral restrain conditions, i.e. to the condition of plane stress or plane strain, particularly in the loading direction.

		Size of the plastic zone [mm]			
		SEB	CT	A-R	A-R
Specimen type		Mode-I		Mode-II	
Plane stress	$\varphi = 0 - \pi$	0.68	0.69	0.68	8.47
	$\varphi = \pm \pi/2$	1.28	1.18	1.34	3.89
Plane strain	$\varphi = 0 - \pi$	0.33	0.32	0.34	7.41
	$\varphi = \pm \pi/2$	0.80	0.71	0.86	3.87

**Table 2.** Approximate sizes of the crack tip plastic zones under plane stress and plane strain for different loading modes and specimen types.

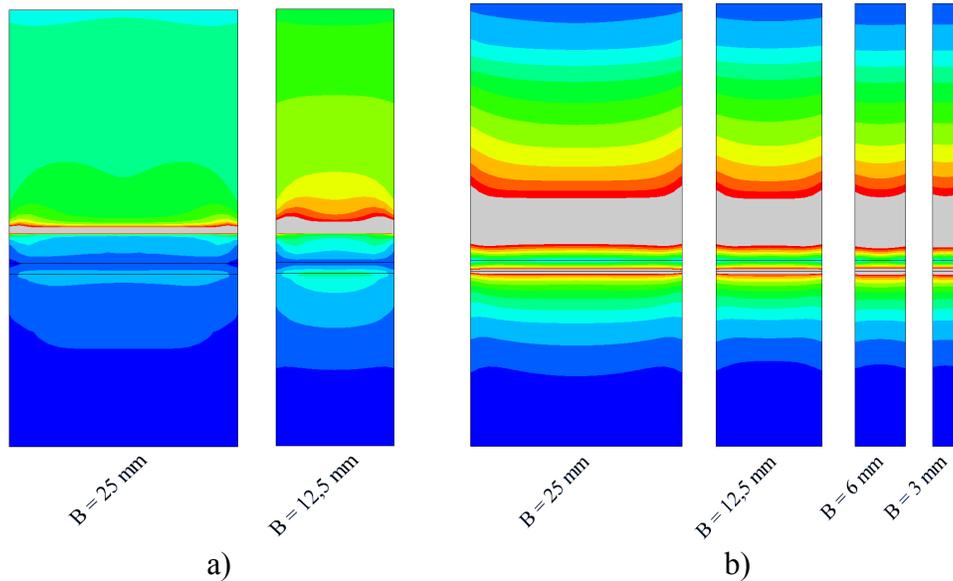
In order to check the results, the stress intensity factors under the condition of the load at fracture, i.e. the fracture toughnesses  $K_{IC}$  and  $K_{IIC}$  were computed on the basis of the numerical data. By use of the J-integral approach the resulting values (expressed in  $K_{IC}$ ) are given in Table 3. The agreement between the numerically calculated  $K_{IC}$ - and  $K_{IIC}$ -toughness-values on the one hand and the experimentally measured data on the other hand is very satisfying.

Specimen type	SEB	CT	A-R	A-R
	Mode-I			Mode-II
J [MPam]	12.09	10.99	11.51	25.58
$K_C$ [MPam <sup>1/2</sup> ]	31.15	29.70	30.40	45.32

**Table 3.**  $K_{IC}$  and  $K_{IIC}$  results deduced from numerical calculations for the three specimen types using the J-integral.

Furthermore, influences of the thickness on the stress distribution along the crack front were studied. Figure 4 shows stress distributions across the thickness of the specimen for various absolute specimen thicknesses B; Fig. 4a gives the results for mode-I loading and Fig. 4b for mode-II loading. One recognizes, plane strain conditions under mode-I loading can only be achieved for large thicknesses, in full accordance with the usual validity

conditions for mode-I testing as mentioned in Section 2. On the contrary, plane strain conditions for mode-II are established for considerably smaller thicknesses already.



**Figure 4.** Stress distribution in the crack plane of A-R-specimens for different specimen thicknesses: a) for mode-I, b) for mode-II.

From the foregoing argumentation it is evident that specimen size requirements for a valid mode-II test should notably differ from the ones postulated by the current standards for a mode-I test. Specifically, larger specimen dimensions in in-plane directions are needed to fulfill the conditions of limited plastic zone sizes, whereas the specimen dimension in thickness direction can be drastically reduced.

Lastly, the crack tip plastic zones have been computed on the basis of the real stress-strain behaviour measured for the material Al 7075. The numerical calculations show differences with respect to the results obtained for ideal elastic behavior, both for mode-I and mode-II loading. In general, however, agreement between calculations of crack tip plastic zones using an ideal elastic and the real stress-strain behavior of the material is satisfying.

#### 4. REQUIREMENTS FOR MEASURING THE MODE-II FRACTURE TOUGHNESS $K_{IIc}$ AND IMPLICATIONS ON A-R-SPECIMENS

On the basis of the performed numerical calculations a comparison is made between the crack tip plastic zones at failure for mode-I and mode-II conditions of loading. In Table 4 the maximum total extensions of the crack tip plastic zones are given, as derived from Table 2.

Mode-I	Mode-II
Maximum Extension of Plane Strain Crack Tip Plastic Zone	
$\varphi = \pm \pi/2$ 0.86 mm	$\varphi = 0 - \pi$ 7.41 mm
Minimum Thickness for Plane Strain Dominance	
12.5 - 25 mm	~ 3 mm

**Table 4.** Data on the size of the crack tip plastic zones and plan strain dominance for mode-I and mode-II conditions of loading at fracture.

For mode-I loading, the plastic zone extends largest in y-direction ( $\varphi = \pm \pi/2$ ) by a length of 0.86 mm, for mode-II loading the largest extension is obtained in x-direction ( $\varphi = 0, \pi$ ) and amounts to 7.41 mm. Considering that these zones apply for different toughnesses,  $K_{IIc} = 1.48 K_{Ic}$  (see section 3) and, furthermore, that plastic zones scale with  $(K_{Ic}/R_{p0.2})^2$  or  $(K_{IIc}/R_{p0.2})^2$  respectively, the mode-II plastic zone is roughly four times larger than the mode-I crack tip plastic zone, when the sizes of the zones are expressed in  $(K_{Ic}/R_{p0.2})^2$  or  $(K_{IIc}/R_{p0.2})^2$  respectively. Consequently, the in-plane dimensions of a  $K_{IIc}$ -specimen would have to be about four times larger than of an  $K_{Ic}$ -specimen, expressed in terms of the corresponding scaling unit.

Table 4, in conclusion of the results given in Fig. 4, additionally lists the critical specimen thicknesses from which on plane strain dominance along the crack front is guaranteed for both, mode-I and mode-II loading. Although the data are somewhat vague because of the limited variations in specimen thicknesses considered, the small thickness of about 3 mm for mode-II compared to an assumed averaging thickness for mode-I, indicate, that the minimum thickness for a plane strain dominance of a  $K_{IIc}$ -specimen

would be about one order of magnitude smaller than for a  $K_{IC}$ -specimen, again expressed in  $(K_{IC}/R_{p0.2})^2$  or  $(K_{IIC}/R_{p0.2})^2$  respectively.

These size requirements for measuring mode-II fracture toughnesses  $K_{IIC}$ , introduced before in a similar form by Hiese and Kalthoff [5,6], follow the same principles for limiting the size of the crack tip plastic zones and for establishing plane strain dominance as the standard E 399 for measuring the mode-I fracture toughnesses  $K_{IC}$ . But, the different coefficients in the conditions lead to severely different absolute specimens sizes for  $K_{IIC}$ -specimens when compared to the usual  $K_{IC}$ -specimens.

	Mode-I	Mode-II
H	$\geq 5 (K_{IC}/R_{p0.2})^2$	$\geq 20 (K_{IIC}/R_{p0.2})^2$
W-a	$\geq 2.5 (K_{IC}/R_{p0.2})^2$	$\geq 10 (K_{IIC}/R_{p0.2})^2$
a	$\geq 2.5 (K_{IC}/R_{p0.2})^2$	$\geq 10 (K_{IIC}/R_{p0.2})^2$
B	$\geq 2.5 (K_{IC}/R_{p0.2})^2$	$\geq 0.25 (K_{IIC}/R_{p0.2})^2$

**Table 5.** Minimum size specimen requirements for measuring valid mode-I and mode-II fracture toughnesses  $K_{IC}$  and  $K_{IIC}$ .

## 5. CONCLUDING REMARKS

As regards the suitability of the AR-specimen for measuring mode-II, mixed mode and mode-I fracture toughnesses a consideration of the following remarks in conclusion of the reported behaviour may be helpful:

For measuring mode-I fracture toughnesses  $K_{IC}$  the fracture loads when using SEB- or CT- (or DCT- or AT- specimens, although not considered in this paper) are similar in magnitude, fracture loads using the A-R-specimen, however, are much larger. This can be inconvenient when large A-R-specimens are needed. The crack tip plastic zones in the standard CT- and the SEB-specimen practically coincide with the one in the A-R-specimen for mode-I loading. This demonstrates the suitability of using the A-R-specimen for determining the mode-I fracture toughness  $K_{IC}$  according to the usual standards.

A-R-specimens allow to determine fracture toughnesses under mixed mode and, in particular, under pure mode-II and also mode-I conditions of

loading, using the same fixture system. But, special attention has to be given to the size of the system. In general, for A-R-specimens of different sizes loading fixtures of corresponding sizes are needed and have to be built. If the same relation between plastic zone and specimen size has to be preserved, as is the case for determining valid fracture toughnesses  $K_{IIc}$  and  $K_{Ic}$ , the in-plane dimensions of the A-R-specimen must be considerably larger for mode-II than for mode-I. On the other hand, plain strain dominance for mode-II is guaranteed at much thinner specimens already than for mode-I. In principle, the determination of fracture toughnesses under mixed mode loading would require specimens of different dimensions in in-plane directions and different thicknesses depending on the ratio of mode mixity.

Thus, loading conditions can very easily be varied with the A-R-system but special care is needed to establish agreement with the corresponding specimen size requirements.

## 5. REFERENCES.

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