Integrity Assessment of Pressure Vessels Containing Random Number of Defects

V. Bína and O. Bielak

BiSAFE s.r.o., Malebná 1049, 140 00 Praha 4

ABSTRACT: *The paper provides an in-depth probabilistic integrity assessment of pressure vessels. Although the engineering integrity criteria have been fully observed, it has to be taken into consideration that the fracture properties, fracture quantities (the stress intensity factor, J-integral) are random quantities or stochastic processes (time-dependent degradation of fracture properties, crack size and crack growth). These principles form the basis for equations from which a single defect failure risk is calculated, both for elastic and elastic-plastic state. The second part of the paper contains a calculation of the integral failure risk for random number of possible structural defects.*

INTRODUCTION

There are various integrity-assessment procedures. For fracture mechanics, the issue has been dealt with in great detail. For calculation procedures, conventional methods have been used which are based on safety factors. These methods cannot give answer to question: how great is the failure risk of each single structural member or even of the structure as a whole.

In order to solve the issue the probabilistic theory has been used. The fracture-mechanics quantities, material properties, crack growth are considered to be random or stochastic processes which form the basis for the calculation of defect-based failure-risk initiation as well as for the assessment of the number of defects and for the complex integrity assessment.

ENGINEERING INTEGRITY CRITERIA

Our probabilistic assessment is based on basic principles of fracture mechanics where the most important quantity is considered to be the stress-intensity factor K (SIF) or J-integral. If the quantity K or J reaches respective critical values K_c or J_c, failure of material occurs. Hence, the integral state is characterized by

$$K(a,\sigma) < K_c \quad \text{or} \quad J(a,\sigma) < J_c \tag{1}$$

where $K(a,\sigma)$, $J(a,\sigma)$ are SIF and J-integral as the crack-depth a and stress σ functions.

For elastic-plastic state, two quantities K_r and S_r are used. Material characteristics is described by a limit curve $K_{rC} = C(S_r)$. The integral state is characterized by a point described by coordinates (K_r, S_r) and consistent with service conditions and lying under the limit curve $C(S_r)$ i.e. [1]

$$K_r = \frac{K(a,\sigma)}{K_c} \quad , \quad S_r = \frac{S(a,\sigma)}{\sigma_f} \quad , \quad K_r < C(S_r) \tag{2}$$

PROBABILISTIC INTEGRITY ASSESSMENT

Failure risk of elastic-state material

If we consider that a structural member is in elastic state, the relevant criterion is described by relationships (1). Then the probability that the member is in the integral state is given by probabilities [2]

$$P(K(a,\sigma) < K_c) \text{ or } P(J(a,\sigma) < J_c) \ . \tag{3}$$

As the material properties certainly exhibit some amount of variability, fracture toughness K_c (or J_c-integral) can be considered to be a random quantity. The other quantity $K(a,\sigma)$, (or $J(a,\sigma)$) can also be considered to be a random one as a result of randomness of crack depth a. To solve the relationship (3) we introduce variable $Z = K_c - K(a,\sigma)$ or $Z = J_c - K(a,\sigma)$ the probability distribution function of which is given by

$$P(Z \le z) = \int_0^\infty P[K_c \le z + k]dF(k) = \int_0^\infty H(z+k)dF(k) \tag{4}$$

where $H(c) = P[K_c \le c]$ is a fracture-toughness K_c probability distribution function, $F(k) = P[K(a,\sigma) \le k]$ is distribution function of SIF. The distribution function of $K(a,\sigma)$ is derived from the distribution function $G(x)$ of the crack-depth a.

The service-ability criterion described by relationship (1) is identical with relationship (4) for $Z > 0$ and the failure risk calculation is given by

$$P(R=1|\sigma) = 1 - P(Z>0) = \int_0^\infty H(k)dF(k) = \int_0^{s_0} H[K(x,\sigma)]dG(x) \qquad (5)$$

where $P(R = 1|\sigma)$ is failure risk under constant stress σ, s_o is the nominal (measured) wall thickness of the structural member.

Assume that a stochastic degradation of material occurs. Then, if critical values decrease to the fracture-quantity value (SIF, J-integral), fracture is initiated. Fracture properties, when degraded, are likely to suffer from a monotone decrease and hence (see Figure 1): event $\mathbf{C} = \{K_c(t) > c\}$ implies event $\mathbf{D} = \{\tau > t \mid c\}$ where τ is time to fracture. From the equivalence of both random events **C** and **D** it follows the equality of probabilities $P(\mathbf{C}) = P(\mathbf{D})$ or $P\{K_c(t) > c\} = P\{\tau > t \mid c\}$. And the failure risk before time t under constant stress intensity factor $K(x,\sigma)$ is

$$P(R=1|x,\sigma) = P(\tau \le t \mid c) = 1 - P(\mathbf{D}) = 1 - P(\mathbf{C}) = H(c;t) \qquad (6)$$

where $P(R = 1| x,\sigma)$ is failure risk under constant crack depth x and constant stress σ, $H(c;t) = P(K_c(t) < c)$ is probability distribution function of fracture properties $K_c(t)$.

In practice, however, cracks may grow and there may also be corrosion. Corrosion may cause reduction in wall thickness, which may increase stress that, together with the growing crack, may increase the level of fracture quantity. Assume another random process $K[a(t),\sigma(t)]$. Again, introduce a random function $Z(t) = Z[a(t),\sigma(t)] = K_c(t) - K[a(t),\sigma(t)]$. As $K[a(t),\sigma(t)]$ is a monotone-growing function of crack depth and stress (and thus also time), fracture starts at the moment when both processes meet, or the other way round, no fracture occurs if the random function $Z[a(t),\sigma(t)] > 0$ (see Figure 2), i.e. event $\mathbf{C} = \{Z[a(t),\sigma(t)] > 0\}$ implies event $\mathbf{D} = \{\tau > t\}$. Thus the risk (probability) of fracture initiation starting before time t is

$$P(\tau \le t) = P(Z[a(t),\sigma(t)] \le z) = \int_0^\infty H(z+k;t)dF(k;t) \qquad (7)$$

where $H(c;t) = P[K_c(t) \leq c]$ is the distribution function of fracture characteristics - of the process $K_c(t)$, $F(k;t) = P(K[a(t),\sigma(t)] \leq k)$ is the distribution function of the process $K[a(t),\sigma(t)]$.

The fracture-characteristics distribution function $H(c;t)$ of the process $K_c(t)$ is likely to be available as a material characteristics. Distribution function $F(k;t)$ can be derived from the distribution functions $G(x;t)$ and $D(\sigma;t)$ of the stochastic processes $a(t)$ and $\sigma(t)$, respectively [3]. The resulting formula of life-time and risk-calculation can be written as follows

$$P(\tau \leq t) = \int_0^{\sigma_0}\int_0^{s_0} H[K(x,\sigma);t]\,dG(x;t)\,dD(\sigma;t). \tag{8}$$

Failure risk of elastic-plastic-state material

If we assume that the material is in elastic-plastic state, the integral-state criterion is given by relationship (2). The probability that the structure member is in this state is given by probabilities

$$P\left(\frac{K(a,\sigma)}{C(S_r)} < K_c\right) \quad \text{or} \quad P\left(\frac{J(a,\sigma)}{C(S_r)} < J_c\right) \quad \text{where} \quad S_r = \frac{S(a,\sigma)}{\sigma_f}. \tag{9}$$

First, assume that crack depth a, flow stress σ_f and stress σ are constant and that fracture properties are a random quantity. The integral state can be formulated by event **B**

$$\mathbf{B} = \left\{\frac{K(a,\sigma)}{C(S_r)} < K_c \middle| x,\sigma_f,\sigma\right\}, \quad \text{where} \quad x,\sigma_f,\sigma = \text{const.} \tag{10}$$

The failure risk is given by probability

$$P(R = 1 | x,\sigma_f,\sigma) = 1 - P(\mathbf{B}) = H(c), \quad c = \frac{K(x,\sigma)}{C(S_r)}, \quad S_r = \frac{S(x,\sigma)}{\sigma_f}. \tag{11}$$

Consider that material properties K_c and σ_f are subjected to time-dependent degradation, i.e. they are a decreasing function of time $K_c(t)$, $\sigma_f(t)$. Crack growth $a(t)$ (as a result of fatigue processes) and stress increase

$\sigma(t)$ (eg., as a result of wall-thickness reduction caused by corrosion) are also taken into consideration. Then, the load trajectory $[S_r(t)\,,K_r(t)]$ is a monotone-growing curve – see Figure 3. The limit curve $C(S_r)$ is a decreasing function. It means that both curves intersect in one point only. Next, conditional probabilities shall be applied [3]. Then, fracture risk occurring before time t is calculated by integration from

$$P(\tau \le t) = \int_0^{\sigma_m} \int_0^{\sigma_{fm}} \int_0^{s_0} P\left(R = 1 \middle| x, \sigma_f, \sigma\right) dG(x;t)\, dW(\sigma_f;t)\, dD(\sigma;t) \tag{12}$$

where $P(K_c \le c)$ is fracture-properties distribution function $H(c)$, c see Eq. (11), $G(x;t)$, $W(\sigma_f;t)$, $D(\sigma;t)$ are the probability distribution of crack depth, flow stress and effective stress in time t, respectively.

FAILURE RISK OF STRUCTURE SYSTEM

Dealing with exactly defined number of critical locations, the classical probability procedures can be applied. However, in practice the number of critical locations is not always known exactly, like the number of defects in structure.

Probability of defect occurrence

We can assume that the occurrence of defects is stationary, proportional to the volume increase ΔV and depending on the weld size (volume), i.e. $P(\Delta V) = \psi.\Delta V + o(\Delta V)$ – ψ is defect-occurrence intensity.

There are only three possible ways of m defects occurrence in volume $V+\Delta V$ and the probability $P(m,V+\Delta V)$ of m defect occurrence is

$$P(m, V + \Delta V) = P_1 + P_2 + P_3 = P(m,V).[1 - \psi.\Delta V] + P(m-1,V).\psi.\Delta V + o(\Delta V)$$

After a modification and having used a limit $\Delta V \to 0$ we obtain a differential equation for the Poisson probability distribution which gives probability of m defects occurrence in volume V (ψ is the mean number of defects per volume unit)

$$P(m,V) = P(M = m| V) = \frac{(\psi.V)^m}{m!} \exp(-\psi.V). \tag{13}$$

Complex Integrity Assessment of Structure

The reliability of the system $P[\mathbf{A}(m)]$, which consists of just m critical locations can be expressed by the relationship

$$P[\mathbf{A}(m)] = \prod_{j=0}^{m} P(\mathbf{A}_j) \;,\; P(\mathbf{A}_o) = 1 \,,\, \mathbf{A}_j = \{R = 0\} \text{ or } \{\tau_j \geq t\} \qquad (14)$$

where $P(\mathbf{A}_j)$ is the reliability of the j-th critical location.

Relationship (14) can be considered to be the probability of a random event (integral state) for the m-number of defects. If m-number is a random variable, theorem for conditional probability can be applied [3] and, after substitution (13) into (14), the resulting equation is obtained which takes account of the random number of defects.

The relationship determining the reliability of the system can be modified further if the structure consists of several homogenous sections, as far as defect distribution, strain and operation conditions are concerned

$$P(\mathbf{A}) = \prod_{i=1}^{n} \exp\left(-\psi_i . V_i P(\mathbf{R}_i)\right) = \exp\left[-\sum_{i=1}^{n} \psi_i . V_i . P(\mathbf{R}_i)\right] \qquad (15)$$

where $P(\mathbf{A})$ is the reliability of the system as a whole, $P(\mathbf{R}_i)$ is the defect failure risk, ψ_i is the mean number of defects per volume unit, V_i is the volume of the i-th section, n is the number of sections in the structure.

APPLICATION OF PROBABILISTIC MODEL

As an example let us have a failure risk calculation of a pressure-vessel shell, of both circumferential and axial welds. The calculations are based on relationship (8). First, the pressure-vessel shell must be divided into homogenous sections $i = 1, 2, \ldots n$, i.e. into members with identical calculation conditions, as shown in Table 1.

For the defect-size distribution function, we directed our attention to the two-dimensional normal distribution function the application of which allows taking account of the correlation between the crack depth and crack length.

The failure risk $P(\mathbf{R}_i) = P(\tau_i \leq t)$ can be calculated from the relevant distribution-functions shown in Table 1 after estimation their parameters.

TABLE 1: Pressure vessel shell sections

Structure Homogenous Sections	Nominal Stress	Welding technology	Distribution function $K_c(t)$ relation (7)	Defect-size $a(t)$ distribution function [3]	Defect-number distribution $P(m,V)$ relation (13)
Axial weld	σ_1	shop weld	$H(c;t)$	$g_1(x_1,x_2)$	ψ_1 , V_1
Crcumferencial weld 1	σ_2	shop weld	$H(c;t)$	$g_2(x_1,x_2)$	$\psi_2 = \psi_1$, V_2
Circumferencial weld 2	σ_2	site weld	$H(c;t)$	$g_3(x_1,x_2)$	ψ_3 , V_3

Now it is easy to calculate the pressure-vessel-shell integrity risk $P(\tau \leq t)$ from relationship (15). And the dependence of failure risk on service time is obtained from the calculation for various t.

CONCLUSIONS

The paper is aimed at creating a probabilistic-model conception in which calculation quantities (understood as random variables or stochastic processes) are integrated into a unified context. As basic principles, engineering integrity criteria have been used, i.e. the elastic criterion and the two criteria method. The basic quantities are: material properties – fracture toughness and flow stress, crack size which are considered to be random variable or stochastic processes with the parameter of time. The failure risk calculation for randomly occurring crack-shaped defects is given in the last part of the paper.

REFERENCES

1. Milne I., Ainsworth R. A., Dowling A. R. and Stewart A. T. (1988) *Assessment of the Integrity of Structures Containing Defects*, Int. J. Pres. Ves. & Piping, **32**, pp 3 – 104
2. Provan, J.W. (1987) *Probabilistic Fracture Mechanics and Reliability*. Martinus Nijhoff Publisher, Dordrecht.
3. Feller, W. (1977) *An Introduction to the Probability Theory and its Applications*. John Willey and Sons. Inc., London.

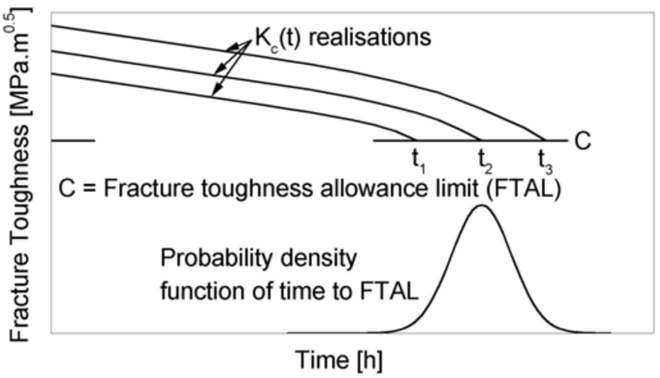

Figure 1: Fracture toughness $K_c(t)$ degradation and time-decrease to level C

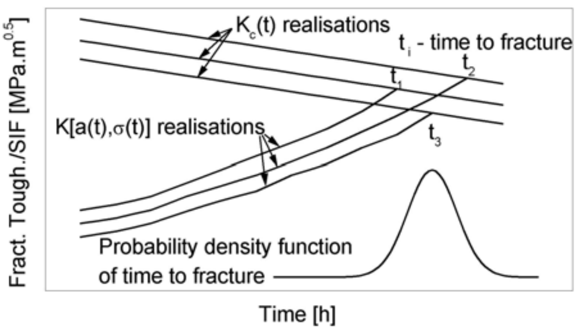

Figure 2: Fracture toughness $K_c(t)$ degradation and SIF increase as a result of crack-growth $a(t)$ and stress $\sigma(t)$

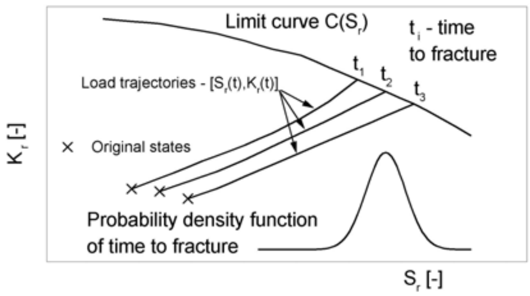

Figure 3: Load trajectory $[S_r(t), K_r(t)]$ as a result of crack propagation $a(t)$ and fracture toughness $K_c(t)$ and flow stress σ_f degradation