SIMULATION OF THERMOMECHANICAL BEHAVIOUR OF BIMATERIAL CONTAINING AN INTERFACE CRACK WITH ROUGH FACES

A. Kryshtafovych, R. Martynyak, S. Nagalka

Institute for Applied Problems of Mechanics and Mathematics of National Academy of Sciences of Ukraine, Lviv, Ukraine

ABSTRACT

The paper deals with the bimaterial containing interface crack closed under an applied load. Thermal contact of the flaw's faces is non-ideal due to their roughness. Corresponding problem of heat conduction and thermoelasticity is formulated. The statement takes into account dependence of contact thermal conductivity of the closed crack on the contact pressure of its faces and thermal conductivity of a medium filling the cavity. The methods of complex potentials and singular integro-differential equations are employed to solve the problem. Numerical results are illustrated in graphs. They show dependence of a temperature jump between the faces and stress intensity factors on parameters of surface roughness and thermal and mechanical load applied.

INTRODUCTION

Roughness is a feature of surfaces of inner flaws [1]. Its influence is especially of a great weight if the applied load governs flaws' closing. Then surface microroughness results in a discreteness of contact and real contact area depends on the contact pressure and changes with the load changing. Influence of phenomena that are connected with this fact on strength of materials being under a mechanical loading was a subject of a series of both theoretical and experimental papers (see, e.g. [2-4] and the literature cited there). If the body is under a thermal load, discreteness of mechanical contact of surfaces results in an imperfectness of thermal contact [5], which is characterized by either a thermal resistance or thermal conductivity - the quantity which is the reciprocal of the thermal resistance. A presence of contact thermal resistance causes a temperature jump between the conjugated surfaces.

As it was revealed in [6], an essential dependence of thermal resistance on contact pressure affects parameters of thermoelastic interaction of bodies. But thermally induced stresses and thermal strength of bodies containing closed cracks with the thermal resistance depending on the applied load have not been studied in the literature yet. In the paper [7] it was investigated thermal stresses in bimaterial containing closed interface crack with the thermal resistance independent of pressure. Formation of interface plastic strips near the tips of the aforementioned crack has been studied in [8]. It was shown that thermal resistance of the interface crack can provoke its opening [9]. Research in the branch of thermoelasticity of bodies with opened cracks filled by a heat conducting medium was pioneered in the paper [10] and developed in [11]. In a series of publications [11-14] it was analyzed a thermostressed state of the flawed structures possessing both zones of direct contact and contact fault. Our paper is aimed to study thermal stresses in the neighborhood of the crack with rough surfaces that is closed under the applied load. It is employed the model of imperfect thermal contact of crack faces allowing the dependence of its thermal conductivity on a contact pressure of faces and thermal conductivity of the medium filling the cavities between contacting microunevenness. The solution to the problem is presented through the jumps of temperature and tangential displacements of crack's faces. To evaluate these jumps a system of integro-differential equations is obtained. Temperature of surfaces and stress intensity factors are determined. It is revealed new phenomena caused by imperfectness of thermal contact of the crack faces.

STATEMENT OF THE PROBLEM

Let us consider a 2a-length crack located at the interface of two half-planes D_1 and D_2 (Fig. 1). Uniformly distributed compressive load p and heat flow q are applied at infinity. Compressive load causes crack's closure. We assume that mechanical contact of crack faces is frictionless and thermal contact is imperfect due to microunevenness of faces and contact discreteness. According to the theory of contact heat exchange [5] we describe crack's thermal conductivity by a function $\lambda_n(x)$ such that

$$\lambda_n(x) = \lambda_c + KP(x). \tag{1}$$

Here the first term λ_c allows thermal conductivity of crack's filler. The second term $KP(x)$ describes increasing of conductivity with increasing of contact pressure $P(x)$ of crack's faces. Coefficient $K > 0$ depends on mechanical and thermophysical properties of the material as well as on the geometric characteristics of the faces.

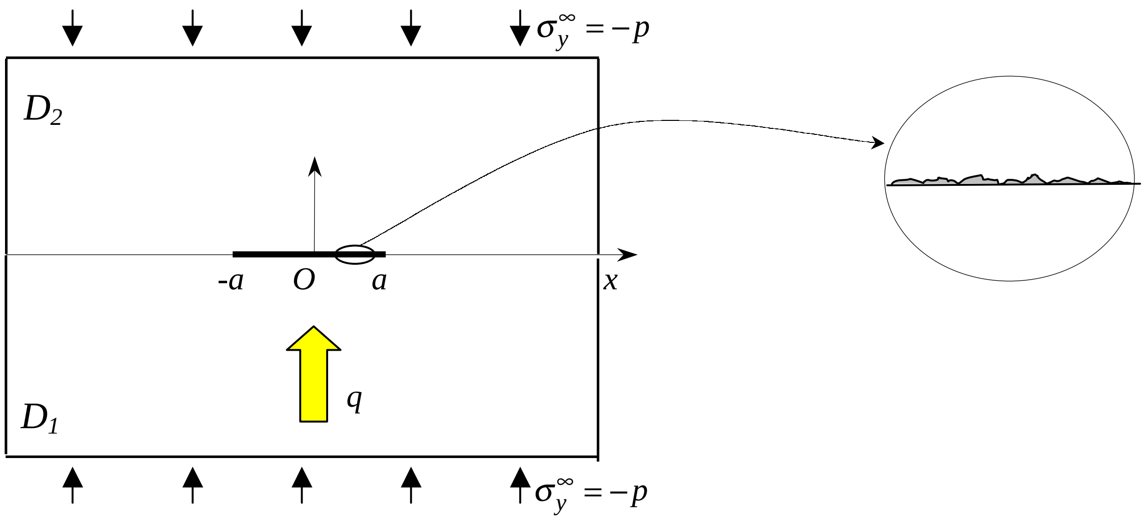

Figure1: Model of the problem

Ideal thermal and mechanical contact of the half-planes is in forth outside the crack.

Boundary conditions of the problem are the following:

— in regions of ideal contact outside the crack ($y = 0$ and $|x| > a$)

$$T^+ = T^-,\ q_y^+ = q_y^-,\ \sigma_y^+ = \sigma_y^-,\ \tau_{xy}^+ = \tau_{xy}^-,\ \ u^+ = u^-,\ v^+ = v^-; \tag{2}$$

— in a zone of crack's occupation ($y = 0, |x| \le a$)

$$q_y^+ = q_y^-,\ \ \lambda_n(x)(T^- - T^+) = q_y^+,\ \ \sigma_y^+ = \sigma_y^-,\ \ v^+ = v^-,\ \tau_{xy}^{\pm} = 0 \tag{3}$$

— at infinity

$$q_x = 0, \quad q_y = q, \quad \sigma_y = -p, \quad \tau_{xy} = 0\,. \tag{4}$$

Here T is a temperature; q_x, q_y are components of heat flow vector; $\sigma_x, \sigma_y, \tau_{xy}$ are components of a stress tensor; u, v are components of displacement vector. Superscripts "+" and "−" denote boundary values of functions at the interface in the half-planes D_1 and D_2 respectively.

METHOD OF SOLUTION

In order to solve the problem let us present temperature, heat flows, displacements and stresses inside both the upper and the lower half-plane through the Muskhelishvili complex potentials [15] in the form [7]

$$T = \mathrm{Re}\left[F_k(z) + \frac{iqz}{\lambda_k}\right], q_x - iq_y = -\lambda_k F_k'(z) - iq, 2G_k(u' + iv') = \kappa_k \Phi_k(z) + \Phi_k(\bar{z}) - (z - \bar{z})\overline{\Phi_k'(z)} + \beta_k F_k(z),$$
$$\sigma_x + \sigma_y = 4\,\mathrm{Re}[\Phi_k(z)] - p,\ \sigma_y - i\tau_{xy} = \Phi_k(z) - \Phi_k'(\bar{z}) + (z - \bar{z})\overline{\Phi_\kappa'(z)} - p,\ z \in D_k, k = 1,2, \quad (5)$$

where $F_k(z), \Phi_k(z)$ are piece-wise holomorphic functions in the whole plane (line $y = 0$ is the jump line for them). Using boundary conditions of the problem we can present these functions in terms of jumps of temperature $\gamma(x)$ and shear displacements $U(x)$ of crack's faces:

$$F_k(z) = -\frac{\lambda}{2\pi i\lambda_k}\int_{-a}^{a}\frac{\gamma(t)dt}{t - z},\ \Phi_1(z) = -\Phi_2(z) = \frac{(-1)^k G_1 G_2}{G_{*k}}\left[\frac{\beta_k}{G_k}F_k(z) + \frac{1}{\pi i}\int_{-a}^{a}\frac{U'(t)dt}{t - z}\right], z \in D_k, k = 1,2, \quad (6)$$

where $\gamma(x) = T^- - T^+$, $U(x) = u^- - u^+$; $G_{*1} = G_1 + G_2\kappa_1, G_{*2} = G_2 + G_1\kappa_2$; $\kappa = 3 - 4\nu, \lambda = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$, λ_k is the coefficient of heat conductivity; ν is Poisson's coefficient; α is the coefficient of linear thermal expansion; $\beta = 2\alpha(1 + \nu)G$. Index k indicates that the quantity is related to the half-plane D_k $(k = 1,2)$.

Presentations (5),(6) satisfy all boundary conditions of the problem except for conditions $(3)_2$ and $(3)_5$. Satisfying these equalities yields a system of singular integro-differential equations in functions γ and U:

$$(\lambda_c + KP(x))\gamma(x) - \frac{\lambda}{2\pi}\int_{-a}^{a}\frac{\gamma'(t)dt}{t - x} = q, |x| \le a, \gamma(\pm a) = 0, \quad (7)$$

$$\int_{-a}^{a}\frac{U'(t)dt}{t - x} - \frac{\lambda\eta_{12}}{G_*^+}\int_{-a}^{a}\frac{\gamma(t)dt}{t - x} = 0, |x| \le a, U(\pm a) = 0. \quad (8)$$

Solving equation (8) we can represent function $U'(x)$ through the function $\gamma(x)$:

$$U'(x) = \frac{\lambda\eta_{12}}{G_*^+}\left[\gamma(x) - \frac{\Gamma}{\pi\sqrt{a^2 - x^2}}\right], \Gamma = \int_{-a}^{a}\gamma(t)dt, \quad (9)$$

where $G_*^+ = G_{*1} + G_{*2}, \eta_{12} = \eta_1 G_{*2} + \eta_2 G_{*1}, \eta_k = \alpha_k(1 + \nu_k)/\lambda_k$, $(k = 1,2)$.

As one can see from formulae (5), (6), (9), temperature field and stressed state of bimaterial are completely defined by a single function $\gamma(x)$. In particular, a contact pressure of crack's faces is in the form:

$$P(x) = p - \frac{\lambda G_1 G_2}{G_*^+}\left[2\eta^-\gamma(x) - \frac{G_*^-\eta_{12}\Gamma}{G_{*1}G_{*2}\pi\sqrt{a^2 - x^2}}\right], \eta^- = \eta_2 - \eta_1, G_*^- = G_{*2} - G_{*1}. \quad (10)$$

Thus, the problem has been reduced to evaluation of the function $\gamma(x)$ from the equation (7), where $P(x)$ is in the form (10).

As one can see from the formula (10), the contact pressure of crack's faces qualitatively depends on the relationship between thermal and elastic parameters of materials. Specifically, it can be available such a combination of these parameters that $P(x)$ becomes negative and this will point on the crack opening. Therefore, the consequent investigation should be carried out separately for the each range of material parameters, where contact pressure $P(x)$ qualitatively behaves in the similar manner, i.e. ensures contact of crack faces. Below we consider two certain examples, namely cases of mechanically different and identical materials.

EXAMPLES AND NUMERICAL RESULTS

Case 1. Different shear moduli of the half-planes.

Let us study interaction of two half-spaces possessing different shear moduli ($G_1 \neq G_2$) but the same Poisson's coefficients and thermal characteristics $\nu_1 = \nu_2 = \nu, \alpha_1 = \alpha_2 = \alpha, \lambda_1 = \lambda_2 = \lambda$. Suppose that a medium is absent inside the crack ($\lambda_c = 0$) and its thermal conductivity depends on contact pressure of crack's faces only ($\lambda_n(x) = K\,P(x)$). The materials are assumed not to be under an action of any external mechanical load ($p = 0$) but subjected to the heat flow q at infinity. Then the expression for the pressure of crack's faces is in the form:

$$P(x) = \frac{G_1 G_2 (1+\nu)\alpha \bar{G}_*}{G_{*1} G_{*2} \pi a} \Gamma \frac{1}{\sqrt{1-(x/a)^2}}. \tag{11}$$

As it follows from (11), contact pressure of the faces is positive if

$$q(G_1 - G_2) > 0. \tag{12}$$

In what follows we assume that the condition (12) is fulfilled. To this end the heat flow is required to be directed from the material with the greater shear modulus to the material with the less one.

In this case the singular integro-differential equation (7) takes the form

$$\frac{K S_1 \Gamma}{\sqrt{1-(x/a)^2}} \gamma(x) - \frac{\lambda}{2\pi} \int_{-a}^{a} \frac{\gamma'(t)dt}{t-x} = q,\ S_1 = \frac{1}{a\pi} \frac{G_1 G_2 (1+\nu)}{G_{*1} G_{*2}} \alpha \bar{G}_*. \tag{13}$$

The solution to this equation is the following function:

$$\gamma(x) = 4aq\sqrt{1-(x/a)^2} \Big/ \left(\lambda\left(1+\sqrt{1+8a^3 q\pi K S_1/\lambda^2}\right)\right). \tag{14}$$

Taking into account (5), (6), (9), (14) the interface stress intensity factors (SIFs), which are defined by the expressions $k_1^{\pm} = \lim_{x\to\pm a}\left\{\sqrt{2(\pm x - a)}\sigma_y(x,0)\right\}, k_2^{\pm} = \lim_{x\to\pm a}\left\{\sqrt{2(\pm x - a)}\tau_{xy}(x,0)\right\}$, can be rewritten as:

$$k_1^{\pm} = 0,\ k_2^{\pm} = \mu \frac{\lambda(1+\nu)(1+\kappa)(G_1+G_2)G_1 G_2}{G_{*1} G_{*2}\sqrt{a}} \Gamma. \tag{15}$$

Numerical calculation is carried out in dimensionless quantities: $\bar{q}=\frac{q}{q_0}, \bar{\gamma}=\frac{\gamma}{\gamma_0}, \bar{P}=\frac{P}{G_1\alpha(1+\nu)\gamma_0}$, $\bar{k}_2^{\pm}=k_2^{\pm}/[G_1\alpha(1+\nu)\gamma_0\sqrt{a}]$, $\bar{x}=x/a$, $\bar{G}_2=G_2/G_1$, $\bar{K}=[8a^2K\alpha(1+\nu)q_0G_1]/\lambda^2$. Here q_0 is a heat flow of unit density $(q_0=1W/m^2)$, γ_0 is the temperature jump in the center of thermally insulated crack being under the heat flow q_0 at infinity $(\gamma_0=2aq_0/\lambda)$.

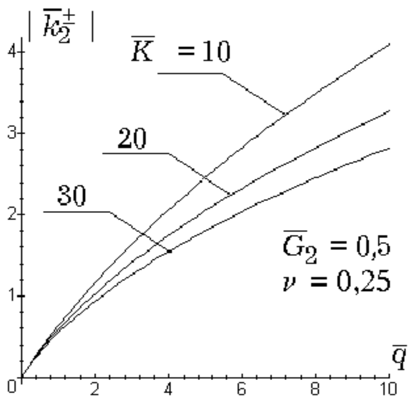

Figure 2: Dependence of the stress intensity factor $\bar{k}_2$ on the heat flow $\bar{q}$

Influence of a heat flow on the SIF $\bar{k}_2^{\pm}$ is shown in Fig. 2 for various values of parameter $\bar{K}$. One can see that $|\bar{k}_2^{\pm}|$ increases with $\bar{q}$ increasing, decreases with $\bar{K}$ increasing and non-linearly depends on $\bar{q}$.

Dependence of $\bar{k}_2^{\pm}$ on parameter $\bar{G}_2$ (from the range $0\le\bar{G}_2\le1$), which characterize respective rigidity of the upper material, is shown in Fig. 3 for various values of parameter $\bar{K}$. SIF $\bar{k}_2^{\pm}$ increases with increasing of $\bar{G}_2$ and decreases with increasing of $\bar{K}$.

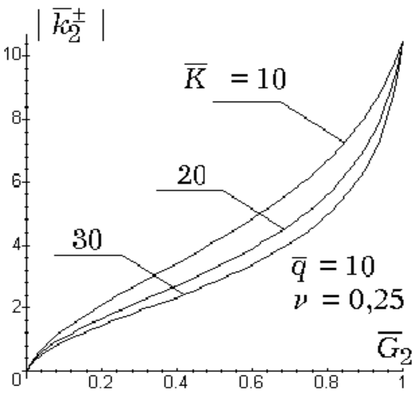

Figure 3: Influence of the relative shear modulus $\bar{G}_2$ of the upper half-plane on the SIF $\bar{k}_2$

Case 2. Identical materials.

Let us consider a crack in a homogeneous material ($\alpha_1=\alpha_2=\alpha$, $\lambda_1=\lambda_2=\lambda$, $\nu_1=\nu_2=\nu$, $G_1=G_2=G$). The crack is closed due to an action of two concentrated forces of intensity P_0 that are applied on opposite sides of the crack in the points $z=\pm id$ and directed perpendicularly to the crack (Fig. 4).

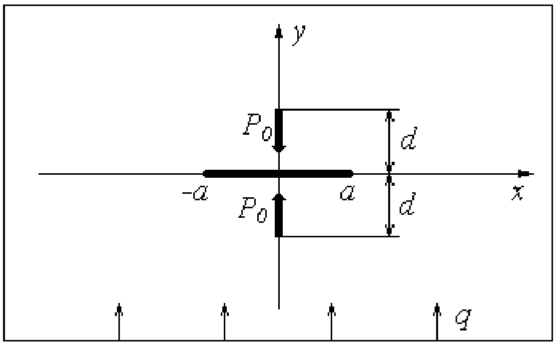

Figure 4: A closed crack in a homogeneous plane

Complex presentations of temperature, displacements and stresses are in the form:

$$T=\mathrm{Re}\left[F(z)+\frac{iqz}{\lambda}\right],\quad 2G(u'+iv')=\kappa\Phi_k(z)+\Phi_k(\bar z)-(z-\bar z)\overline{\Phi_k'(z)}+\beta F(z)-P_d\frac{2(z^2+d^2)-4d^2}{(z^2+d^2)},$$

$$\sigma_x-i\tau_{xy}=\Phi_k(z)-\Phi_k(\bar z)+(z-\bar z)\overline{\Phi_k'(z)}-P_d\frac{(\kappa-1)(z^2+d^2)-4d^2}{(z^2+d^2)^2},\quad \sigma_x+\sigma_y=4\,\mathrm{Re}\left[\Phi_k(z)-\frac{P_d}{z^2+d^2}\right],$$

$$F(z)=-\frac{1}{2\pi i}\int_{-a}^{a}\frac{\gamma(t)dt}{t-z},\quad \Phi_1(z)=-\Phi_2(z)=\frac{(-1)^k}{1+\kappa}\left[\frac{G}{\pi i}\int_{-a}^{a}\frac{U'(t)dt}{t-z}-\frac{\beta}{2\pi i}\int_{-a}^{a}\frac{\gamma(t)dt}{t-z}\right], z\in D_k,\ k=1,2,$$

where $\beta=2\alpha(1+\nu)G,\ \kappa=3-4\nu,\ P_d=P_0 d/\pi(1+\kappa)$.

The temperature jump $\gamma(x)$ can be deduced from the equation

$$\left(\lambda_c+KP_d\frac{(\kappa-1)(x^2+d^2)+4d^2}{(x^2+d^2)^2}\right)\gamma(x)-\frac{\lambda}{2\pi}\int_{-a}^{a}\frac{\gamma'(t)dt}{t-x}=q,\ |x|\le a,\ \gamma(\pm a)=0. \quad (16)$$

Introducing dimensionless functions and constants $\bar\gamma=\frac{\lambda\gamma}{aq},\bar\lambda_c=\frac{\lambda_c a}{\lambda},\bar P_0=\frac{P_0}{aG},\bar K=\frac{KaG}{\lambda},\bar d=\frac{d}{a},\xi=\frac{x}{a},\zeta=\frac{t}{a}$, let us present dimensionless temperature jump as a sum [16]

$$\bar\gamma(\xi)=\sqrt{1-\xi^2}\sum_{m=1}^{M}X_m U_{2(m-1)}(\xi), \quad (17)$$

where $U_m(\xi)=\sin((m+1)\arccos(\xi))/\sqrt{1-\xi^2}$ are the Chebyshov polynomials of the second kind, X_m are the unknown coefficients. Substituting expression (17) into equation (16) yields a functional equation. Satisfying it in M collocation points chosen as zeroes $\xi_k=\cos(k\pi/2M)$ of the polynomial $U_{2M-1}(\xi)$ we arrive at the system of linear equations on the unknown coefficients X_m:

$$\sum_{m=1}^{M}A_{km}X_m=1,\ A_{km}=\left[\left(\lambda_c+\overline{KP_0}\bar d\frac{(\kappa-1)(\bar d^2+\xi_k^2)+4\bar d^2}{\pi(1+\kappa)(\bar d^2+\xi_k^2)^2}\right)\sin\left(\frac{k\pi}{2M}\right)+\frac{2m-1}{2}\right]U_{2(m-1)}(\xi_k),\ k=\overline{1,M}. \quad (18)$$

Grounding on the numerical solution of the system (18) it has been carried out a parametrical analysis of the temperature jump between the crack faces and the SIF $k_2^{\pm}$ for $\nu = 0{,}25$ and $\overline{K} = 2$. Note, that dimensionless SIF is defined as $\overline{k}_2^{\pm} = \frac{1}{2\pi}\int_{-1}^{1}\overline{\gamma}(\xi)d\xi$.

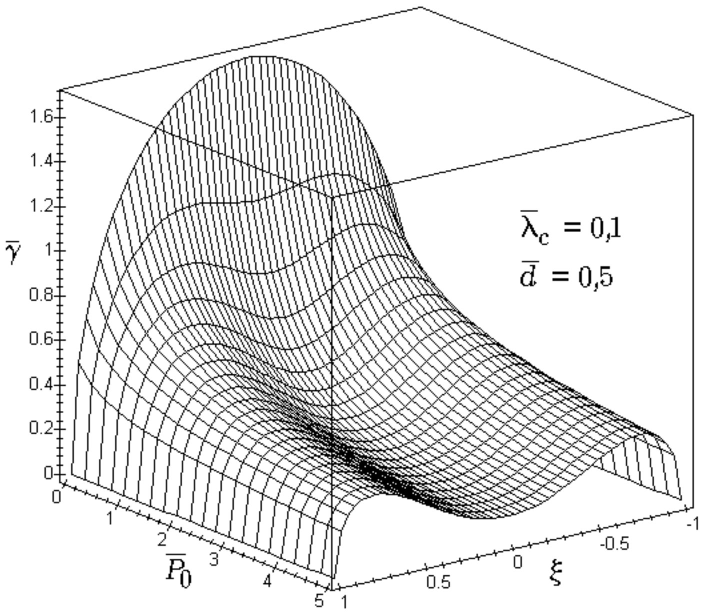

Figure 5: Dependence of the temperature jump $\overline{\gamma}$ on the magnitude of the concentrated force $\overline{P}_0$

Fig. 5 illustrates dependence of temperature jump between crack's faces on intensity of concentrated forces provided the distance from the points of the force applying to the crack is fixed. Increasing of the force results in overall reducing of temperature jump. In so doing the local minimum arises in the center of the crack ($\xi = 0$), where the function $\overline{\gamma}(\xi)$ attains its maximum in the case of load absence. The maximum of the temperature jump $\overline{\gamma}_{max}$ is reached in the vicinity of crack's faces (at the points $\xi \approx \pm 0{,}77$, if $\overline{P}_0 = 5$).

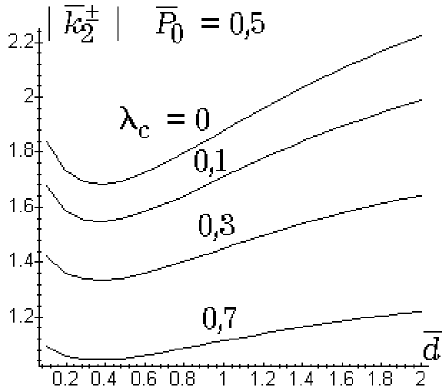

Figure 6: The SIF $\overline{k}_2$ versus the distance $\overline{d}$.

Figure 6 illustrates a relationship between the distance from the points of applying of fixed forces to the crack ($\overline{d} \geq 0{,}1$) and the SIF $\overline{k}_2^{\pm}$ for various conductivities of a medium in the crack. It is easily seen that the SIF reaches its minimum at $\overline{d} \approx 0{,}38$. The greater conductivity the less SIF.

CONCLUSIONS

Imperfect thermal contact of crack's faces, which is caused by their roughness and a presence of a filler inside the crack, induces qualitatively new rules of thermomechanical behavior of the material that are not observed in the case of opened thermally insulated cracks. In particular, temperature field and stress intensity factor k_2 non-linearly depend on the mechanical load normal to the crack, while in the case of opened crack [17] neither T nor k_2 depend on this load. In the case of interface crack the SIF k_2 and temperature jump between the crack's faces are non-linear functions of physical parameters of bimaterial's components and a heat flow. Increasing thermal conductivity of the crack's filler causes decreasing temperature jump between the faces and as a result decreasing stresses and the SIF k_2. The results obtained show that intensity of thermal stresses in bimaterial containing interface closed crack essentially depends on material combination. Thus by appropriate choosing of the bimaterial components it is possible to reduce stresses if necessary. Another way to reduce thermal stresses is to apply mechanical load resulting in a compression of crack's faces.

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