Three-dimensional boundary element analysis of the behaviour of quarter-elliptical corner cracks emanating from fastener holes

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ABSTRACT: *In this paper, the behaviour of double quarter-elliptical corner cracks emanating from fastener holes is numerically analyzed by the boundary element method. The stress intensity factors along the crack front are calculated for different lengths and depths of the crack. The effect of hole radius is also examined. The obtained results are compared with those obtained with the finite element method by Lin & Smith and those calculated by the Newman & Raju's equations.*

INTRODUCTION

Presence of fastener holes is one of common causes of failure in metallic structures, as the metal becomes more brittle. A fastener hole acting as stress raiser concentrates stress in a manner witch introduces pseudo-brittleness. The corner cracks typically occur along the bore of fastener holes [1]. The Knowledge of the behavior of cracks emanating from fastener holes is frequently required for predicting fatigue life in structures containing these kinds of holes. The estimation of the stress intensity factors is particularly required for such prediction. Many efforts have been made during the past to study the behavior of corner cracks emanating from fastener holes by evaluating their stress intensity factors. The methods used for this evaluation are either by experiment or by calculation. Estimation of the stress intensity factors by calculation has been widely used in literature. In general, three techniques have been exploited to calculate the stress intensity factors for cracks emanating from fastener holes: the approximate analytical approach, the weight function technique and the finite element analysis. Among the authors whom used the analytical approach, we can quote: Newman & Raju [2] and Vainshok & Varfolomeyev [3] have used the weight function technique. The finite element method was the most frequently used to calculate the stress intensity factors. In the case of cracks emanating from fastener holes, Kullgren et al [4], Grandt & Kullgren [5], Alturi & Nishioka [6], Raju & Newman [7] and lin & Smith [8] used the finite element analysis.

In this work, we studied the behavior of double quarter-elliptical corner cracks emanating from fastener hole in thick plate under remote tension by the three-dimensional boundary element analysis. While varying different geometrical ratios of the crack and the hole, such as: depth ratio of the crack a/t, aspect ratio of the crack a/c and hole radius ratio r/t, we calculated the stress intensity factor along the crack front. The obtained results are compared to those obtained with the finite element method by Lin & Smith [8] and those calculated with the analytical equation of Newman & Raju [2].

BOUNDARY ELEMENT MODELING

Geometrical model

Let's consider a finite thickness plate with a circular through hole situated at its centre. The height (2H) and width (2r+2W) of the plate were selected to be much larger (H/t =32; W/t=40) than the plate thickness (t) in order to reduce the influence of both sizes on stress intensity factors results to a negligible extent. Two symmetric quarter-elliptical corner cracks were assumed to exist near the hole edge, as shown in figure 1.

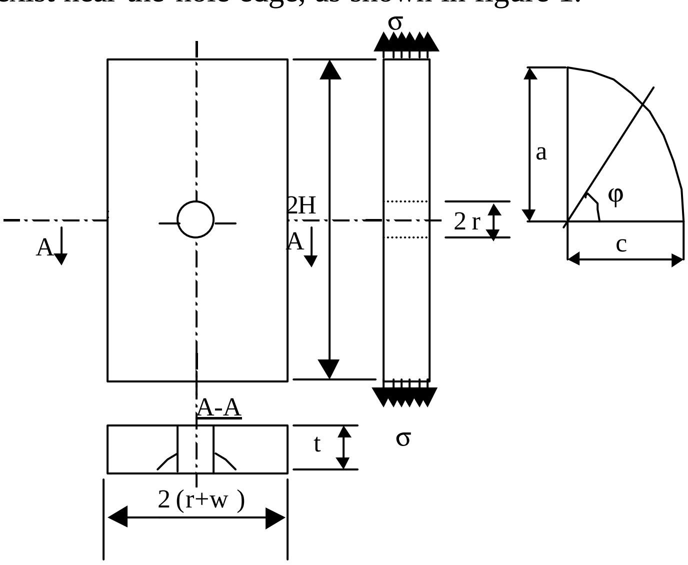
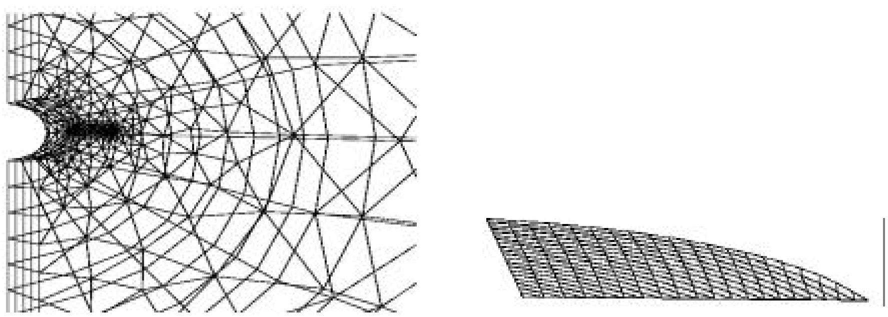


Figure 1: Geometrical model

A remote tension is considered. Considering the symmetry of geometry and loading, only one half of the plate was analyzed. Wide ranges of configuration parameters are considered. They are all combinations of the following ratios: crack surface length to plate thickness, a/t=0.2, 0.4,0.6, 0.8 and 1; crack surface length to depth, a/c=0.5,1,1.5,2 and 2.5; and hole radius to plate thickness r/t=0.5,1,2 and 3. The material Poisson ratio ν was assumed to be 0.3 for all calculation.

Creation of the BE mesh

The geometrical model of the half of the plate was developed using OSM code version 2.0 and the pre processor function of FRANC3D has been used to integrate material properties, boundary conditions and the mesh model. The structure and the crack surface were meshed by eight node isoparametric quadrilateral elements as shown in figure 2. Stress fields and displacements have been calculated using BES (boundary element system) code version 5.0. From the results given by the BES code, we used the post-processor function of FRANC3D to compute the stress intensity factors along the crack front. There are number of methods for extracting the stress intensity factors from a numerical simulation; FRANC3D uses displacement correlation [9]. Displacement correlation is a simple technique and is easy to implement for non-planar crack geometries. The displacement extrapolation technique is available, but it has been shown to give erratic results [10].



(a) (b)

Figure 2: (a) Boundary element mesh of the structure near the hole
(b) Boundary element mesh of the crack surface

The displacements on the crack surface-opening, sliding and tearing- can be directly related to the three stress intensity factors. In pure mode I, stress intensity factor is evaluated from the crack displacement at the quarter-point located at the first row of nodes from the crack front [11]:

$$K_I = \frac{\mu}{2(1-\nu)} \Delta u \sqrt{\frac{2\pi}{L}}$$

where μ and ν are the shear modulus and the Poisson ratio of the material respectively, L is the element width and Δu the normal opening displacement. The normalized SIF is given by K_I/K_0 where $K_0=\sigma\sqrt{\pi a}$.

ANALYSIS AND RESULTS

Presentation Of Results

In figure 3, we plotted the variation of the normalized mode I SIF's along the crack front (as a function of angle φ for different values of hole radius r/t, for a/c=2.5 and for a small value of depth a/t=0.2). In figure 4, we present the same variation but for a higher value of at (a/t=0.8) and for a/c=1. We can see on both figures 3 and 4 that a larger r/t ratio has a larger SIF value for the same crack configuration, but the rate of increase seems to be reduced as the r/t ratio becomes large. This is because the crack surface receives a high normal stress component for a large r/t and its increase with the r/t ratio shows down as the r/t ratio rises. Several authors such as Lin & Smith [8] noticed this behavior. While analyzing figures 3 and 4 simultaneously, we note that the SIF is more important at $\varphi = 0°$ when the depth ratio a/t is weak (a/t=0.2) and the SIF becomes higher at φ=90° when the a/t ratio takes high values.

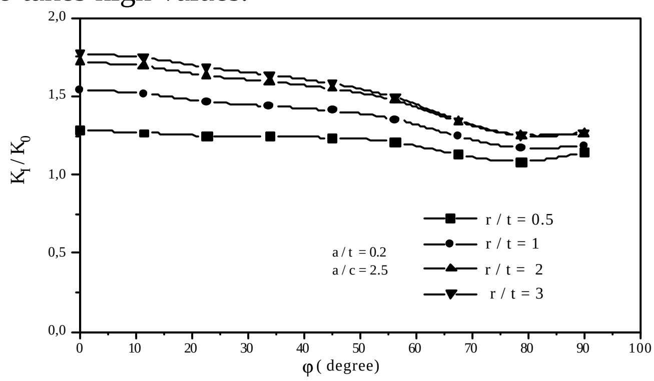


Figure3: Variation of the normalized stress intensity factor along the crack front for different r/t ratios.

It means that the crack growth is more important in the width direction (c direction) for weak values of a/t ratios and its more important in the depth direction (a direction) for the high values of a/t.

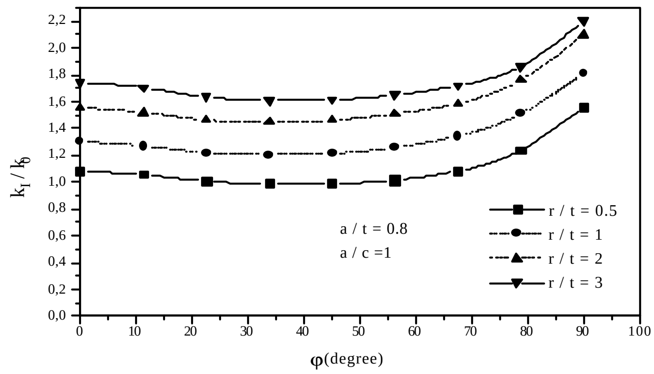


Figure 4: Variation of the normalized stress intensity factor along the crack front for different r/t ratios.

Figures 5 and 6 presents the normalized SIF variation according to the depth ratio a/t at the crack tips φ = 90° and φ =0° respectively. It should be noted that the SIF is more significant for the extreme values of a/t. The behavior of the crack for a/t=1 is interesting to analyze. We note a significant deviation of K_I/K_0 = f (a/t) curve when a/t approach the unit value, the through cracks propagate then more rapidly than the corner cracks. Kullgen et al [4] and Lin & Smith [8] announced this deviation in their works.

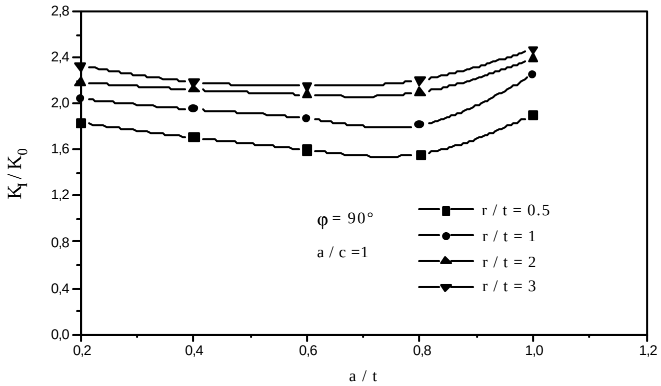


Figure5: Variation of the stress intensity factor for different r/t ratios. Vs a/t.

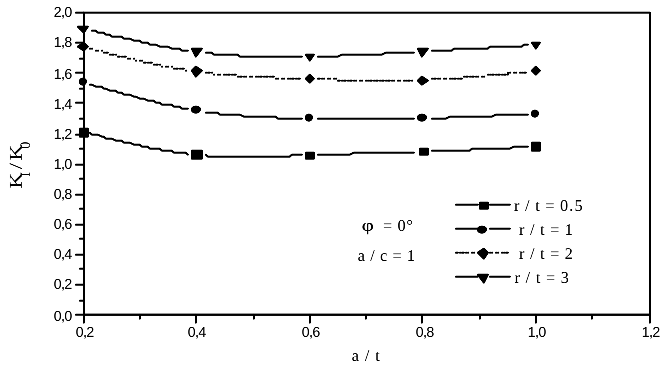


Figure 6: Variation of the stress intensity factor for different r/t ratios Vs a/t.

Comparison with other existing solutions

In Figure 7 the SIF variations along the crack front, obtained by the present authors are compared with those of Lin & Smith [8] obtained by the finite element method. The curves were plotted for r/t=1 and 3; a/c=1; and a/t=0.8. The difference between the two models is included between 0.1% and 15% with an average of 9%. It's seen clearly that there is a good agreement with the two models, which confirm the advantages of the boundary element methods for the resolution of fracture mechanics problems compared with the finite element method which uses more elements.

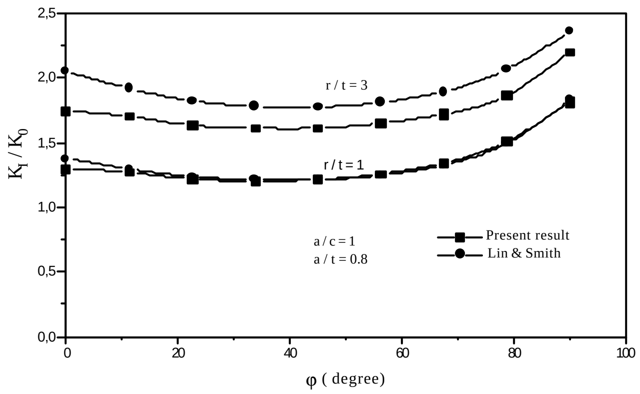


Figure7: Comparison with the boundary and the finite element methods.

In figure 8, the normalized SIF variations at the extremes of the crack front with the depth ratio a/t for r/t =0.5 and a/c=1, obtained by the present solution are compared with those of Lin & and Smith [8] and those calculated by Newman & Raju's equations [2]. At the extremity of the crack

front (φ =90°), the difference lies between 2 and 9.5 % with an average of 6% for r/t=0.5 and between 0.5 and 3.5 with average of 1.5 for r/t=2.
The comparison with Newman & Raju's equations the difference does not exceed the 4%. It lies between 0.9 and 2% for r/t=0.5 and between 1.4 and 4% for r/t=2. We can conclude that the present results are in good agreement with the finite element solution and the Newman & and Raju's solutions except for r/t=0.5 and a/t<0.4 where the difference with the finite element solution exceed the 10%.

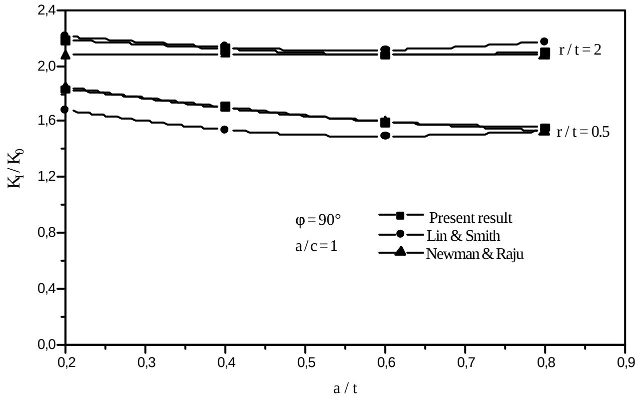


Figure 8: Comparison of the stress intensity obtained with the BEM, the FEM and the Newman and Raju's equations

CONCLUSION

The analysis of the obtained results and the comparison with other existing solutions allow us to extract the following conclusions:

-The stress intensity factor for a quarter-elliptical corner crack emanating from fastener hole is higher at the extremes of crack front facilitating the crack propagation.

-When the depth ratio of the crack is equal to unit value, the stress intensity factor increases significantly. The through crack propagates more rapidly than the corner crack.

-The good agreement of our results with those of Lin & Smith and Newman & Raju, shows that the boundary element method is a promising alternative to the finite element method for resolution of fracture mechanics problems.

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