# A Formalism to Model Environment-Sensitive Fatigue Crack Growth

**F. Bergner**

Dresden University of Technology, Institute of Materials Science, Germany

***ABSTRACT:*** *The paper deals with the effect of 3.5% NaCl solution on long fatigue crack growth in the thin sheet wrought aluminium alloy 6013 T6. A formalism aimed to provide a quantitative model of the environmental effect is summarized and extended. The formalism is essentially based on the concept of a rate-controlling step and the application of the method of dimensional analysis to the individual steps. The sensitivity of the resulting model to distinct modifications of the underlying assumptions is discussed.*

## INTRODUCTION

The starting point of the present investigation is the observation of fatigue crack growth in the aluminium alloys 6013 T6 in 3.5% NaCl solution at rates in excess of those in both laboratory air and deionized water [1,2]. As stress corrosion cracking cannot be the reason for the increased fatigue crack growth rates [3], an alternative LEFM-based model was developed and applied to a set of measurements of constant-amplitude long fatigue crack growth in the range of frequencies, f, from 25 mHz to 40 Hz and in the $\Delta K$ range from about 8 to about 30 MPa $m^{1/2}$ [1]. The model is based on the assumption of a single multiple-step mechanism being responsible for the increased crack growth rates. Of these steps the slowest one is assumed to control the crack growth rate [4]. The expressions derived were then fitted to the measurements. However, as mentioned in [1] a good fit alone cannot be considered enough evidence for both the validity of the model assumptions and the interpretation of the rate-controlling steps.

In the present paper, the formalism applied in [1] is investigated in some more detail. First, a list of assumptions along with comments is given. Both the formalism and its application to the aluminium alloy 6013 T6 are summarized. The model is then extended in order to empirically take into account the simultaneous contribution of more than one step to the rate control of environmental fatigue crack growth. Finally, the sensitivity of the resulting model to distinct modifications of the assumptions is considered.

## ASSUMPTIONS

### K-Validity

The assumption of applicability of linear-elastic fracture mechanics (K-validity) is discussed by McClintock [5]. It is required that cracks are long compared to the microstructural size and applied stresses are low enough to limit the size of the plastic zone relative to both crack length and ligament. In the case of K-valid fatigue crack growth the stress intensity factor range, $\Delta K$, is the first parameter required. Implicitly, another consequence is the requirement to incorporate the modulus of elasticity, E, in the analysis. It is understood that the application of $\Delta K$ (instead of the effective value, $\Delta K_{eff}$) and E (instead of other elastic moduli or the yield stress, $\sigma_y$) may be the subject of some debate. As the formalism does not fundamentally depend on these details, this matter will be discussed later.

### Number of Rate-Controlling Steps

As mentioned in the introduction a single multiple-step mechanism of the effect of the environment on fatigue crack growth is assumed. The concept of a rate-controlling step [4] means that under certain conditions the slowest one of several successive steps (i. e. the one that imposes the strongest restriction to the environment-assisted fatigue crack growth rate) is the rate-controlling one. In the original version of the present model [1] a number of three potential rate-controlling steps have been taken into account. These steps had been assumed to act independently of each other. The extended version to be presented will take into account the possibility of simultaneous contribution to the rate control by more than one step. The effect of incorporating a forth potential rate-controlling step will be touched.

### Nature of Rate-Controlling Steps

Each one of the potential rate-controlling steps is assumed to be characterized by a single (or effective) physical quantity. The formalism turns out to depend only on the physical dimensions of these quantities but not on the nature of the underlying processes. Therefore, this assumption is related to the possibility to combine two steps characterized by physical quantities of the same dimension into a single step characterized by an effective quantity. In this way, the formalism is strongly simplified, whereas possible complications are transferred to the interpretation of the results. In the original version of the model the three potential rate-controlling steps are assumed to be characterized by quantities of physical dimensions $T^{-1}$, L/T, and $L^2/T$, where T and L denote the physical dimensions of time and length, respectively. These quantities can be interpreted to be a rate of production, q, of directly damaging hydrogen or indirectly damaging (by destabilizing the surface layer) chloride ions, the rate of thickness increase, v, of a surface layer on the freshly formed metal surface, and an effective coefficient of diffusion, D, of hydrogen to the embrittlement sites in front of the crack tip or to transporting dislocations. However, it is important to note that the formalism does not depend on these interpretations.

### The Reference Environment

The proper selection of a reference environment is related to the assumption of a single (if multiple-step) mechanism already made above. The effect of the environment is isolated by means of the introduction of the dimensionless damage ratio, $\eta=(da/dN)_e/(da/dN)_{ref}$. In [1] data measured in laboratory air was taken as reference with arguments supporting this idea given. In particular, deionized water might be a better reference, but no significant differences between fatigue crack growth rates in air and deionized water were found at least at f=20 Hz [2]. Thus the present approach is supported.

## FORMALISM AND MODEL

### Summary of the Original Formalism

The formalism consists of the following steps [1]:

- According to the experimental observations and the above assumptions the damage ratio is a function of $\Delta$K, E, f, and either q, or v, or D.
- The $\Pi$-theorem of dimensional analysis is applied to these three cases. The results are representations of $\eta$ as a function, $\Phi$, of dimensionless power-law monomials of the quantities considered [6]. The results are:

$$\eta = \Phi_a\left(qf^{-1}\right) \tag{1a}$$

$$\eta = \Phi_b\left(\Delta K^2 E^{-2} f v^{-1}\right) \tag{1b}$$

$$\eta = \Phi_c\left(\Delta K^2 E^{-2} f^{1/2} D^{-1/2}\right) \tag{1c}$$

- At this point additional information on the type of dependence is needed. If the dependence of $\eta$ on one particular parameter is known, the dependence on the other ones will also be known simultaneously. In the original version of the model, $\eta$ was assumed to be proportional to q, inverse proportional to v, and proportional to the mean diffusional path, $(D/f)^{1/2}$, respectively. This yields expressions for the functions, $\Phi$:

$$\Phi_a = C_a f^{-1} q \tag{2a}$$

$$\Phi_b = C_b \Delta K^2 E^{-2} f v^{-1} \tag{2b}$$

$$\Phi_c = C_c \Delta K^{-2} E^2 f^{-1/2} D^{1/2} \tag{2c}$$

$C_a$, $C_b$, and $C_c$ are dimensionless constants.

- In order to identify the ranges of $\Delta K$ and f, where a certain step is the slowest one, the above expressions are equated to one another. In a double-logarithmic f-$\Delta K$ plot the resulting power-law equations between $\Delta K$ and f are represented by straight lines. These lines separate the regions in the f-$\Delta K$ plane, where the respective steps are rate controlling.
- The lines defined by the conditions $\Phi$=1 bound these regions against the region outside, where no environmental effect is present.
- In order to obtain a graphical representation of the „rate-control map“, frequency, f, and stress intensity factor range, $\Delta K$, have to be normalized by the model parameters, $f_S$ and $\Delta K_S$. Furthermore, a particular base, b, of the logarithm has to be introduced:

$$f_S = \left[ (C_a q)^4 \left( C_b^{-1} v \right)^2 \left( C_c^2 D \right)^{-1} \right]^{1/5} \tag{3a}$$

$$\Delta K_S = E\left[ (C_a q)^{-3} \left( C_b^{-1} v \right) \left( C_c^2 D \right)^2 \right]^{1/10} \tag{3b}$$

$$b = C_a q / f_S \tag{3c}$$

The resulting rate-control map is presented in Fig. 1. If in a fatigue crack growth test (at varying $\Delta K$ and/or f) one of the lines in Fig. 1 is crossed, a transition in the crack growth behaviour is expected. Such transitions have indeed been observed in [1]. By fitting individual lines of the map to the measured locations of the corresponding transitions the model parameters $f_S$, $\Delta K_S$, and b, have been estimated. The physical quantities characterizing the individual rate-controlling steps can then be calculated via Eqs. 3. In this way the general model, Eqs. 1 or 2, can be applied to a particular material, if the model assumptions are prooved to be valid.

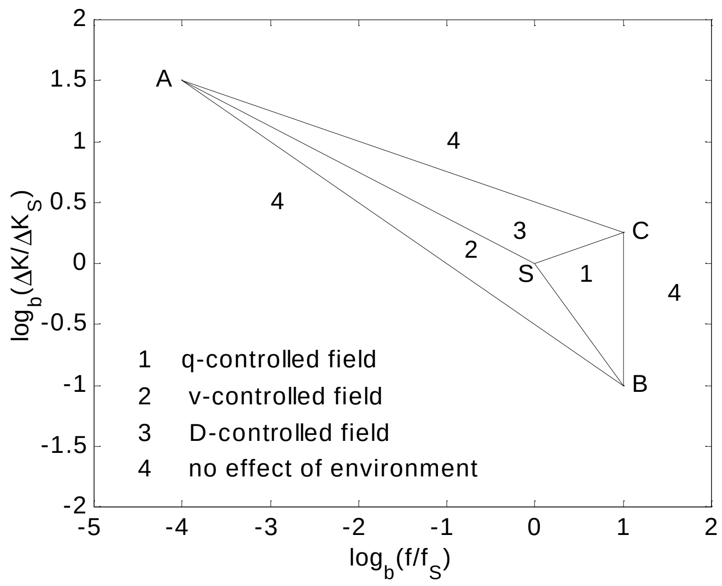


**Figure 1:** Normalized double-logarithmic rate-control map [1]

## Extension of the Original Formalism

As a single rate-controlling step was assumed, the model outlined above is not able to predict crack growth rates in the vicinity of the bounds of the rate-control map. In order to take into account the simultaneous operation of more than one rate-controlling step the damage ratio is considered the analogue of the spring constants (or conductivities) in a series of springs (or electric resistors). The resulting model reads:

$$(da/dN)_e = \eta (da/dN)_{ref} \tag{4a}$$

$$\eta = 1 + \frac{1}{1/\Phi_a + 1/\Phi_b + 1/\Phi_c} \tag{4b}$$

The process-competition model for the crack growth mechanisms in the aggressive and in the reference environment applied to obtain the rate-control map is substituted by the superposition model represented by the first plus sign in Eq. 4b. The functions $\Phi_a$, $\Phi_b$, and $\Phi_c$ can be replaced by the expressions given in Eqs. 2, if the corresponding assumptions are justified. Using the constants obtained by the fit performed in [1] for the Al alloy 6013 T6 a 3D representation of Eq. 4b is obtained (Fig. 2).

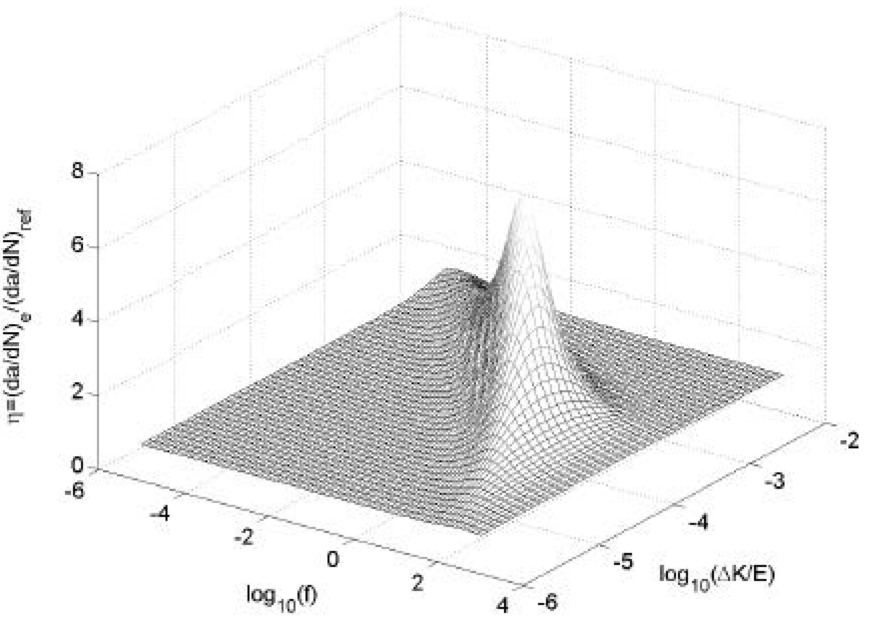


**Figure 2:** 3D mesh representation of Eq. 4b applied to 6013 T6

## DISCUSSION

### K-Validity

Within the range of K-validity the opened-crack part of $\Delta K$ may be a better driving force parameter than the nominal value of $\Delta K$. For the particular case of the aluminium alloy 6013 T6 at f=20 Hz no significant effect of a 3.5% NaCl solution on the amount of crack closure with respect to air was observed [1]. Furthermore, the plateau of the crack growth curves obtained for 6013 T6 at f=20 Hz are in the same $\Delta K$ range for R=0.1 and R=0.5 [1]. Thus the use of $\Delta K$ is justified at a constant value of the stress ratio, R.

The modulus of elasticity, E, the shear modulus, μ, the yield stress, $\sigma_y$, (and others) are interchangable material parameters from the viewpoint of dimensional analysis. Nevertheless, the values of the fitting constants are essentially affected. If E is used, the estimated value of $D \sim 10^{-13}$ $m^2/s$ is of the order of magnitude of the coefficient of diffusion of H in the Al lattice [7]. In this respect the alternative use of μ or $\sigma_y$ would cause deviations by many orders of magnitude. Another argument for the use of E instead of $\sigma_y$ is given by the similarity of the environmental effects for the alloys 6013 T6 and 6013 T4, which are represented by equal values of the elastic moduli but different values of the yield stress.

The assumption of K-validity can be violated by high stresses and/or short cracks. Under certain conditions the problem can be solved by using $\Delta J/E$ or $\Delta CTOD$ [5] instead of the term $(\Delta K/E)^2$ (the dimension of each is a length) throughout the dimensional analysis. Typically, however, additional parameters, such as yield stress, hardening exponent, and a microstructural size (short cracks) have to be introduced. Therefore, the functions, Φ, in Eqs. 1 become multiparametric ones and the present formalism will fail to work.

**Number of Rate-Controlling Steps**

The formalism applied in the present paper is generally applicable to any number of rate-controlling steps. In the present power-law approximation each one of the rate-controlling steps can be represented by a plane in a 3D f-ΔK-η (x-y-z) co-ordinate system. The possible projective relations between three planes in 3D are well manageable but the introduction of any new plane introduces a variety of new relations. Some types of relations between four planes are indicated in Fig. 3. While the graphical appearence of the rate-control map is strongly affected by the number of rate-controlling steps, the analytical representation of the model in terms of Eq. 4b is only affected by the number of terms in the denominator.

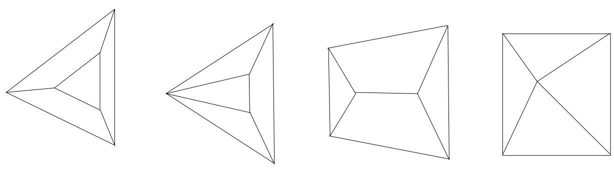

**Figure 3:** Possible projective variants of the rate-control map in the case of four rate-controlling steps

**Nature of Rate-Controlling Steps**

First, the present model of environment-sensitive fatigue crack growth is affected by the nature of the rate-controlling steps through the physical dimensions of the characteristic quantities. The formalism works for any combination of independent physical dimensions. In general, a modification of the physical dimension of one of the characteristic quantities affects the exponents in each one of the Eqs. 1 and 2 simultaneously.

Second, the model is affected by the nature of the rate-controlling steps via the type of function, Φ, in Eqs. 1. The power-law dependencies of Eqs. 2 are only a special case partly motivated by mechanism-based arguments and partly supported by the agreement of the theoretical slopes of lines AB and

AC in Fig. 1 with the measured frequency dependence of the respective transition points [1]. The formalism works independently of the type of function, but for the formalism to be explicitly applied the functions have to be known. The validity of a power-law approximation is limited by the presence of both a threshold and a saturation concentration of the damaging species. The threshold condition can be identified with lines AB, BC, and CA in the rate-control map (Fig. 1). The error of $\eta$ according to Eq. 4b caused by a threshold is small. The effect of saturation is a cut-off of the surface in Fig. 2 by a plane parallel to the f-$\Delta$K plane. In terms of the rate-control map this effect is represented by the left graph in Fig. 3.

### Reference Environment

The present formalism can be applied, if the increase of the crack growth rates with respect to vacuum is due to a single mechanism. Otherwise, the individual contributions have to be carefully separated prior to application. In this respect deionized water would be the favourable reference medium but referring to vacuum data afterwards may provide additional information.

## CONCLUSION

The sensitivity of the formalism and the resulting model of environment-assisted fatigue crack growth to modifications of the assumptions has been analysed. The formalism is characterized by a broad applicability. The model as applied to the aluminium alloy 6013 T6 is partly supported.

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