FINITE ELEMENT STUDY OF A DOUBLE EDGE CRACK SPECIMEN

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***ABSTRACT*:** The elastic *T*-stress is known as an important parameter for evalulating structural integrity of cracked specimens. Although many researchers have attemped to determine the *T*-stress for various specimens, most of the previous studies are based on two-dimensional models. The aim in this paper is to calculate the *T*-stress for double-edge cracked specimens using a three-dimensional model. A linear elastic finite element analysis is employed to determine the stress field around the crack front. The *T*-stress is computed directly from stresses along the crack faces and across different planes through the thickness. The variation of *T*-stress with different parameters like the Poisson's ratio, the ratio of crack length to the specimen width (*a/W*) and the specimen thickness are also studied.

INTRODUCTION

The constant elastic term of stress, often known as the *T*-stress, has received much attention in last two decades by researchers working in the field of fracture mechanics. It is now well established that the *T*-stress can be used as a measure of constraint around the crack tip [1,2]. The significant influence of the *T*-stress on the size and shape of plastic zone around the crack tip has also been shown by Larsson and Carlsson [3]. The *T*-stress can also be used to determine the stability of crack growth path in brittle fracture [4].

The finite element technique can be used as a general method for calculating the *T*-stress for cracked specimens [5,6]. However, almost all of the data available in papers for the *T*-stress are related to two-dimensional models of cracked specimens. Very limited results have been reported in the past for the *T*-stress in three-dimensional crack geometry shapes. For instance, Nakamura and Parks [7] studied stresses around the crack front for a single edge specimen subjected to bending and tension to determine the *T*-stress in a three-dimensional model. A similar investigation was conducted by Henry and Luxmoore [8] for center crack plates.

In this paper, the *T*-stress is calculated for a three-dimensional double edge crack specimen subjected to uniform tension. The variation of *T*-stress through the thickness of plate is studied using finite element analysis. The effects of Poisson's ratio, the thickness of plate and the length of crack on the *T*-stress are also investigated.

STRESS FIELD ALONG A THREE-DIMESIONAL CRACK FRONT

The stresses in an isotropic linear elastic body containing a crack can be written as series expansions [9]. If the geometry and loading configurations are symmetric relative to the crack line, the body is subjected to mode I loading. For a three dimensional mode I crack, the leading terms of the series expansions of stresses are

$$\sigma_{xx}(r,\theta,z)=\frac{K_I(z)}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+T_{xx}(z) \tag{1}$$

$$\sigma_{yy}(r,\theta,z)=\frac{K_I(z)}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{2}$$

$$\sigma_{zz}(r,\theta,z)=\frac{K_I(z)}{\sqrt{2\pi r}}2\nu\cos\frac{\theta}{2}+T_{zz}(z) \tag{3}$$

$$\sigma_{xy}(r,\theta,z)=\frac{K_I(z)}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{4}$$

$$\sigma_{xz}=\sigma_{yz}=0 \tag{5}$$

where ν is the Poisson's ratio and K_I is the stress intensity factor. The coordinates *x, y, z* and r, θ, z are related to the Cartesian and cylindrical coordinate systems attached to the crack front (see Figure 1). The terms T_{xx} and T_{zz} in the stress components σ_{xx} and σ_{zz} are independent of distance from the crack front and are only functions of *z*. The *T*-stress in two-dimensional crack specimens corresponds to T_{xx}. However, in 2D models the *T*-stress is always assumed to be independent of specimen thickness. Since the aim is to compare the results of 2D and 3D models, only T_{xx} is analyzed in this paper.

T-STRESS IN A THREE-DIMENSIONAL CRACK

Equation (1) is used here for determining T_{xx}. Since $\sigma_{xx}(r,\theta,z)$ comprises of the singular term and T_{xx}, the stress T_{xx} can be calculated along any direction θ where the singular term of $\sigma_{xx}(r,\theta,z)$ vanishes or can be set to zero by superposing with an appropriate fraction of $\sigma_{yy}(r,\theta,z)$. This corresponds to different angular positions around the crack tip. For example:

$$T_{xx}(z)=\sigma_{xx}(r,\frac{\pi}{3},z)-\frac{1}{3}\sigma_{yy}(r,\frac{\pi}{3},z) \tag{6}$$

$$T_{xx}(z)=\sigma_{xx}(r,\frac{\pi}{2},z)-\frac{1}{3}\sigma_{yy}(r,\frac{\pi}{2},z) \tag{7}$$

$$T_{xx}(z)=\sigma_{xx}(r,\pi,z) \tag{8}$$

One can provide a three-dimensional model of the cracked specimen and perform an elastic analysis to determine different stress components near the crack front. The *T*-stress then can be calculated from finite element results and by using one of the equations (6 to 8). It is preferable to use equation 8, as it needs only one component of stress. It should be noted that very near the crack front the results are affected by numerical errors normally expected from finite element results in the zones of high stress gradiant. In contrast, at distances too far from the crack front the effects of higher order terms in William's series expansion become significiant. Therefore, the *T*-stress must be determined from a region where σ_{xx} is a constant.

DOUBLE EDGE CRACK SPECIMEN

A double edge crack specimen shown schematically in Figure 2, was simulated using finite element method. The three-dimensional model of specimen had a total length of *2h*=200mm and a total width of *2W*=200mm. Different values of crack length were considered in the simulation to provide a crack aspect ratio *a/W* ranging from 0.3 to 0.7. A uniform distributed stress *S*=50MPa, was applied normal to the top end plane of the specimen. Symmetry boundary conditions were applied on the planes *x*=*W*-*a*, *y*=0 and *z*=0.

To study the effects of different geometrical and material parameters on T_{xx}, a series of finite element analyses were performed for different magnitudes of: the plate thickness 10mm≤t≤50mm, the crack length 0.2≤a/W≤0.7 and the Poisson's ratio 0.2≤v≤0.4.

To construct the finte element mesh, a two-dimensional model of specimen was first provided. Then the 2D model was extruded through the thickness to generate the three dimensional model. Six element layers were considered through the half-thickness of the specimen. Each mesh approximately consisted of 13000 nodes and 3200 twenty-noded three-dimensional quadratic elements.

RESULTS AND DISCUSSION

The stress T_{xx} along the half-crack front, normalized by S is shown in Figure 3 for thickness 2t of 20 and 100mm. In this Figure $a/$W=0.4 and the Poisson's ratio is 0.3. For both thin and thick specimens, the variation of T_{xx} through the thickness is not considerable in mid-thickness region. The highest value of T_{xx} through the thickness, takes place on the free surface where $z=t$. Depending on the thickness of specimen and the Poisson's ratio, T_{xx} calculated in the region near the free surface exhibits a sudden variation for some specimens. This feature can be seen for example in Figure 3 for t=50mm. A similar finding has been reported by Nakamura and Parks [7] for single edge crack specimens.

Figure 4 shows the variation of normalized T_{xx} in the mid-thickness plane versus the Poisson's ratio for a/W=0.4 and $2t$=100mm. It is ovserved that the value of T_{xx} increases by rising Poisson's ratio. However, this increase in T_{xx} is not considerable for 0.2≤v≤0.4. Figure 5 indicates the variation of normalized T_{xx} against various values of the thickness in the specimen.

According to this Figure, T_{xx} increases with thickness only for thin specimens (t≤30mm). For thick specimens, T_{xx} is almost independent of thickness. The effect of thickness on T_{xx} for double edge crack specimen is opposite to that for the single edge crack (SEN) specimens, as Nakamura and Parks [7] have reported that for SEN specimen T_{xx} decreases by increasing the thickness. Figure 5 also shows that the variations of T_{xx} with the specimen thickness have little difference for ν=0.3 and ν=0.4. Finally, Figure 6 shows how $T_{xx}/$ S varies for different values of normalized crack length a/W. In Figure 6 the thickness of the plate and the Poisson's ratio are

100mm and 0.3, respectively. Similar to the results for two-dimensional model [9], there is a significiant increase in the normalized T_{xx} for larger values of *a/W*. Figure 6 also shows that the results of T_{xx} at the mid-thickness plane of specimen are slightly higher than the results of *T*-stress in a 2D model.

CONCLUSIONS

A three dimensional finite element model was employed to obtain the elastic stress T_{xx} in a double edge crack specimen. The variation of T_{xx} with various parameters was investigated. It was shown that in contrast to two-dimensional models, both the Poisson's ratio and the specimen thickness can influence T_{xx} in three-dimensional case. However, the effect of Poisson's ratio on T_{xx} is not significant. It was also shown that the variation of T_{xx} through the thickness of specimen is noticeable only near the free surfaces.

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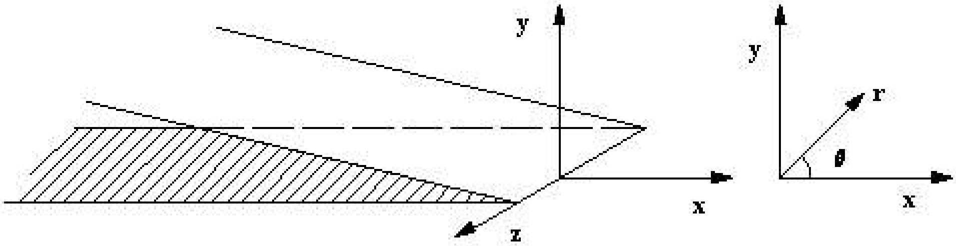

Figure 1: Coordinate systems and directions.

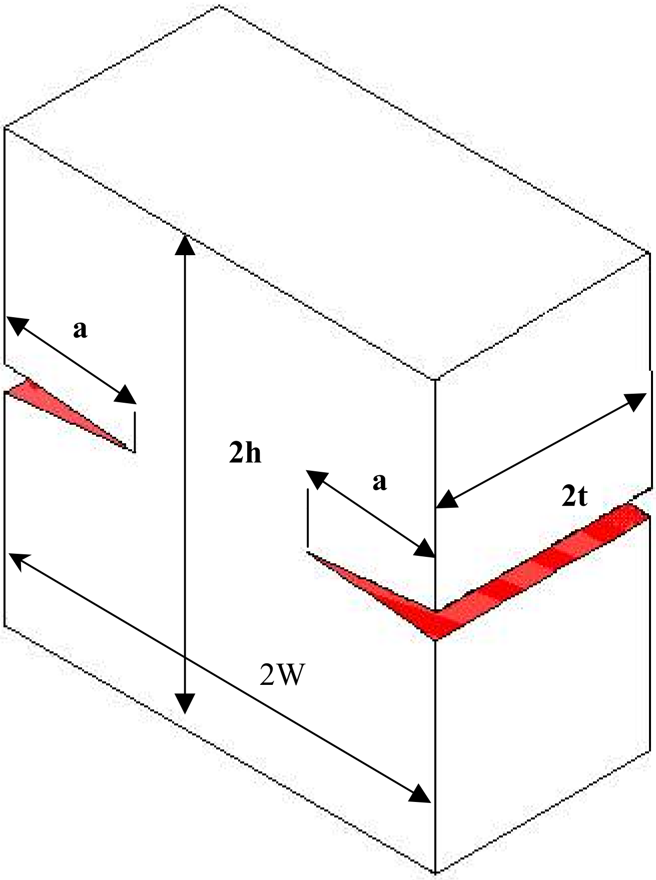

Figure 2: The double edge crack specimen.

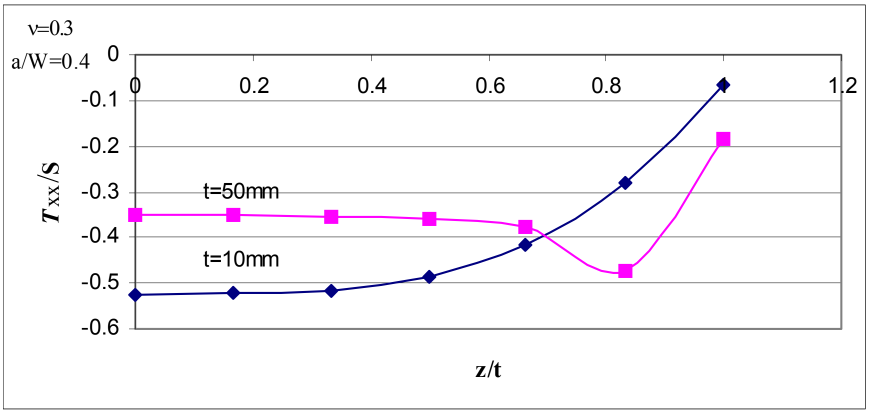

Figure 3: Variation of normalized T_{xx} along the crack front versus normalized thickness.

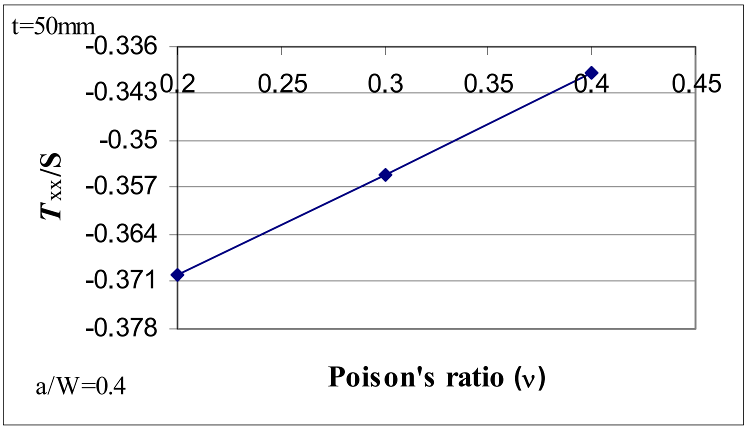

Figure 4: Variation of normalized T-stress versus the Poisson's ratio.

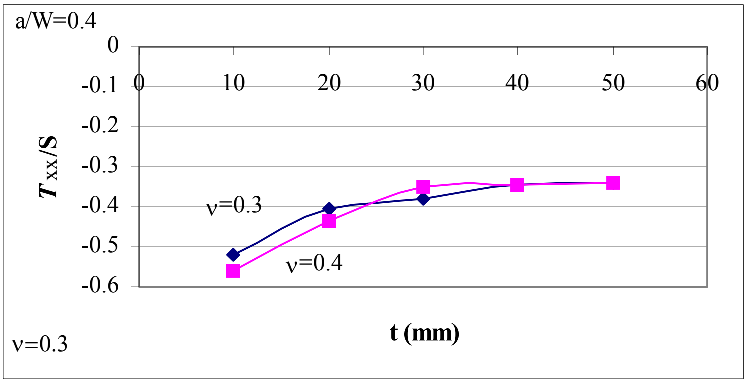

Figure 5: Variation of normalized T_{xx} versus thickness of specimen.

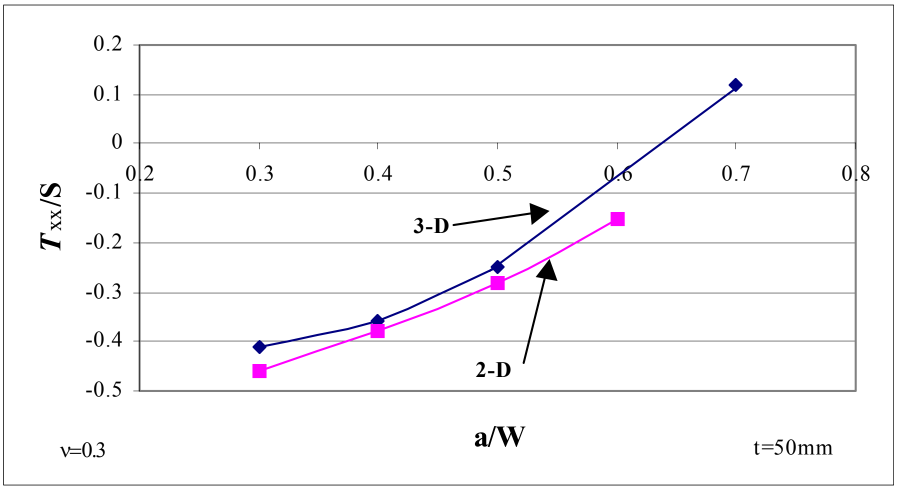

Figure 6: Variation of normalized T_{xx} with crack length ratio *a/W* compared with 2D results [10].