

# Small Crack under Cyclic and Static Loading

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## ABSTRACT

The experimental data on small crack growth (from several shares of millimeter up to several millimeters) in aircraft Al-alloys are presented. The method is proposed to define residual strength for elements with small cracks based on replacement of a real crack by the equivalent through-thickness-crack equal to the real one in the area and it is also based on modifying fracture toughness function limit  $I_c = K_c(\sigma_c)$ . The calculated and experimental residual strength curves have been compared for small cracks at various notches.

The features of initial fatigue crack sites at various types of cyclic loading as well as the feature of small crack growth have been considered. The experimental data of crack growth rate and their scatter are presented for small crack range. The model of submicrocracks growth prediction is suggested. It is based on crack-closure model and change of stress intensity factor threshold ( $K_{th}$ ) in small crack range. The complete fatigue life for “strip with a hole” specimens has been calculated under various load using the suggested model.

## INTRODUCTION

Development of fracture mechanic methods alongside with the more and more increasing requirements to reliability and economic efficiency of modern designs has now given a significant push to research of fatigue initial stage and development of more perfect approaches to its description. Small crack behaviour analysis at crack length from the decimal parts of a millimeter to a few millimeters became the necessary part in the study of fracture mechanics and fatigue life predictions. Some typical properties of small cracks different from those of well-investigated long cracks can be noted:

- the growth of small cracks occurs within the limits of one or several grains, that defines their dependence on microstructural parameters of a metal
- if the cracks are formed at notches their growth occurs in plastic deformations zone due to these notches
- as a rule there are some crack initiation sites that results in occurrence of a grid of fatigue microcracks
- small crack cannot be completely defined by one parameter, i.e. by length; crack front shape should be accounted contrary to the long primary crack
- small size of microcracks requires the development of special means and methods of control even in laboratory conditions.

Therefore the task of structural element crack resistance determination without simplifying and any assumptions is difficult to solve. It is important for practical applications that the approaches used for calculation of both small and long cracks were grounded on one methodology and used whenever possible same characteristics. Figure 1 shows the photos of the typical cracks initiation sites.

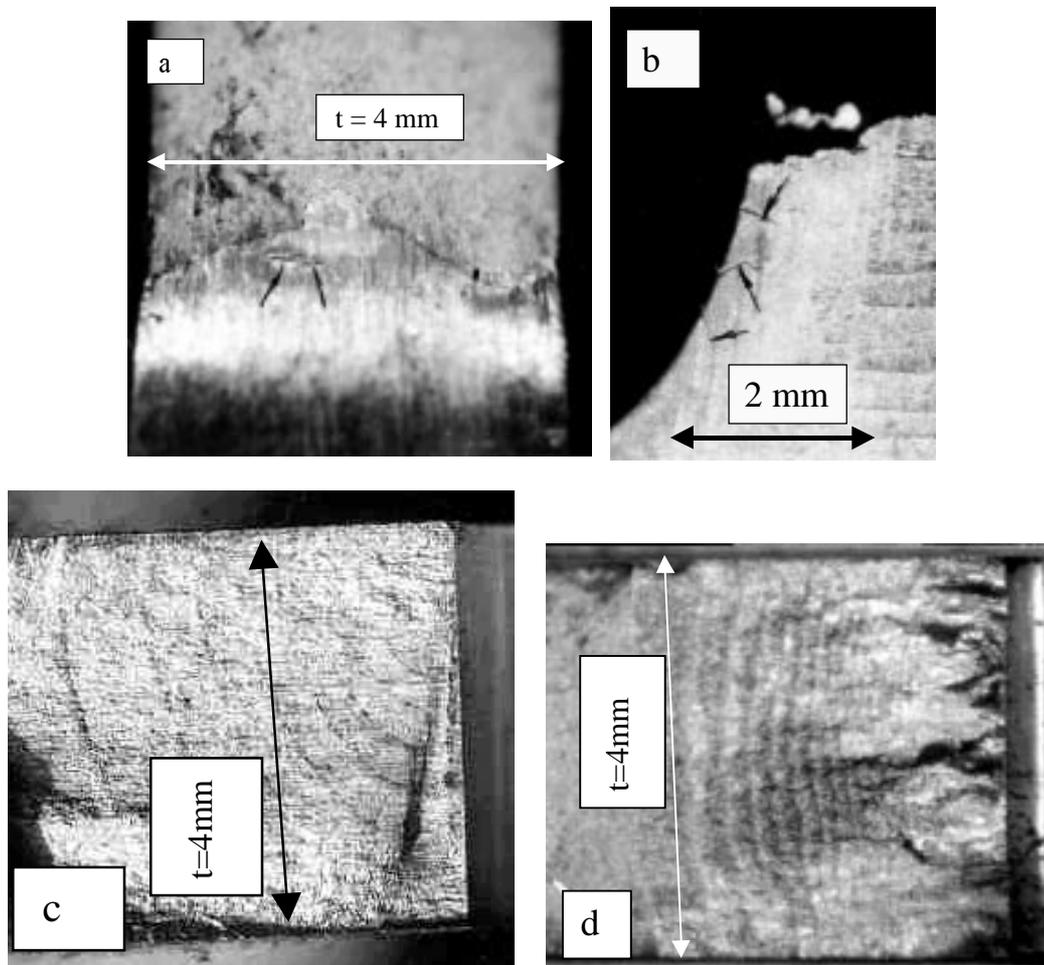


Figure 1: Initial fatigue crack growth. a), b) - multiple crack sites at the notches; c), d) – crack growth under constant (c) and random (d) loading.

A method for determining crack resistance of plain Al-alloy structural elements with small cracks is given here based on stress intensity factors (SIF) and material properties, taken from standard tests. Some aspects of this approach dealing with the small crack under static and cyclic loading are presented earlier by Zverev et al [1, 2]. In addition to the problems discussed here some data dealing with crack model presentation and with  $v - \Delta K$  curve approximation for submicrocracks and complete fatigue life prediction by small crack model are considered.

## STATIC LOADING

Small crack shape analysis based on test results of Al-alloy specimens with different geometric stress concentrators ( $K_t = 1-6$ ) shows that the crack shape with stress concentrator at  $K_t < 2.5$  is nearly angular covering a quarter of a circle. It should be noted that one or two cracks usually initiate near notches. The number of fatigue crack initiation sites increases at  $K_t$  increase and finally these cracks can be considered a through-thickness crack at large enough  $K_t$  ( $K_t > 5$ ) (see Figure 1). Similar changes take place at the transition from regular to random loading.

To unify the description of small crack sizes at crack resistance analysis it is proposed to simulate the actual crack (or cracks) as a through-thickness one equal to the actual crack in area [1] (see Figure 2). The adequacy and assumptions of the proposed approach is considered using the analysis of criteria relation for the body with a crack of arbitrary configuration written down in energetic form [3]:

$$d(U+H-W)/da = 0 , \quad (1)$$

where  $U$ ,  $H$ ,  $W$  are, respectively, the body strain energy, the of work external forces and the energy of crack surface forming.

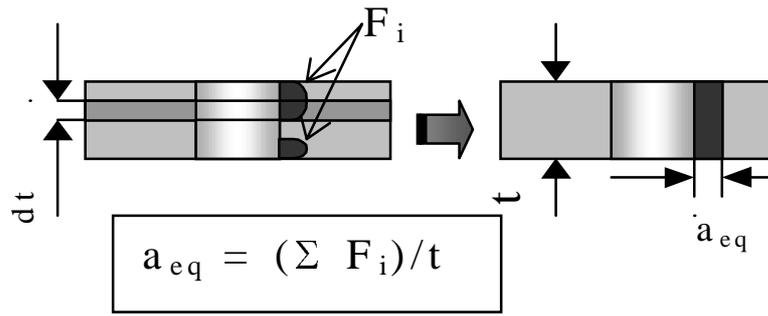


Figure 2: Simulated presentation of small cracks.

Dividing the body with a crack into single thickness layers (see Figure 2) and accounting for the following relationships from Broek [4]

$$d(U^* + H^*)/da = (\pi \cdot \sigma_c^2 \cdot h \cdot f_c)/E \text{ и } dW^*/da = \gamma ,$$

where  $U^*$ ,  $H^*$ ,  $W^*$  are the respective energies per thickness unit;  $\gamma$  is specific density of fracture surface formation energy ( $\gamma = K^2/E$  in the case of the combined plain stress state);  $h$  is a crack front point coordinate;  $f_c$  is the function including the layer interrelation;  $\sigma_c$  is failure stress.

The criteria relation (1) can be written as

$$(\pi \cdot \sigma_c^2 / E) \cdot \int_0^t (h \cdot f_c) dt - \gamma \int_0^t dt = 0 \quad (2)$$

Assuming that single layers affect each other insignificantly due to extensive plastic area near the crack front the function  $f_c$  is considered constant in terms of the specimen thickness and equal to 1. Equation (2) can now have a form of

$$(\pi \cdot \sigma_c^2 / E) \cdot \sum F_i / t - \gamma = 0, \quad (3)$$

where  $\sum F_i$  is cumulative crack area.

Equation (3) is similar to the criteria relation for the body with a through-thickness-crack of length  $a_{eq}$  outlined from equation (1) [4]

$$(\pi \cdot \sigma_c^2 / E) \cdot a_{eq} - \gamma = 0$$

The multiplier  $\cdot \sum F_i / t$  in the left-hand part of relation (3) may be considered as the through-thickness-crack reduced length  $a_{eq}$ . The test results confirming the opportunity of crack area usage as damage measure are presented in Figure3.

Each point here coincides with two specimens carrying the same failure load, but having different crack shapes. The crack shape after the crack growth before the failure moment is also shown in this figure (white zone), that is at different initial crack shapes the cracks have similar shapes after the growth before the failure and propagate through the whole specimen thickness.

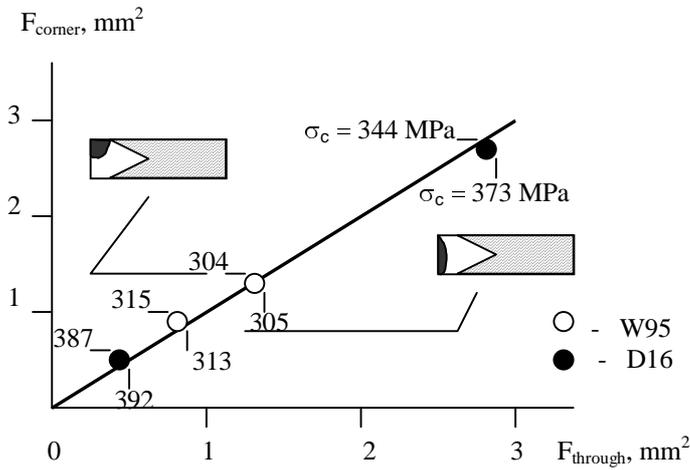


Figure3: Area relation of through-thickness and corner cracks at the near  $\sigma_c$  values

To describe the critical failure curve at small cracks the function of crack resistance limit  $I_c$  introduced by Morozov [5] should be used:  $I_c = K_c(\sigma_c)$ , that according to Zverev at all [1] be written as

$$I_c = K_c (1 - (\sigma_c / (Y \cdot \sigma_b))^2)^{0.5}$$

where  $I_c$  is a fracture toughness function limit (limit values  $K_c$  complex);  $Y = [1 - (1 - \sigma_b / \sigma_o^c) / (1 + 1/\rho)^2]^{-1}$  is the correction for the presence of stress concentrator;  $\sigma_b$  is ultimate strength;  $\sigma_o^c$  is failure stress of element without a crack ;  $\rho$  is radius at the stress concentrator tip;  $K_c$  is critical value of stress intensity factor at common specimen test with large cracks and  $2a/B = 1/3$ ;  $B$  is specimen width.

The design critical failure curves for various types of notches are shown in Figure 4.

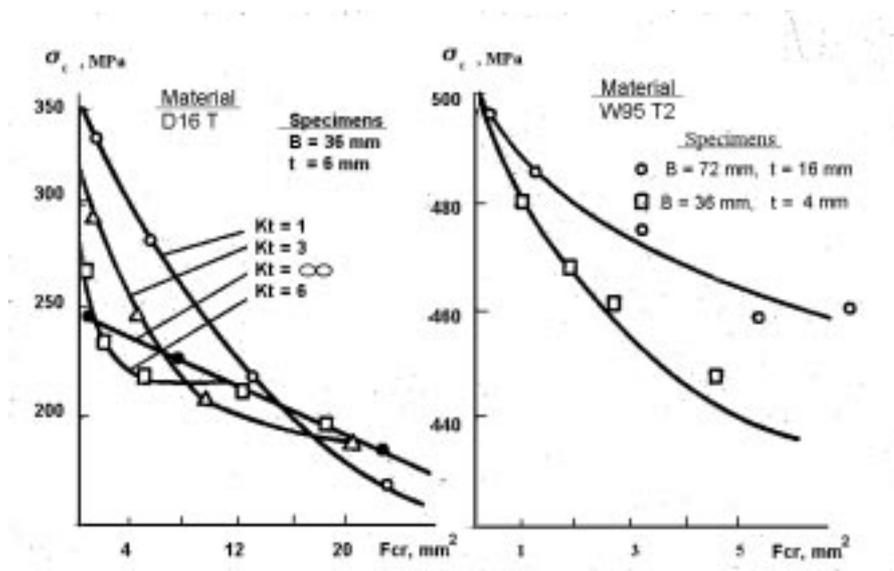


Figure 4: Experimental (points) and calculated (lines) failure stress for specimens with notches. ( $t$  is thickness of specimens,  $F_{cr}$  – cracks area).

While calculating failure strength curves all necessary parameters were taken from standard fracture toughness tests.

## CYCLE LOADING

The simulation of small crack shapes in through thickness ones was carried out for the description of their growth rate. The  $v - \Delta K$  curve for small cracks at the hole is shown in Figure 5 [2]. The experimental data of small crack growth were obtained by eddy current facility [2].

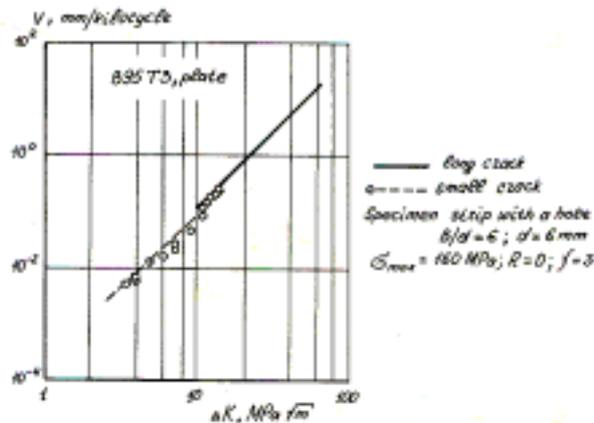


Figure 5: Long and small crack growth rates.

The method of crack length record during the cyclic loading (from  $F_{cr} = 0.05 \text{ mm}^2$ ) is described by Zverev et al. [2]. The comparison of crack growth rates for small and long cracks shows that  $v - \Delta K$  curve for small cracks is considered as linear continuation of the middle part in the relationship  $v - \Delta K$  for long cracks. This gives an opportunity to use the results of standard tests for the evaluation of small crack growth rate at  $F_{cr} = 0.05 \text{ mm}^2$ .

The use of a tool means at small cracks growth tests [2] has enabled to receive the statistical characteristics of their growth rate [6]. Table 1 presents the scatter of small crack growth rates and parameters of Paris equation ( $m$  and  $C$ ) for small cracks in four Al-alloys.

TABLE 1  
Parameters of crack growth rate ( $C$  and  $m$  in Paris equation,  $t_{spec}$  – specimen thickness).

Material	$t_{spec.}, \text{MM}$	$c \cdot 10^{-9}$	$m$	$c \cdot 10^{-7}$	$m$	$S_{lgv}$ at $\Delta K=5$ $\text{MPa} \cdot \text{m}^{1/2}$
		corner		through		
AK4-1 T1 (plate)	4	3.1	4.37	2.2	2.49	0.08
W95 T2 (plate)	4	4.9	4.02	3.5	2.32	0.03
1973 T2 (plate)	4	2.4	4.25	2.5	2.41	0.04
D16 T(extrusion)	5	13.7	3.58	5.3	2.13	0.06
1163 T (sheet)	2	1.3	4.41	1.6	2.51	0.05
W93 T2(forging)	4	2.5	4.35	2.8	2.47	0.07

However the life prediction of small crack growth for smaller  $\Delta K$  using equivalent areas model has not demonstrated enough steady results. It is necessary to use in this area more complex representations about behaviour of small crack. The approach allowing to present the known assumptions as a model of submicrocracks growth and finally to predict complete fatigue life by through fracture mechanics is suggested below.

The basis of suggested model is crack-closure model develop by Newman [7]. According to this model the real scope of SIF in a cycle of loading is replaced by the effective one.

$$\Delta K_{ef} = (K_{max} - K_{cl}) \quad \text{at} \quad K_{min} < K_{cl} \quad (4)$$

where  $K_{max}$ , and  $K_{min}$  - maximal and minimal values of SIF in a loading cycle respectively;  $K_{cl}$  – SIF of crack opening start.

The expression for calculating  $K_{ef}$  and  $K_{cl}$  depending on asymmetry  $R$ , stress strain type and load level for the large cracks is given in [8]. For small cracks the dependence of  $K_{cl}$  or crack opening stress ( $S_{cl}$ ) from the length of a corner small crack is corrected by data from [9]. The form of correction is illustrated by an example of the data for 7075-T6 material in Figure 6.

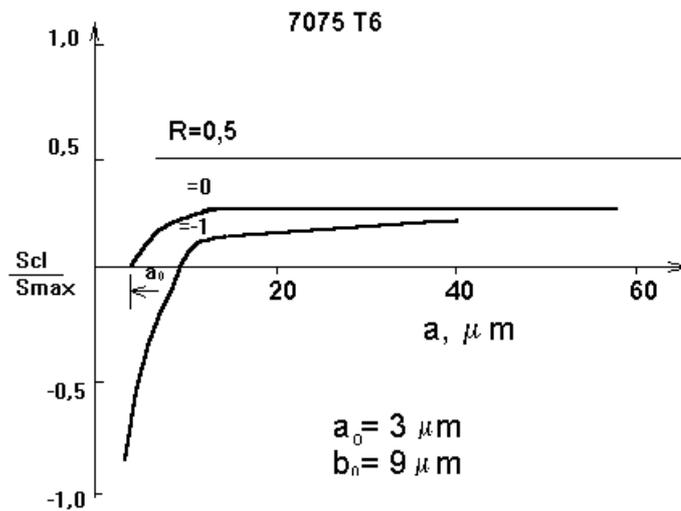


Figure 6: Crack-opening stresses for constant amplitude loading as a function of corner crack lengths [9]. ( $a_0$  and  $b_0$  are initial sizes for elliptic crack shape)

Besides the suggested model takes into account the feature of  $K_{th}$  reduction at the decrease in small crack length [2, 10]. The dependence of  $K_{th}$  from small crack length is illustrated in Figure 7.

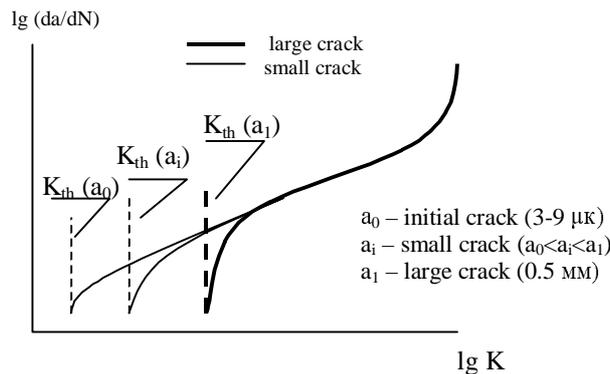


Fig. 7 Dependence of  $K_{th}$  on small crack length  $a$ .

Here the limits of SIF  $K_{th}$  change for small cracks are the following. The minimal value of  $K_{th}$  will be determined as

$$K_{th}^{(min)} = Se \cdot K_q(a_0),$$

where  $K_q(a_0)$  is SIF at unit load for a crack length  $a_0$ ;  $a_0$  is initial crack length.

The maximal value of  $K_{th}^{(max)}$  is equal to  $K_{th}$  at standard tests with a crack length  $a > 0.5$  mm.

Thus the basis of the suggested model for submicrocrack growth rate includes the following positions:

- $v$ - $\Delta K$  curve for small cracks is defined from  $v$ - $\Delta K$  standard test data in view of  $K_{th}$  change depending on crack length. For example according to Collipriest equation:  $C \cdot (\lg(K_{th}(a))/\lg(K_c/K_{ef}))^m$ , where  $m$  and  $C$  – parameters,  $K_{th}(a)$  is function of  $K_{th}$  related to  $a$ .
- $K_{th}$  varies for small cracks with changing crack length from 0.5 mm till  $a_0$  (see Figure 7)
- The  $K_{ef}$  value is determined by equation (4) and by  $K_{cl}$  dependence from crack length (see Figure 6).

The presented algorithm is realized in a special program SCG. Some examples of complete fatigue life using SCG program are given below.

Figure 8 shows the calculations and experimental data on fatigue life for the “strip with a hole” specimens. The initial data for the calculations are taken from [7]

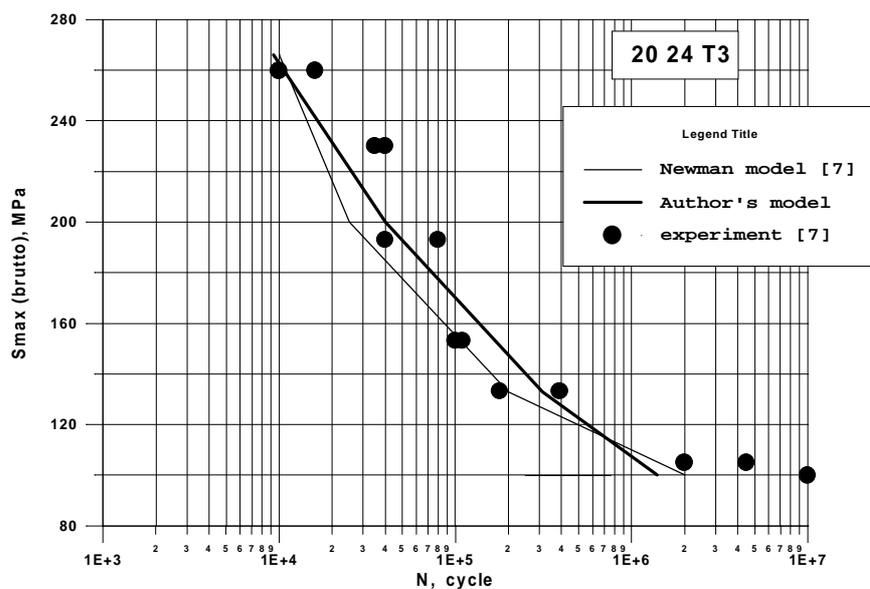


Figure 8: S – N curves for 2024 T3 calculated using small crack model and experimental data. (R = 0)  
 “Strip with a hole” specimens: thickness is 2.3 mm, width is 25.4 mm, hole diameter is 3.2 mm..

The initial corner crack length was assumed equal to 6  $\mu$ m [7]. Other parameters of suggested model (calculated by SCG) and  $v$ - $\Delta K$  curve are also determined based on [7] data.

The complete fatigue lives of “strip with a hole” specimens - (width – 36 mm, thickness - 4 mm and hole diameter – 6 mm) have been also calculated for two materials: D16T sheet and 1163T plate. The results are shown in Figure 9. The initial corner crack length was also assumed equal to 6  $\mu$ m.

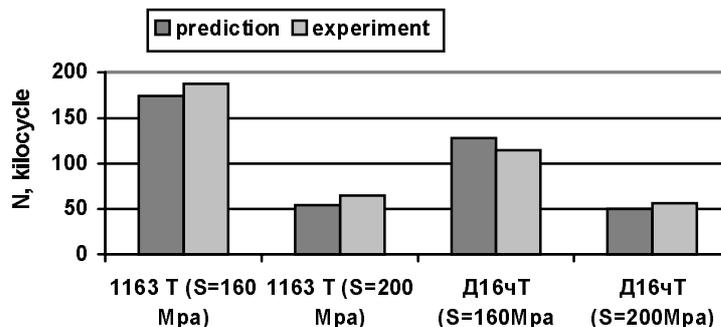


Figure 9: Calculated and experimental fatigue lives for the specimens “strip with a hole” at R = 0.

The received results testify to enough good correlation of the suggested model with the experimental data. Thus the basic parameters of the model were determined basically from standard tests.

## CONCLUSION

1. It is rather difficult to predict accurately the shape, the quantity of initial fatigue defects and behaviour of small fatigue cracks. It is necessary to assume the appropriate simplifications for engineering estimations.
2. The model of equivalent areas can be applied for estimations of residual strength.
3. The crack growth rate calculations for crack length equal to 0.2 – 0.5 mm in sheet elements can also be based on equivalence of the areas. So the  $v$ - $\Delta K$  curve of these equivalence through-thickness-cracks is the continuation crack growth diagram of long cracks.
4. For the description of submicrocracks or complete fatigue life a more completed method based on a) crack-closure model develop by Newman [7] and b) model of  $K_{th}$  change depending on crack length is used.

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## ACKNOWLEDGEMENT

This work has been supported by the International Science and Technology Center, Project #808-99

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