

# Interaction of two hard indenters in the cutting process of quasi-brittle materials

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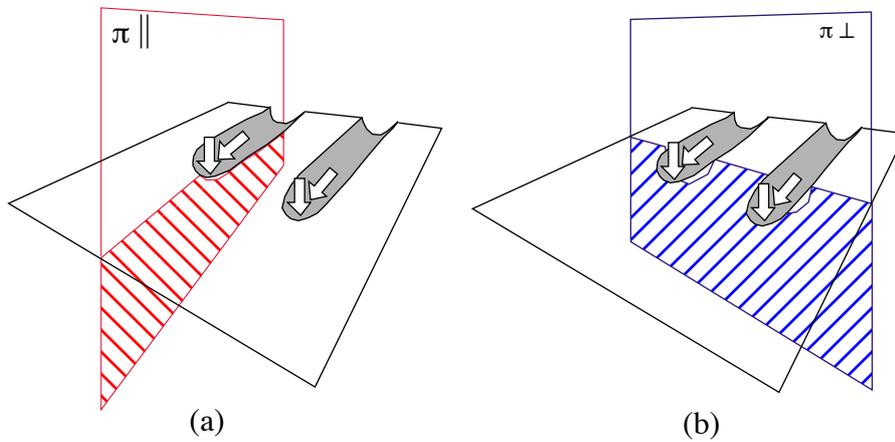
***ABSTRACT:** In this paper, an attempt is made to find some general relations for the micro-cutting process in brittle or quasi-brittle materials, under different hypotheses of microscopic failure behaviour. Fracture patterns in homogeneous brittle solids are obtained by the Finite Element Method in the framework of Linear Elastic Fracture Mechanics (LEFM). On the other hand, microstructural heterogeneities are taken into account by Lattice Model simulations. When two indenters are acting in parallel, their mutual distance plays an important role. If the indenters are very close, they behave like a unique large indenter, whereas if the distance is relatively large, their mechanical interaction vanishes. In addition, when the distance is approximately three or four times their radius, the mechanism of chipping (with formation of secondary chip between the two parallel grooves) can take place, improving the ratio of removed volume to spent energy and then the demolition ability of the two indenters.*

## INTRODUCTION

Many technological operations involve two or more contacting bodies sliding with respect to one another. A series of damage mechanisms can occur in these situations, for instance fretting fatigue and wear. In a totally different context, scratching and cutting represent fundamental manufacturing processes, as in the case of cutting precious and ornamental stones and of rock excavations and drilling. Therefore, the mechanics of these processes has been an important subject of research in the last few years.

In the following, the cutting process in brittle or quasi-brittle materials is analysed numerically under different hypotheses for the microscopic failure behaviour. Fracture patterns in homogeneous brittle solids are obtained by the Finite Element Method in the framework of Linear Elastic Fracture Mechanics (LEFM). On the other hand, microstructural heterogeneities are taken into account by Lattice Model simulations.

Although the problem is actually three-dimensional, it is possible to consider two plane-strain schemes in order to simplify the analysis, as shown in

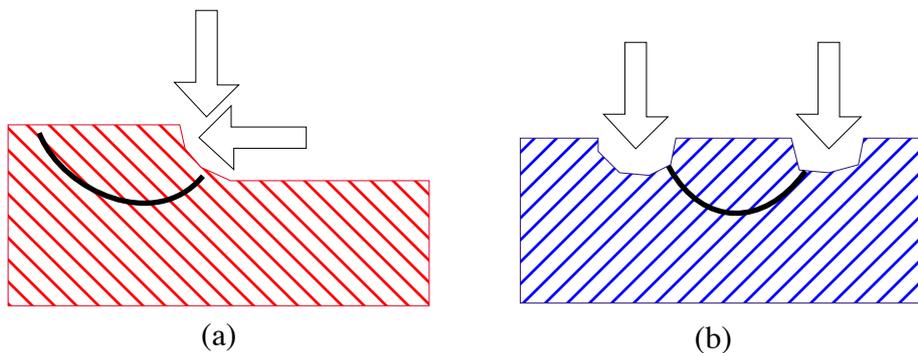


**Figure 1:** Plane parallel to the indenter motion (a); Plane perpendicular to the indenter motion (b).

Figure 1. In the plane parallel to the indenter motion (Figure 2a) the normal and tangential indentation mechanisms interact in the so called *cutting process*. On the other hand, in the plane perpendicular to the indenter motion the interaction is mainly between the two normal forces (Figure 2b) so that, if the distance is optimal, the coalescence of Hertzian cracks leads to the formation of chips between the two main grooves (*chipping mechanism*). This effect is exploited, for instance, by hammering operations.

## NONDIMENSIONAL ELASTIC SOLUTIONS FOR INDENTATION

Although the unilateral contact problem is inherently nonlinear, within certain limits it is possible to substitute the action of the punch with a concentrated

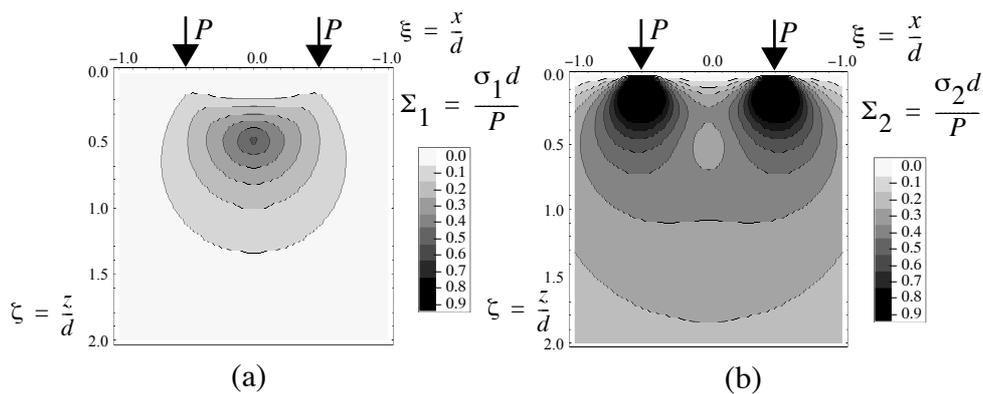


**Figure 2:** Cutting mechanism in front of the indenter in the plane  $\pi \parallel$  (a); chipping mechanism between parallel indenters in the plane  $\pi \perp$  (b).

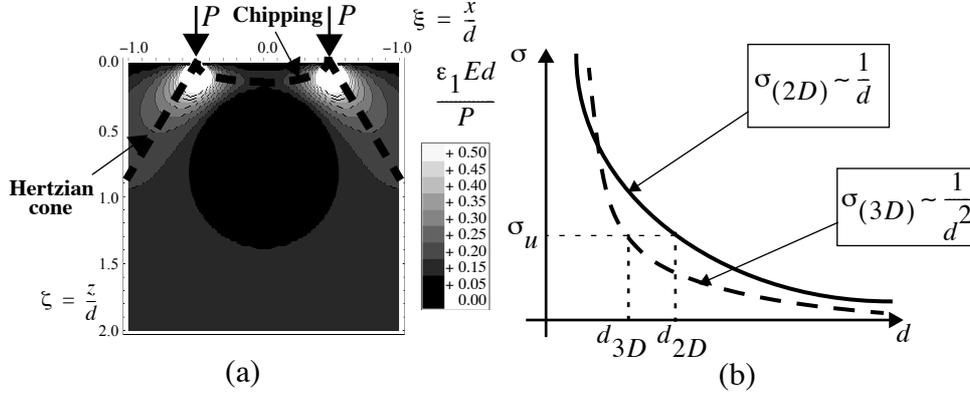
force acting directly on the elastic half-plane or half-space. In this way, the problem is linearized. Moreover, point force solutions and extremely sharp indenters provide exactly the same results.

Analytical solutions for the stress and strain fields due to normal and tangential point forces are available in the literature [1]. The 2D plane strain analytical solution is known as the Flamant solution. The concentrated force  $P$  is actually a distributed load per unit thickness. Since the problem is self-similar, there is no characteristic length, and the stress and strain fields are the same under any length magnification. The problem of a vertical point load acting on a semi-infinite elastic half-space is commonly addressed as the Boussinesq problem.

The problem of two indenters acting at a distance  $d$ , respectively upon an elastic half-plane or half-space, can be easily solved by superimposing two shifted elastic solutions valid for the single force. Moreover, it is useful to introduce the nondimensional group  $\Sigma = \sigma d / P$  (or  $\Sigma = \sigma d^2 / F$  in the 3D case). In this way, the variation of both  $P$  and  $d$  is taken into account, and diagrams can be plotted with respect to the normalised variables  $\xi = x/d$  and  $\zeta = z/d$ . Although the stress field is everywhere of compression (see Figure 3), considerable dilations occur due to Poisson effect. In Figure 4a, it is worth noting the steep slope of the dilation zone outside the point forces, that corresponds to the formation of the so-called Hertzian fracture cone. On the other hand, the slope of the dilation zone between the forces is notably smaller. The  $\varepsilon_1$  principal strain field suggests the shape of the chip formation due to the interaction of the two point forces.



**Figure 3:** Contour plot of the normalized maximum principal stress (a); Contour plot of the normalized minimum principal stress (b).



**Figure 4:** Contour plot of the normalized maximum principal strain (a); scaling of the maximum chip size (b).

For the problem under consideration there are three relevant dimensional quantities:

$$[\sigma] = [F][L]^{-2}, \quad [d] = [L], \quad (1a)$$

$$[P] = [F][L]^{-1}, (2D), \text{ or } [P] = [F], (3D). \quad (1b)$$

Thereby, according to the Buckingham's  $\pi$ -Theorem, only one nondimensional group can be obtained. This group depends on the dimension of the problem, and his expression is the following:

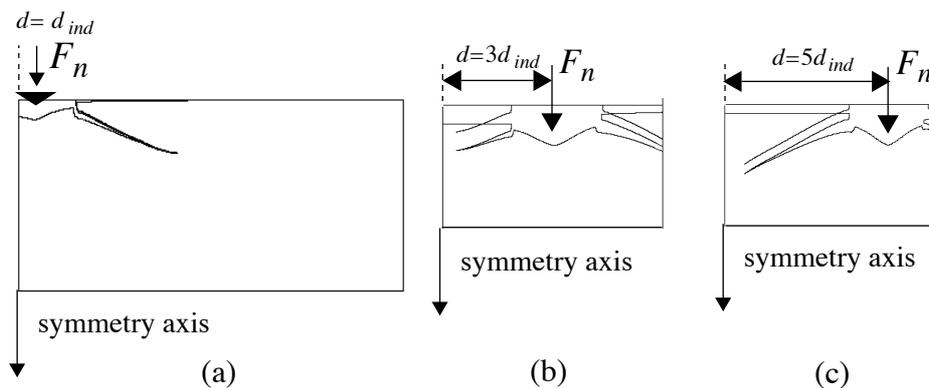
$$\Sigma_{(2D)} = \frac{\sigma d}{P}, \text{ or } \Sigma_{(3D)} = \frac{\sigma d^2}{P}. \quad (2)$$

A corollary of the  $\pi$ -Theorem states that, when the nondimensional group is only one, it must be constant with respect to the position, and this is the case. Some remarks can be made. First, in both cases, the stress must be proportional to the applied forces, if the distance  $d$  is kept constant. Furthermore, once the loads have been assigned, in the 2D case the stress is inversely proportional to the distance  $d$  between the loads, whereas in the 3D case, the stress is inversely proportional to the square of the distance  $d$ .

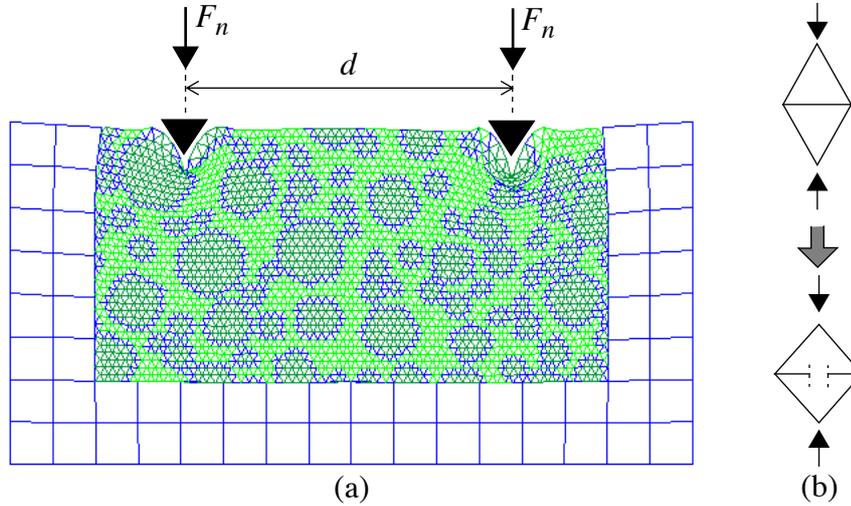
If it is assumed, as reasonable, that chip formation occurs as soon as the stress overcomes the strength of the material, the relation between the maximum chip size (which is proportional to  $d$ ) and the material strength can be obtained (Figure 4b).

## NUMERICAL FRACTURE PATTERNS BETWEEN THE INDENTERS

The FRANC2D [2] simulations of homogeneous microstructures have been carried out by exploiting the symmetry with respect to the Z-axis. Thus, only the right side of the specimen has been modelled. It is implicitly assumed that a symmetrical notch is present and that a crack is contemporarily propagating along a symmetrical path from the other indenter. As usual, the FEM approach to LEFM problems requires a pre-existing notch in order to activate a macrofracture. This aspect of the problem has been already considered in a previous work on single indenter fracture [3]. Three different distances  $d$  between the indenters have been tested, respectively equal to  $d=d_{ind}$ ,  $d=3d_{ind}$  and  $d=5d_{ind}$ , where  $d_{ind}$  represents the reference size of the indenter. In Figure 5a, the case of  $d/d_{ind} = 1$  is shown. Analysis of the elastic stress field showed that a wide (biaxial) compression bulb is formed beneath the indenters, which hinders formation of cracks in that zone. Thus, the only possibility for a crack to propagate is in the form of a large Hertzian cone outside the bulb. In Figures 5b,c, the crack patterns obtained by increasing the distance between the indenters are reported. In these cases, two distinct compression bulbs are formed, and cracks can propagate in the internal area between the indenters, finally coalescing on the symmetry axis. Of course, the finite boundary impedes to follow the entire path. Moreover, due to reciprocal shielding effects, the two approaching tips would eventually diverge from each other. However, it is interesting to note that, in the case of  $d=3d_{ind}$ , the tendency for the internal crack to merge with the symmetric one is more evident.



**Figure 5:** Two indenters:  $d=d_{ind}$  (a);  $d=3d_{ind}$  (b);  $d=5d_{ind}$  (c).



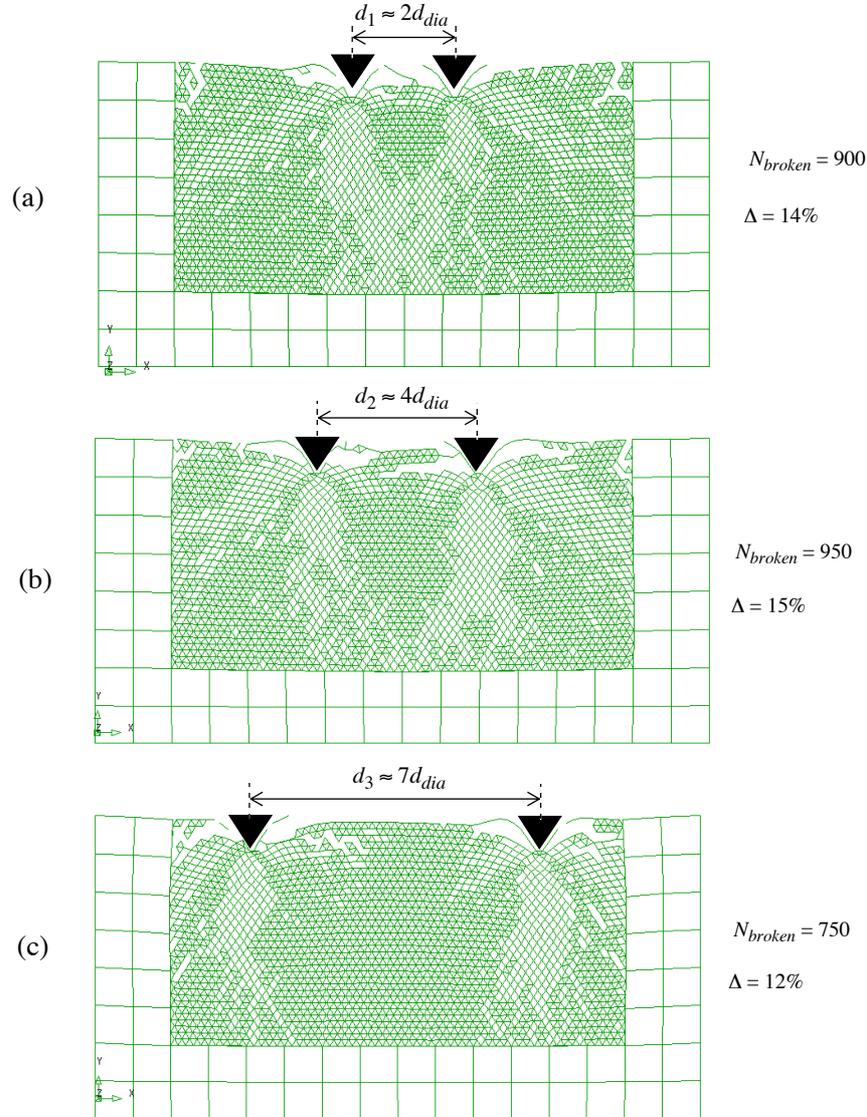
**Figure 6:** DIANA lattice mesh used for the indentation simulations (coarse microstructure) (a); Local splitting mechanism induced by compression (b).

Lattice simulations have been carried out on the full scheme with the software DIANA [4]. Three different configurations have been analysed, namely  $d/d_{ind} \approx 2, 4, 7$ . A coarse microstructure (with volumetric percentage of grains equal to 80%) has been generated. In this way, heterogeneous mechanical properties can be mapped onto the lattice mesh (Figure 6).

A preliminary linear elastic analysis confirmed that compression fields are dominant and that they superpose when the distance between the diamonds is sufficiently small. However, contrarily to the LEFM simulations, the lattice analyses permit to capture also the damage occurring within the compression bulbs.

It is worth noting that, while tension failure is locally well represented by the tensile rupture of lattice micro-beams, compression failure takes place through the so-called *local splitting mechanism* (Figure 6b).

The damage patterns related to the three schemes, under the same value of the load, are shown in Figure 7. When the ratio  $d/d_{ind}$  is small (Figure 7a), the Hertzian cone initially develops. Afterwards, damage tends to concentrate inside the central compression zone below the indenters. Only a small hydrostatic core remains free of damage, and fragmentation is very likely to occur elsewhere. Instead, when the ratio  $d/d_{ind}$  becomes rather large (Figure 6c), the two indenters behave independently, and splitting fractures develop below each indenter. There is also, however, a tendency of nearly horizontal cracks attracted by the adjacent indenter. An intermediate situation is shown in



**Figure 7:** Damage patterns in three lattice schemes for a fixed value of the total load.

Figure 6b ( $d/d_{ind} \approx 4$ ). Interestingly, the maximum relative damage in the lattice mesh ( $\Delta=15\%$ ) is obtained in the intermediate situation ( $d/d_{ind} \approx 4$ ). The case when  $d/d_{ind} \approx 2$  is however very similar. Instead, when the distance between the indenters is larger, the damage index drops to 12%, under the same total load. This is another aspect of the weaker interaction between the indenters.

## CONCLUSIONS

Various mechanisms interact during the cutting process, mainly plastic crushing and brittle chipping. Moreover, cutting performances can be significantly improved by reducing the crushing component and enhancing the chipping ability of the indenters. Therefore, when two indenters are acting contemporarily, their mutual distance plays a crucial role.

LEFM simulations evidenced a clear transition of the chipping interaction effect between two indenters, as a function of the nondimensional ratio  $d/d_{ind}$ . When  $d/d_{ind} < 2$ , compression stresses prevail and there is no chance for brittle cracking between the indenters which behave as a unique large indenter. On the contrary, when  $d/d_{ind} > 5$ , interactions substantially vanish, and the diamonds behave independently of each other. In between these extremes, an effective interaction develops. It can be concluded that  $2 \lesssim d/d_{ind} \lesssim 3$  represents the optimal ratio to enhance brittle chipping.

On the other hand, lattice simulations show a similar behavior in the case of damage patterns and a remarkable difference is indeed observed. The lattice model permits, in fact, to capture also the damage occurring within the compression zone which, in real situations, implies fragmentation and crushing of the base material. In this respect, the coupled action of two close indenters is very effective for the material demolition.

## ACKNOWLEDGEMENTS

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