STATISTICAL FATIGUE PROPERTIES OF ALUMINUM ALLOYS BASED ON THE STATISTICAL ASPECT OF CRACK INITIATION AND PROPAGATION BEHAVIORS

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A large scale database MSDRD on mechanical properties of structural materials was constructed by the Research Group for Statistical Aspects of Materials Strength. In order to extract and analyze the material strength data from MSDRD, the software of STANAD was also developed by this research group. In the present study, fatigue data of aluminum alloys such as 2024-T4 and 5083BE-O were analyzed by using the STANAD. Paying particular attention to the statistical feature of the crack initiation and propagation behaviors, statistical fatigue properties were theoretically analyzed for smooth and notched specimens. It was finally found that *P-S-N* characteristics and fatigue live distributions in a wide stress range were well explained by the present analytical model.

1. INTRODUCTION

Mechanical property data for various kinds of structural materials are required to the safety design of machines and structures. A large scale database of Material Strength Database for Reliability Design of Machines and Structures(MSDRD) on mechanical properties of structural materials was constructed by the Research Group for Statistical Aspects of Materials Strength(RGSAMS)[1]. In order to extract and analyze the material strength data in the database of MSDRD, the software of STANAD was also developed by this research group[1]. Fatigue data of structural steels having various strength levels were analyzed by means of this software from a statistical viewpoint. The results obtained were reported in the last paper by the authors[2].

In the present study, fatigue data of aluminum alloys such as JIS:2024-T4 and JIS:5083BE-O were newly analyzed by the same software. Paying particular attention to the statistical feature of the crack initiation and propagation behaviors, statistical fatigue properties such as *P-S-N* characteristics were theoretically analyzed for smooth and notched specimens. A fundamental viewpoint of this analysis is the fact that the total fatigue life N_f is given by sum of the crack initiation life N_i and the crack propagation life N_p and we have $N_f = N_i + N_p$. Each component of the fatigue lives N_i and N_p has the distinct scatter so that the total fatigue life N_f also has the considerable scatter as reported by many researchers[3,4]. Thus we should combine the distribution characteristics of the respective components N_i and N_p to evaluate the distribution property of the total fatigue life N_f. From this point of view, we can derive the distribution function of N_f at a definite stress level by applying the convolution integral for probability density functions of N_i and N_p at the corresponding stress level.

2. ANALYSIS OF STATISTICAL FATIGUE PROPERTY

Total fatigue life N_f is given by sum of crack initiation life N_i and crack propagation life N_p. Thus we have

$$N_f = N_i + N_p . \qquad (1)$$

Weibull distribution is assumed to represent the distribution pattern of the crack initiation life. Accordingly, probability density and distribution functions of N_i are provided as follows[5];

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$$f_i(N_i)=\frac{\alpha}{\beta}\left(\frac{N_i-\gamma}{\beta}\right)^{\alpha-1}\exp\left\{-\left(\frac{N_i-\gamma}{\beta}\right)^{\alpha}\right\}, \qquad (2)$$

$$F_i(N_i)=1-\exp\left\{-\left(\frac{N_i-\gamma}{\beta}\right)^{\alpha}\right\}, \qquad (3)$$

where α is shape parameter, β scale parameter and γ location parameter, respectively, and these values can be easily obtained from the fatigue life distribution data by means of the correlation coefficient method[6].

When the crack propagation behavior is governed by Paris law of $da/dN=C(\Delta K)^m$, distribution pattern of the crack propagation life N_p is reduced to a log-normal distribution provided that m is a constant and C is a random variable following a log-normal distribution as reported by one of the authors[7]. Hence, we have the following probability density function of N_p;

$$f_p(N_p)=\frac{1}{\sqrt{2\pi}S_pN_p}\exp\left\{-\frac{1}{2}\left(\frac{\log N_p-\mu_p}{S_p}\right)^2\right\}, \qquad (4)$$

where

$$\mu_p=\log\left\{\frac{2}{2-m}\sigma^{-m}\pi^{-m/2}\left(a_f^{\,1-m/2}-a_i^{\,1-m/2}\right)\right\}-\bar{c}\ . \qquad (5)$$

$\bar{c}$ and S_p are mean and standard deviation of the distribution of $c\,(=\log C)$. a_i is the crack initiation size and a_f is the critical crack size causing the fracture. Distribution function of N_p is here given as follows;

$$F_p(N_p)=\int_0^{N_p}f_p(N_p)\,dN_p\ . \qquad (6)$$

Since the probability density functions of N_i and N_p are given by Eqs.(2) and (4) quantitatively, the probability density and distribution functions of the total fatigue life N_f can be given as follows by applying the convolution integral[3,4];

$$f(N_f)=\int_{\gamma}^{\infty}f_p(N_f-N_i)\,f_i(N_i)\,dN_i\ , \qquad (7)$$

$$F(N_f)=\int_{\gamma}^{N_f}f(N_f)\,dN_f\ . \qquad (8)$$

Fatigue life distributions at three different stress levels are schematically shown as Weibull plot in Fig.1, where distribution curves of N_i and N_p are plotted together with the distribution curves of total fatigue life N_f. It is supposed that the fatigue crack tends to occur at early stage at the higher stress level(σ_1) and, therefore, the crack propagation life occupies the main portion of the total fatigue life. On the other hand, the

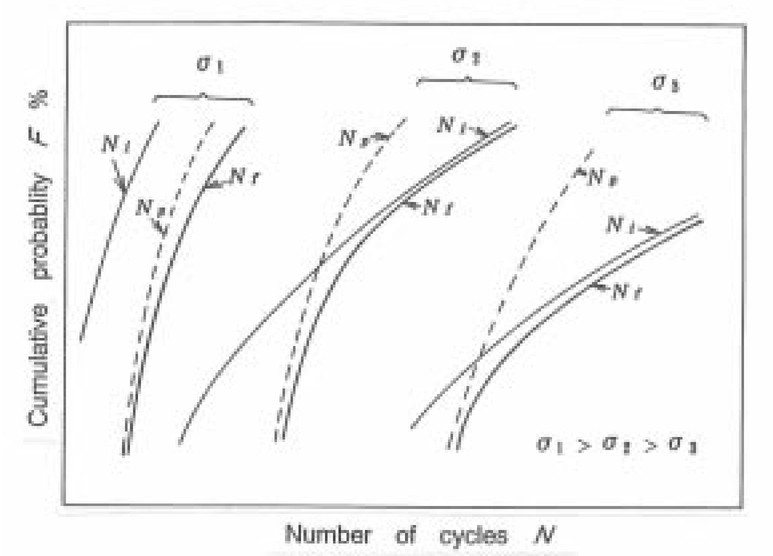


Fig.1 Schematic stress dependence of fatigue life distributions N_i, N_p and N_f.

crack initiation life at the lower stress level(σ_3) tends to increase rather than the crack propagation life. In such a case, the fatigue life is governed by the crack initiation life as indicated in Fig.1. It is also noted that both crack initiation and propagation lives have the comparable effect on the total fatigue life at the middle level of the applied stress(σ_2).

Now it is naturally supposed that the fatigue crack takes place easily for the specimen with the sharp notch and the crack propagation life occupies the main portion of the final fatigue life. In some cases of notched specimen with the notch radius of $\rho \approx 0$, the fatigue crack would occur immediately after the cyclic load was applied. In such a case, the fatigue life N_f can be almost replaced by the crack propagation life N_p. Thus the life ratio of N_p / N_f, is significantly affected by both the applied stress level and the sharpness of the specimen notches. This is a fundamental viewpoint in the present analysis of the statistical fatigue properties of non-ferrous metals extracted from the database of MSDRD.

3. FATIGUE DATA EXTRACTED FROM THE DATABASE OF MSDRD

Based on the database of MSDRD, fatigue test data were extracted by means of the powerful software of STANAD[1]. Thus we have statistical fatigue data for the fourteen different kinds of non-ferrous metals such as aluminum alloys and copper alloys. Among them, the authors have selected the fatigue data for aluminum alloys of 2024-T4 and A5083BE-O due to the sufficient quantity of the data required to the statistical analysis. Material file codes of these data in MSDRD and some reference data are listed in Table 1. These fatigue data were selected as to give the experimental data obtained for both smooth and sharp-notched specimens for the same material. Chemical compositions and typical mechanical properties of the respective materials are also indicated in Table 2 and Table 3 for the sake of reference.

Table 1 List of material file codes.

Material	Data file in MSDRD	Stress concentration factor	Tensile strength (MPa)	Authors of reference
2024-T4	0070310.MTL	$K_b = 1.08$	496	T. Shimokawa et al.
2024-T4	0070313.MTL	$K_b = 3.8$	496	T. Shimokawa et al.
A5083BE-O	0060002.MTL	$K_b = 1.0$	346	S. Nishijima et al.
A5083BE-O	0060003.MTL	$K_b = 2.5$	346	S. Nishijima et al.

Table 2 Chemical compositions of the materials(mass%).

Material	Cu	Si	Fe	Mn	Mg	Zn	Cr	Ti	Al
2024-T4	4.39	0.17	0.32	0.50	1.30	---	---	---	Bal.
A5083BE-O	0.020	0.12	0.21	0.60	4.32	0.019	0.19	0.006	Bal.

Table 3 Mechanical properties of the materials.

Material	0.2% proof stress (MPa)	Tensile strength (MPa)	True fracture stress (MPa)	Elongation (%)	Reduction of area (%)	Vickers hardness Hv
2024-T4	370	496		21.5		
A5083BE-O	182	346	397	13.2	13.5	83

4. ANALYTICAL RESULTS OF STATISTICAL FATIGUE PROPERTIES

4.1 Analytical Results for 2024-T4 ($K_b = 1.08$)

First of all, statistical fatigue properties of smooth specimens would be analyzed by using the fatigue data obtained for the shallow notched specimen having the stress concentration factor of $K_b = 1.08$. In the case of smooth specimen, the crack initiation and crack propagation lives both affect the final fatigue life, since each life component of N_i and N_p has comparable definite value depending on the stress level.

According to the survey[8] on the stress dependence of Weibull three parameters for the crack initiation life, the authors assumed the relationships between the stress level σ and each one of the parameters, α, β and γ, as shown in Fig.2. The other life component of N_p has the peculiar distribution characteristics as explained in Section 2, i.e. the crack propagation life is governed by log-normal distribution. Parameters in the Paris law are determined as $m = 4.6$ and $\bar{c} = 12.35$ making reference to the databook on the crack propagation behavior of metallic materials[9]. Standard deviation of $c\,(=\log C)$ is given as $S_p = 0.04$ based on the earlier work by the authors[7]. Consequently, one can calculate the distribution function of the total fatigue life N_f by Eqs.(7) and (8), if the crack initiation size a_i and the critical crack size causing the catastrophic failure a_f are provided in advance.

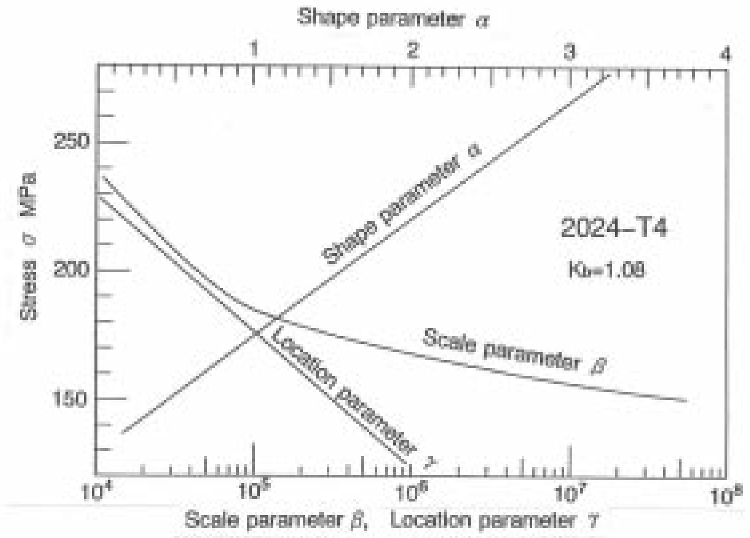


Fig.2 Stress dependence of Weibull Parameters for the crack initiation life of 2024-T4 $(K_b = 1.08)$.

By setting $a_i = 1.0mm$ and $a_f = 9.0mm$ from configuration of the fatigue specimen indicated in the reference, the distribution functions of N_f at four different stress levels were calculated following the procedure explained in Section 2. Distribution curves thus calculated were indicated on Weibull coordinates in Fig.3, where distribution functions for the respective life components of N_i and N_p were also plotted for the sake of comparison with the final fatigue life distribution of N_f. The distribution pattern of N_f is in good agreement with the experimental distributions at the respective stress levels. In Fig.3, fatigue life distributions at only four stress levels were plotted and discussed in order to examine the whole trend of the distribution characteristics for the respective lives N_i, N_p and N_f.

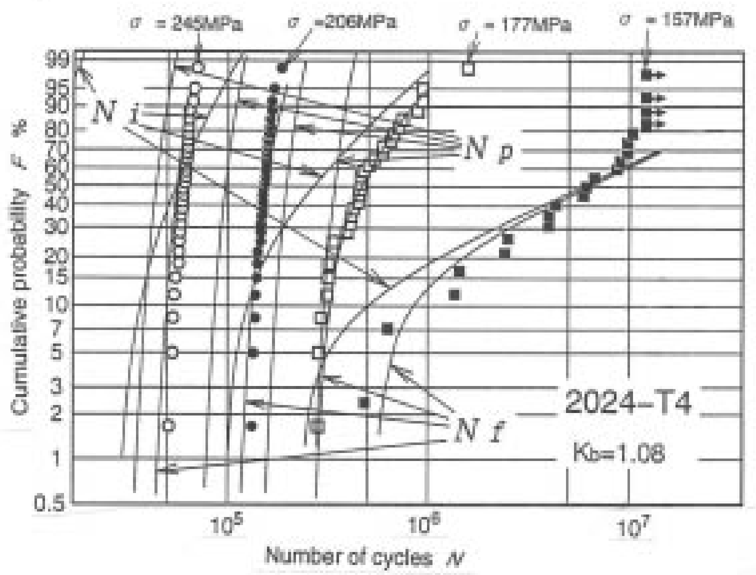
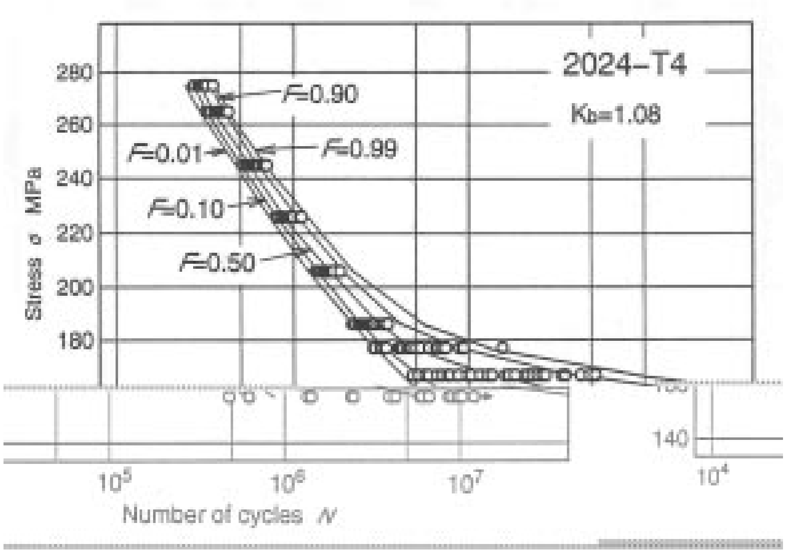
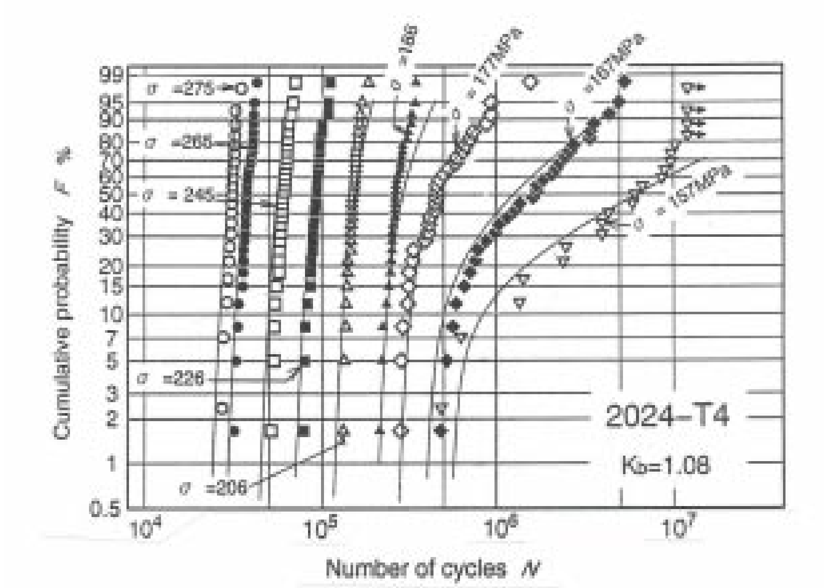


Fig.3 Theoretical and experimental fatigue life distributions at several stress levels.

In actual, fatigue test data obtained at nine stress levels were extracted from the database of MSDRD. All the data were plotted as *S-N* diagram in Fig.4(a) and as Weibull plot in Fig.4(b) together with the theoretical distributions of N_f .



(a) *P-S-N* diagram



(b) Fatigue life distributions

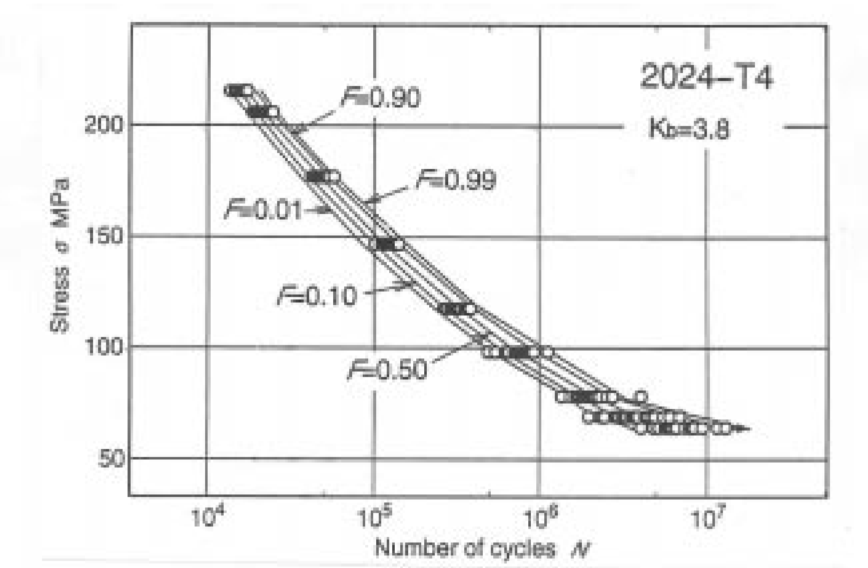
Fig.4 Experimental and analytical results of statistical fatigue property for 2024-T4 $(K_b = 1.08)$.

In Fig.4(a), scatter band of the fatigue life distributions remains narrow in the stress levels higher than 180MPa, but it tends to increase distinctly as the stress level comes down to $\sigma < 180$ MPa. This typical aspect is again observed on the distribution pattern of N_f in Fig.4(b), in which the distribution curves at such low stress levels tend to deflect toward the longer life side at a certain probability level. The probability level to appear the deflection tends to decrease with a decrease of the applied stress level. Theoretical fatigue life distributions calculated by Eqs.(7) and (8) are in excellent agreement with the experimental distributions in a wide stress range. Based on the distribution curves in Fig.4(b), one can read the *S-N* relationship for any level of the cumulative probability. *P-S-N* curves thus obtained for five probability levels of $F = 0.01,\ 0.10,\ 0.50,\ 0.90, 0.99$ are plotted in Fig.4(a). Overall trend of these *P-S-N* curves represents well the statistical feature of the experimental *S-N* data in the entire stress range. Thus one can conclude that the characteristic distribution pattern of N_f and its stress dependence are well explained by combining the distribution properties of the crack initiation and propagation lives.

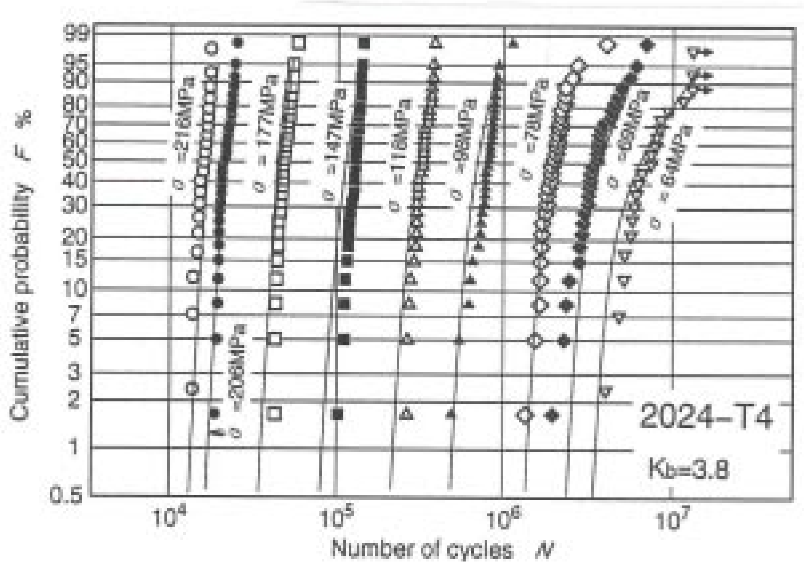
4.2 Analytical Results for 2024-T4 $(K_b = 3.8)$

The crack initiation life of a notched specimen is supposed to be smaller than that of a smooth specimen due to the stress concentration at the notch root. In such a specimen having the sharp notch, the final fatigue life N_f is apt to be governed by the crack propagation life N_p rather than the crack initiation life N_i. From this point of view, probabilistic fatigue properties of notched specimens of the same material(2024-T4) having the stress concentration factor of $K_b = 3.8$ were analyzed by means of the same procedure.

Stress dependence of Weibull parameters α, β, γ for the crack initiation life N_i was similarly assumed



(a) *P-S-N* diagram



(b) Fatigue life distributions

Fig.5 Experimental and analytical results of statistical fatigue property for 2024-T4 $(K_b = 3.8)$.

to give a little short life comparing with that for the smooth specimens. The crack propagation life N_p is governed by parameters in the Paris' law as explained in the previous section. These parameters were assumed here as $m = 4.6$ and $\bar{c} = -12.7$ referring to the databook on the crack propagation behavior of metallic materials[9]. Standard deviation of c was provided as $S_p = 0.04$. The values of m and S_p were fixed to the same values as in the previous analysis, while the other parameter $\bar{c}$ was adjusted as to fit the experimental results and we set the value of $\bar{c} = -12.7$. The initial crack size a_i and the critical size to cause the final failure a_f were given as $a_i = 6.0mm$ and $a_f = 14.0mm$ based on the specimen configuration appearing in the reference.

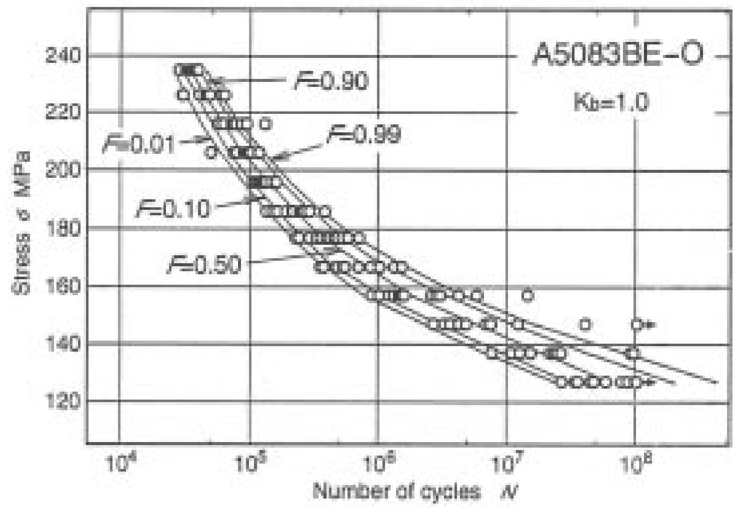
P-S-N curves and fatigue life distributions calculated in the above conditions were plotted by solid curves in Fig.5(a) and (b), where the presentations and calculations are quite similar to the previous results in Fig.4(a) and (b). Theoretical results of the statistical fatigue characteristics thus obtained are in the excellent agreement with the experimental results in Fig.5(a) and (b), respectively. A typical trend of the statistical fatigue property in this case is an aspect that the distribution curve of the fatigue life tends to shift almost parallel with one another in the wide range of the applied stress. This is attributed to the fact that distribution characteristics of the crack initiation life give little effect to the final fatigue life distribution even if the stress level comes down. In other words, the fatigue life distribution of the notched specimens having the sharp notch is governed by the crack propagation life as supposed at the beginning of the analysis.

4.3 Analytical Results for A5083BE-O $(K_b = 1.0)$

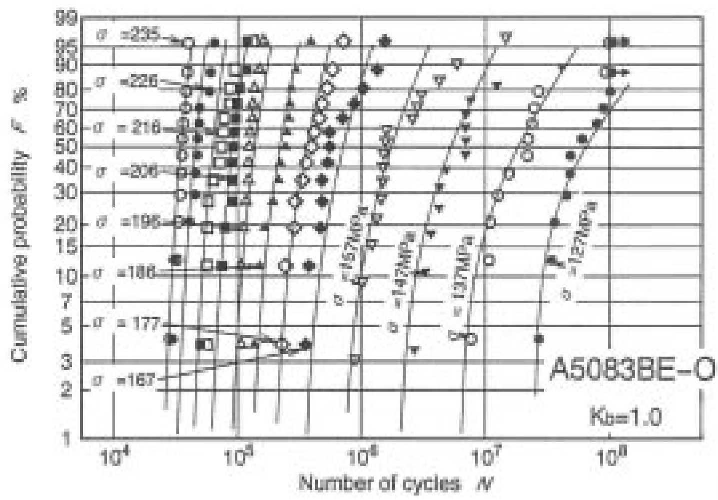
Statistical fatigue property of another kind of aluminum alloy A5083BE-O was similarly analyzed. The stress dependence of Weibull parameters was similarly assumed and the parameters required to the analysis were given as follows;

$m = 5.2$, $\bar{c} = -12.4$, $S_p = 0.06$, $a_i = 0.5mm$, $a_f = 4.5mm$

Analytical results thus calculated are indicated by solid curves in Fig.6(a) and (b) together with the experimental results. Both *P-S-N* characteristics and fatigue life distributions calculated by the present analysis are in good agreement with overall trend of the experimental results. In this case, little deflection of the fatigue life distribution appears on the distribution curves even if the applied stress comes down. This aspect would be attributed to the fact that the crack initiation life for magnesium-aluminum alloy in this section is not so large even for the smooth specimens of $K_b = 1.0$.



(a) *P-S-N* diagram



(b) Fatigue life distributions

Fig.6 Experimental and analytical results of statistical fatigue property for A5083BE-O $(K_b = 1.0)$.

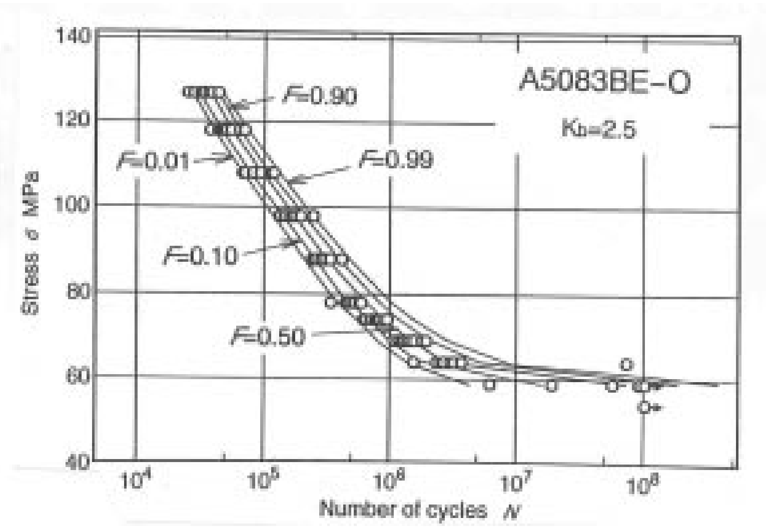
4.4 Analytical Results for A5083BE-O $(K_b = 2.5)$.

Statistical fatigue property for the notched specimens of the same aluminum alloy was also analyzed by using the quite similar procedure. The stress dependence of Weibull parameters in the distribution functions of N_i was similarly given by making reference to the experimental data. A series of other parameters required to the present analysis were provided as follows;

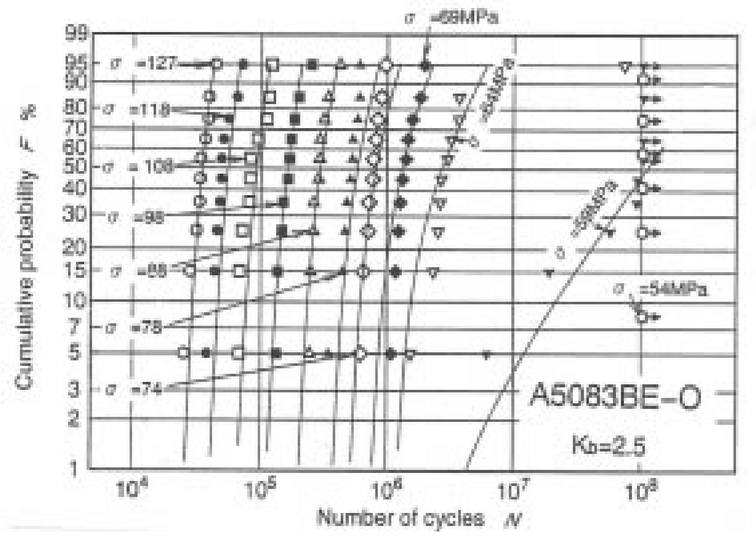
$m = 5.2$, $\bar{c} = -11.2$, $S_p = 0.06$, $a_i = 0.5mm$, $a_f = 4.5mm$

where m, S_p, a_i and a_f were same as the previous calculation, but the value of $\bar{c}$ was a little adjusted as to fit the experimental fatigue life distributions and thus we provided $\bar{c} = -11.2$ instead of $\bar{c} = -12.4$.

Analytical results thus obtained were plotted in Fig.7(a) and (b) together with the experimental fatigue data. *P-S-N* characteristics and fatigue life distributions thus calculated are both in excellent agreement with the experimental results. Another finding in this case is an aspect that the *P-S-N* curves tend to become horizontal rapidly as the applied stress comes down to a critical level. This stress level is supposed to be the critical stress level whether the fatigue crack can take place at the notch root under the cyclic loading.



(a) *P-S-N* diagram



(b) Fatigue life distributions

Fig.7 Experimental and analytical results of statistical fatigue property for A5083BE-O $(K_b = 2.5)$.

5. CONCLUSIONS

(1) A theoretical procedure to analyze the statistical fatigue property of metallic materials was developed by applying the convolution integral for the crack initiation and propagation lives N_i and N_p provided that the final fatigue life was given by $N_f = N_i + N_p$. Applicability of this analysis was confirmed for the fatigue data of aluminum alloys 2024-T4 and A5083BE-O in the database of MSDRD.

(2) In the case of smooth specimens, the crack propagation life N_p is predominant for the final fatigue life at higher stress levels, while the crack initiation life N_i becomes predominant at lower stress levels rather than the crack propagation life. This aspect was well explained by the present analytical model for the statistical fatigue property.

(3) The fatigue crack takes place easily at notch root for the notched specimen having the sharp notch. Accordingly, the crack propagation life N_p becomes predominant for the notched specimens rather than the crack initiation life N_i regardless of the applied stress level. This aspect often reported by many researchers was also successfully explained by the present model.

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