PHENOMENOLOGICAL MODEL OF THE SPECIMEN CONTAINING GROWING CRACK

A.Neimitz

Department of Mechatronics and Machine Design, Kielce University of Technology,
Al.1000 lecia P.P.7, 25-314 Kielce, Poland

ABSTRACT

The aim of the paper is to provide a tool to visualize an influence of the basic geometrical and material parameters on a shape of the force – displacement curves observed during classical fracture experiments performed in order to measure fracture parameters as well as fracture resistance curves. During the process of specimen modeling a classical and modified Dugdale model have been utilized. The analysis although qualitative provides a very interesting information on fracture processes itself and can be a useful tool to “design” experimental program in order to test the fracture resistance of a material to the crack growth.

INTRODUCTION

A nature of physical processes accompanying a stable crack growth has been well understood for most structural materials for years. It is well known that the stable crack growth depends on a variety of material and geometrical factors. They control a nature and extent of dissipative processes during subcritical crack growth. A short but comprehensive discussion concerning this subject is included in Broberg’s book [1] in chapters 1 and 8. However, there is still no reliable theory that could be able to provide a quantitative assessment of the stable crack growth extent and the onset of unstable crack growth. The general structure of the crack growth equation is known (e.g. Neimitz, Molasy [2]). It can be written in terms of the global parameters related to the whole energy dissipated during the crack growth. Such a parameter can be identified with the far field *J* integral that in this case should not be understood as amplitude of a singular HRR field and is not path independent. Except for the far field *J* integral the crack growth equation contains quantity which should characterize material resistance to the crack growth. It should be determined experimentally and it is usually called the *J-R* curve. It turns out however, that the *J-R* curve strongly depends on shape and size of a tested specimen. Thus, the theory although well physically motivated is not attractive from the practical, engineering point of view since the *J-R* curves measured in the laboratory can not be utilized in the practical applications concerning structural or machine members.

Many researchers have tried to find a method of a proper normalization of *J-R* curves (e.g. Turner [3]) to make them geometry or size independent, so far without greater success. Comparison of experimental results obtained by various researchers is often misleading because of different experimental environment and techniques used, as well as different materials or different formulas to compute *J-R* curves. Very often details of experimentation are not available in articles. A wide experimental program to test the influence of size and geometry of the specimen on the *J-R* curves that could lead to a more conclusive results would be a very expensive, time consuming and not necessary successful.

An experimental methodology of the *J-R* curve measurement requires a registration of the force, *P* vs. load point displacement, *u*. Area under the curve provides an information how much energy has been dissipated and how much energy has been stored during the process of the crack growth. In turn, the amount of energies depend on geometry and size of the specimen, the material properties and the crack growth law. Thus, it might be useful to propose a theoretical methodology to compute the $P=f(u)$ curves from a simple model in order to get more information about stable crack growth before an experimental program is undertaken. In this way one may save time and reduce costs of experimental studies in order to predict the influence of various parameters on the *J-R* curves.

In this article such a model is proposed. It does not require complex numerical computations.

DESCRIPTION OF THE MODEL

The general model of the specimen is as follows: it is made of fibers parallel to the direction of external forces. Each fiber consists of segments. Each segment can be modeled by a linear elastic spring placed in series with perfectly brittle element and parallel configuration of linear or non–linear elastic spring and Coulomb friction element. Between fibers "friction forces" should be assumed in order to transmit deformation if it is not uniform. The analysis of deformation of such a specimen requires definition of number of quantities characterizing mechanical elements entering the model and statistical distribution of parameters characterizing onset of plastic deformation and fracture of individual segments. The situation simplifies if in such configuration one can introduce a crack, which cuts a certain number of fibers. In this case the plastic deformation and fracture process become localized in the neighborhood of crack tips. In principle, the plastic deformation in front of the crack tip, the size and the shape of the plastic zone and the shape of the opened crack faces can be known if one knows the solution of the boundary-value problem for given geometry and elastic-plastic material. Such information supplements the model and makes computations much easier.

In this paper the **c**enter-**c**racked specimen in **t**ension (CCT) is assumed although the model can be extended to other configurations. The width of the CCT specimen is $2W$, thickness of the specimen is B, the initial length of the central crack is denoted by $2a_0$, the length of the specimen is L. The coordinate system is located in the center of the specimen in such a way that x-axes is parallel to the crack surfaces and y-axes is perpendicular to the crack surfaces.

According to the model presented, when the CCT specimen is stretched by an external loading the part of the fibers can be totally within the range of elastic deformation and some of the fibers are locally in the state of plastic deformation while the remaining portion of their length is elastically stretched. Finally, some of the fibers are cut by a crack surfaces although they are still stretched by a "friction forces" shaping properly the crack profile.

At the moment there is no solution, in a closed form, of the elastic–plastic problem for a growing cracks for a finite specimens except of the Dugdale model of a crack [4]. We do realize that the Dugdale model of a crack provides a very simplified picture of a real situation. However, it has proven to be an effective tool to characterize *qualitatively* the fracture process. Our goal in this article is to provide such a model that could capture and describe the nature of the basic features of deformation of specimen containing a growing crack. We think, and it will be demonstrated in the subsequent sections of this article, that Dugdale model is a proper tool to reach such a goal.

At this stage of analysis a strain hardening is not introduced into a model.

Basic Relations

In the analysis Dugdale model of a crack [4] has been adopted. Therefore, in principle, the plane stress situation should be discussed only. However, a recent modification of the Dugdale model [5] that takes into account the thickness correction has been adopted, extending analysis to more general cases. Thus, results presented below are not necessarily qualified as a predominantly plane stress cases.

There are a few closed form solutions of the Dugdale problem for finite specimens and some of them can be found in popular handbooks (e.g. [6]). Computations performed in this article have been based on Isida results [6], for a CCT specimen. They have been approximated by closed form formulas. The length of the strip yield zone (*SYZ*), r_p can be computed with a small error (less then 1 per cent) with respect to graphically presented results [6] from relation:

$$r_p = W\left(\frac{a}{W}\right)^{\left(1-\xi^{3.5}\right)} - a\,, \tag{1}$$

where $\xi = \dfrac{\sigma_z}{\sigma_y\left(1 - a/W\right)}$, σ_z is the external traction, σ_y is the yield stress (in the classical Dugdale model it was assumed that material is elastic–perfectly plastic), $2a$ is an actual crack length. The crack tip opening displacement can be approximated by the formula:

$$\delta_T = \frac{2\sigma_y a}{W}\left[\left(\frac{a}{W}\right)^{-\xi^{3.5}} - 1 + 0.1 sign\left(\frac{a}{W} - 0.55\right)\left(\frac{r_p}{a}\right)^2\right], \tag{2}$$

The accuracy of the above formula with respect to Isida results is less then the Eqn. 1. The error is within the range (±7per cent) for $a/W \geq 0.33$. The crack face opening displacement within the strip yield zone $a \leq x \leq (a+r_p)$ or $-(a+r_p) \leq x \leq -a$ can be approximated from a classical solution by a formula:

$$\delta_D(x) = \delta_T\left(1 - \frac{x-a}{r_p}\right)^2. \tag{3}$$

The crack face opening displacement along the crack faces $a \leq |x|$ can be approximated by

$$\delta_s(x) = \delta_0(0) - \left[\delta_0(0) - \delta_T\right]\left(\frac{x}{a}\right)^2, \tag{4}$$

where $\delta_0(0)$ may be found from relation

$$\delta_0(0) = \frac{2\sigma_y}{\pi E}(a + r_p)\cos\theta_2 \ln\left[\frac{\sin\theta_2 + 1}{\sin\theta_2 - 1}\right]^2, \tag{5}$$

where $\theta_2 = \dfrac{\pi P}{4WB\sigma_y}$, P is external loading.

Each elementary fiber in our model transmits load ΔP_i. The resultant force, which is stretching specimen, can be computed as a simple sum of the elementary forces: $P = \sum_{i=1}^{n} \Delta P_i$. In our hypothetical experiment the load point displacement u is controlled and due to the boundary conditions the total elongation of each fiber is the same. Thus, when the plastic zone in front of the crack reaches a fiber being previously in elastic state the plastic deformation must reduce elastic one since $\Delta u = \Delta u_{el} + \Delta u_{pl} = 0$. In our model u_{pl} is identified with δ_D. Similarly, when fiber is cut by a crack faces $\Delta u_{el} + \Delta u_{fr} = 0$ and u_{fr} is identified with δ_S. Consequently the force transmitted by each fiber can be computed from a formula:

$$\Delta P_i = \frac{1}{C_0}(u - \delta(x_i)) \tag{6}$$

where $\delta(x_i)$ should be replaced by $\delta_D(x_i)$ or $\delta_S(x_i)$ depending on the state of a fiber. Integrating forces along the width and thickness of the specimen (Eqns. 2 to 6 have been utilized) one can arrive at:

$$P=\frac{2BW}{C_0}\left[u-\frac{1}{3}\delta_T\frac{a+r_p}{W}-\frac{2}{3}\delta_0\frac{a(u)}{W}\right]H(W-a(u)-r_p)+2B\sigma_y(W-a(u))H(r_p+a(u)-W), \quad (7)$$

where *H(—)* is the Heaviside function, which activates one of two terms within formula. First term is active when elastic zone exists in front of the *SYZ*, the second term is active when the whole-unbroken ligament is in plastic state. In this equation all quantities has already been determined except *a(u).* In principle, the crack extension as a function of a load point displacement should be determined from experiment or from some physically based model. However, in this paper, for a purpose of simulation of *P=f(u)* curves we postulate a general form of *a=f(u)* function:

$$a(u)=a_0+\psi uH(u_i-u)+[\psi u_i+\beta(u-u_i)^{\alpha}]H(u-u_i).. \quad (8)$$

The second term in Eq. (8) represents blunting process, the third - an actual crack growth; α, β, ψ, are some constants (at least in this paper; in general they may be functions of specimen dimensions and material properties), u_i denotes load point displacement at the onset of crack growth. Power α assumes positive values close to 1. It depends on mechanisms of crack growth. Typical experimentally observed[♣] *P=f(u)* curves can be called *pagoda roof* or *round house* (Turner [3]). For *pagoda roof* type curves $\alpha<1$. For *round house* type curves $\alpha>1$. For intermediate shapes $\alpha=1$ (Figure 1a).

Dugdale Model Modification

In his classical model Dugdale assumed that the specimen is in the plane stress situation. He observed in experiment a wedge shaped plastic zone in front of a slit in a thin sheet made of a mild steel. This simple model has been used many times by researchers to discuss various fracture problems. In many cases not necessarily plane stress problems were interpreted using this very useful model. It is easy to show that the *SYZ* in the original model satisfies the Tresca yield criterion for plane stress **only**. The Huber-Mises-Hencky (HMH) criterion is not satisfied for plane stress and for plane strain situation unless one does not assume that stresses within *SYZ* are constant and equal to the yield stress. In the paper [5] author has modified a classical Dugdale model to include other geometrical situations than plane stress. As a one of results of analysis the formula for the level of constant stress distribution within *SYZ* has been approximated:

$$\sigma_{syz}=\sigma_y\phi(\gamma)=\sigma_y\left(e+\frac{f}{1+(\gamma/c)^d}\right) \quad (9)$$

where e=1.081, f=1.734, c=0.0968, d= 0.976 in the case of HMH hypothesis

e=0.999, f=1.564, c=0.102, d= 0.983, in the case of Tresca yield condition, $\gamma=r_p/B$ and r_p can be computed from equation (1) but with the help of iteration procedure (usually two, three iterations are sufficient*)*. With the above modification the specimen thickness can be taken into account in the *P=f(u)* curves simulation in a more appropriate way. Now, the yield stress σ_y should be replaced by σ_{syz} in Eqns (1), (2), (5) and (7).

NUMERICAL RESULTS AND DISCUSSION

As will be shown in this section, many geometrical and material parameters can change the shape of the curves as well as the maximum force and location of this maximum along the *u* axis. However, the most fundamental role in shaping these diagrams plays the crack evolution equation proposed here in this article as an Eqn. (8). In Figure 1a three typical curves are presented: *pagoda roof*, *round house* and intermediate shape. The crack growth equation changes not only the shape of the curves but the areas under the curves as well. They change the maximum force as well as location of this maximum along the *u* axis and location of the moment of crack growth initiation with respect to the location of maximum force. Comparison of the shapes of *P=f(u)* diagrams obtained experimentally and theoretically suggests that the general form of the

[♣] More details concerning the *P=f(u)* curves and associated *a=f(u)* can be found in [7] equation (8) is probably correct. However, an extensive research must be performed to identify the parameters entering this equation. These parameters certainly depend on mechanisms of fracture, material parameters at the micro – meso – macro scale, and geometrical constraints of the fracture process. The fact that three curves in Figure 1a approach almost the same value of displacement *u* when the forces approach zero is quite accidental. In most cases it is not so as shown in Figure 2a. In this plot along with classical Dugdale solution a modified one is presented for a specimen of a thickness *B=5mm.* In Figures 1b and 2b the evolutions of *SYZ's* and crack lengths are plotted. The rapid extension of the *SYZ* takes place until the whole ligament in front of the crack tip is in a plastic state.

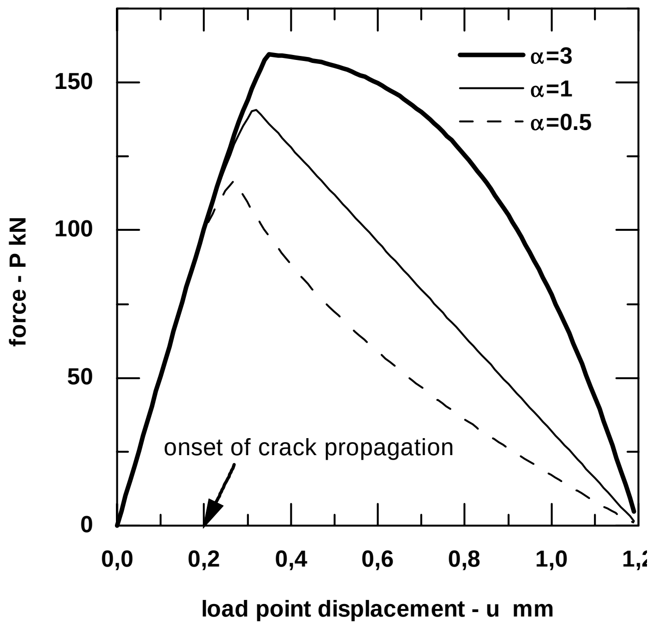

Figure 1a. *P=f(u)* diagrams for different crack growth equations and identical specimens. *B=10mm, L=240mm,* $2a_0$*=20mm, 2W=60mm,* u_i*=0.2mm,* σ_y*=400MPa,* β*=20*

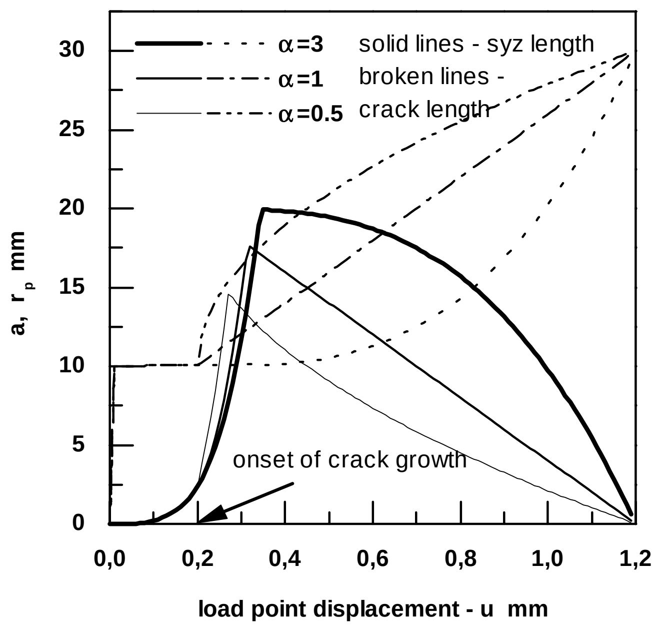

Figure 1b. *a=a(u)* and $r_p=r_p(u)$ diagrams for different crack growth equations and identical specimens. *B=10mm, L=240mm,* $2a_0$*=20mm, 2W=60mm,* u_i*=0.2mm,* σ_y*=400MPa,* β*=20*

Differences in *P=f(u)* plots are not essential for classical and modified Dugdale models. Differences are more pronounced when the length of the *SYZ's* and crack tip displacements are compared [5].

It is the well-known fact that the initial length of a crack strongly influences the shape of the *J-R* curves. Typical diagrams obtained from the model for rather ductile materials *(*α*=2)* are shown in Figure 3. For the same crack growth equation, material parameters and specimen dimensions the character of the curves is preserved, their shapes change as well as initial compliance of the specimen. The observed plateau is due to assumed perfect plasticity inside the *SYZ.* In real specimens the initial crack length can influence the level of three-axiality of stresses, which in turn controls the fracture mechanisms and crack growth equation.

Very often the length of the specimen is simply neglected in fracture analyses. However, it is a geometrical quantity playing an important role in stability analysis of a specimen containing growing cracks. It is the well-known fact that the longer specimen is, sooner after maximum, the global instability takes place. In Figure 4a three typical curves for identical specimens except the length are presented.

The influence of the yield stress level on a maximum value of external load is significant although it decreases with increasing yield strength. Elevation of the maximum of the load is much more pronounced for materials fracturing due to ductile mechanisms *(*α*>1)* (Figure 5). In Figure 6 the *P=f(u)* diagram is presented for a relatively brittle material, which is characterized by a high yield strength – 1000 MPa and crack growth equation containing α less then 1. One may notice a characteristic shape of a curve, which is called pop-in. The pop-in phenomenon is usually understood as a result of a jump – arrest – slow propagation mode of crack growth. Here one may notice that pop-in may also be observed for continuous crack growth as a result of a “competition” between crack and plastic zone growth.

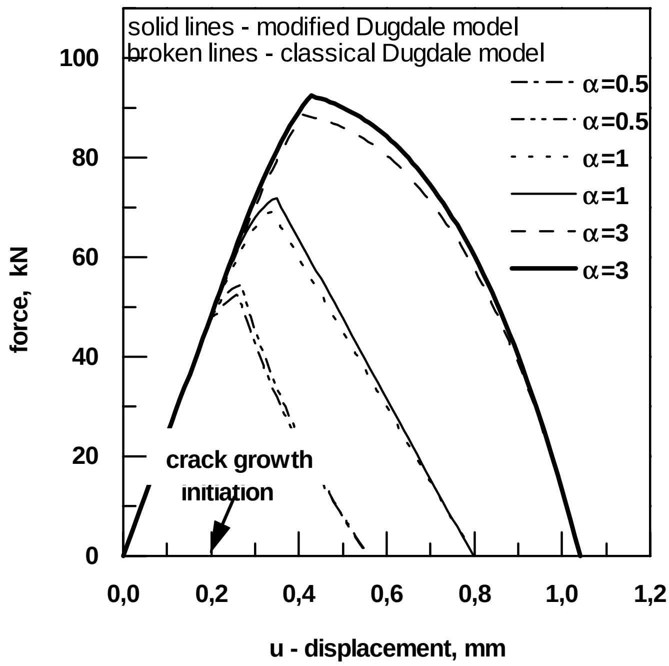

Figure 2a. $P=f(u)$ diagrams for different crack growth equations and identical specimens. $B=5mm$, $L=240mm$, $2a_0=30mm$, $2W=60mm$, $u_i=0.2mm$, $\sigma_y=600MPa$, $\beta=25$

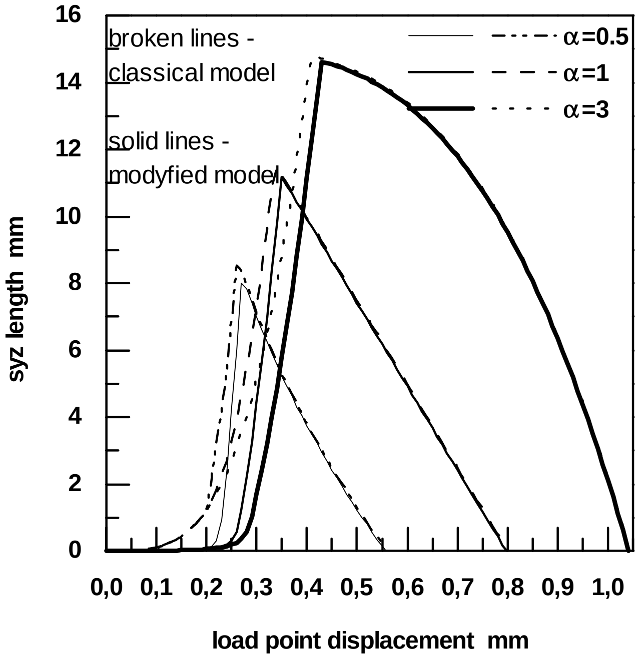

Figure 2b. The SYZ evolution with a load point displacement for three identical specimens with characteristics as in Figure 2a

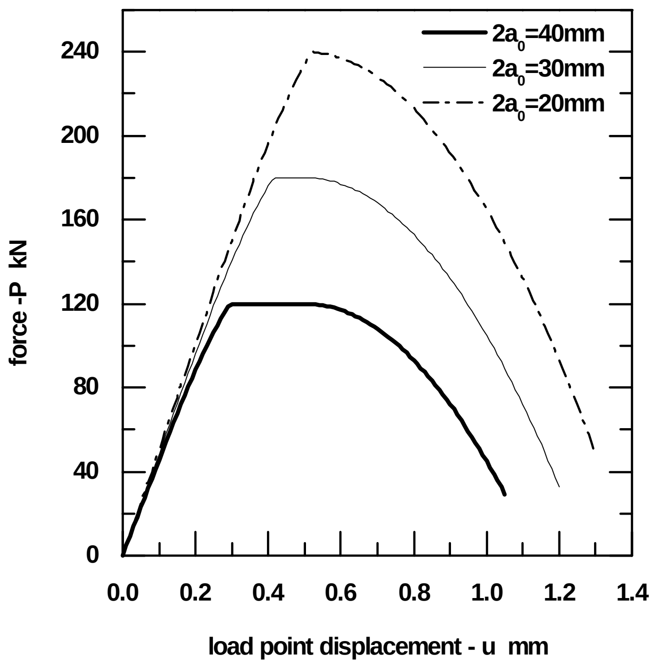

Figure 3. $P=f(u)$ diagrams for different initial crack lengths. $B=10mm$, $L=240mm$, $2W=60mm$, $u_i=0.2mm$, $\sigma_y=600MPa$, $\beta=25$, $\alpha=2$

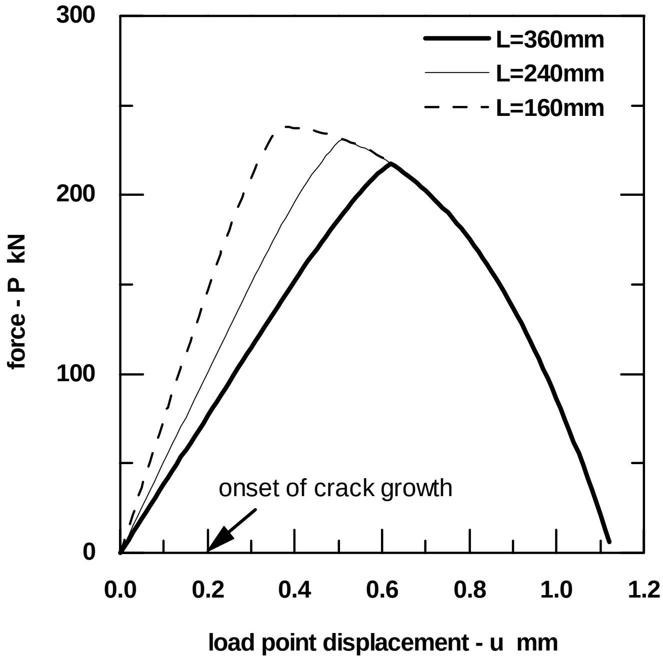

Figure 4. $P=f(u)$ diagrams for different specimen lengths. $B=10mm$, $2W=60mm$, $2a=20mm$, $u_i=0.2mm$, $\sigma_y=600MPa$, $\alpha=3$, $\beta=25$.

Simulations made with the help of the classical Dugdale model in order to test the influence of a specimen thickness on a *P=f(u)* indicates that both maximum force and specimen compliance change in proportion with the specimen thickness (the cross – sectional area of an unbroken material in front of a crack). In fact, the real influence is much more significant since the specimen thickness changes extent and direction of plastic flow as well as fracture mechanisms. In Figure 7 the diagrams are presented both for a classical and modified Dugdale models. Modified Dugdale model [5] reflects the geometrical constraints on plastic flow *only* (within the framework of a simplified model). As one may notice, for thin specimens the results converge for modified and classical model.

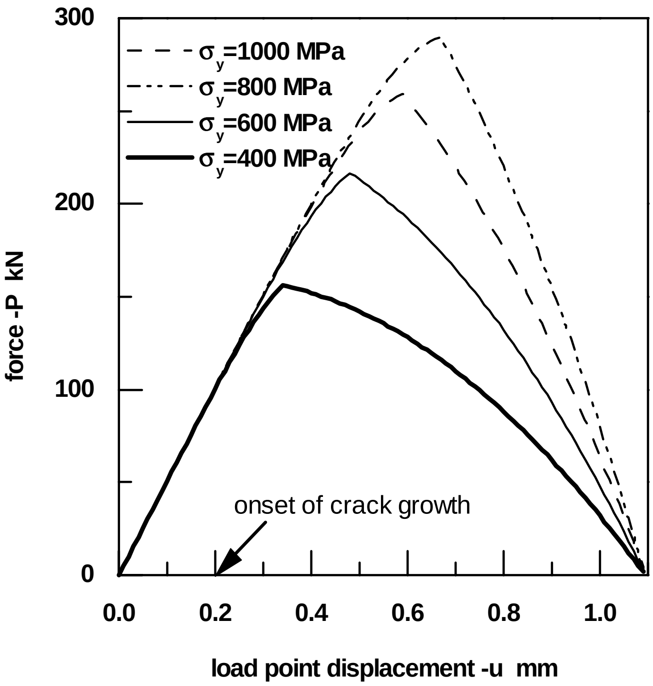

Figure 5a. *P=f(u)* diagrams for different yield strengths. *B=10mm, 2W=60mm, 2a=20mm, L=240mm,* u_i*=0.2mm,* α*=2,* β*=25.*

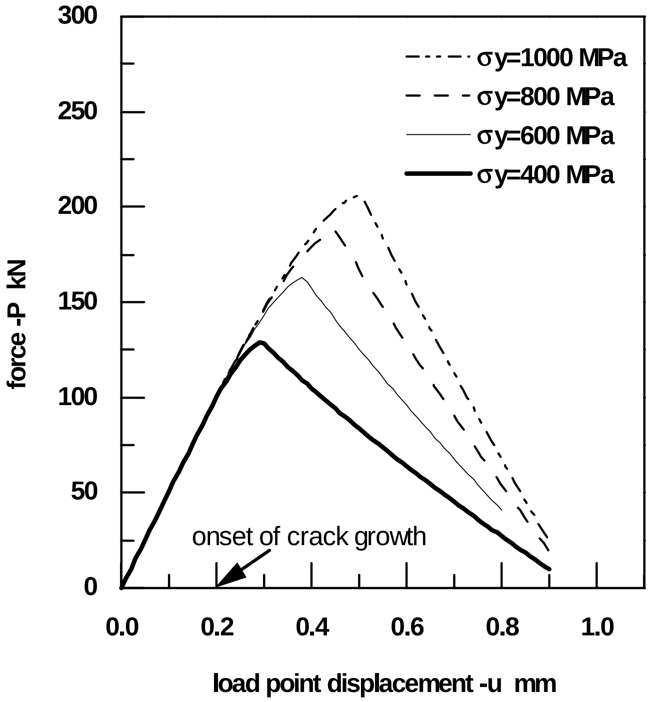

Figure 5b. *P=f(u)* diagrams for different yield strengths. *B=10mm, 2W=60mm, 2a=20mm, L=240mm,* u_i*=0.2mm,* α*=0.8,* β*=25.*

In the paper [7] much more examples of numerical calculations based on the model proposed has been included along with more detailed analysis. They include e.g. the analysis of the influence of the moment of the crack growth initiation on a shape of the *P=f(u)* curves. Also the unloading was taken into account in plotting the *P=f(u)* curves . Conclusions concerning the onset of the crack growth has been presented.

CONCLUDING REMARKS

In the paper a simple model of the specimen made of elastic–perfectly plastic material containing a growing crack has been presented. The *P=f(u)* diagrams obtained during simulation can serve various purposes. They can give a qualitative picture of the fracture process for a variety of specimen sizes and material plastic and fracture properties. Thus, they can be helpful to *design* an experiment of the stable crack growth, undertaken e.g. to normalize the *J-R* curves or to propose a *key curves* characterizing the material resistance to the stable crack growth. They can also serve as guide to understand the influence of various material and geometrical parameters on a stable crack growth. The computations can be extended to other specimen geometries. Simulations can be performed with the help of widely accessible popular computer programs. The *P=f(u)* curves obtained during simulations (only some of the curves have been included in this paper) reflect all basic features of a stable crack growth observed by author in many laboratory tests. Thus, we conclude that

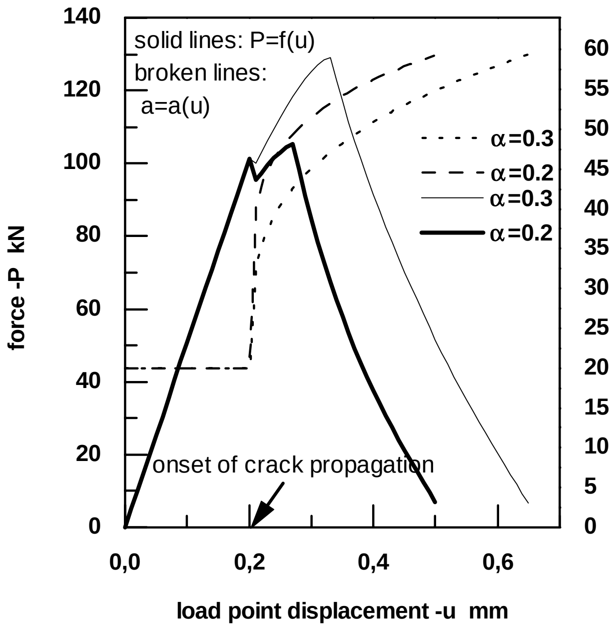

Figure 6. The pop-in phenomenon, *B=10mm, L=240mm, 2W=60mm,* $2a_0=20$*,* σ_y*=1000MPa,* u_i*=0.2mm.*

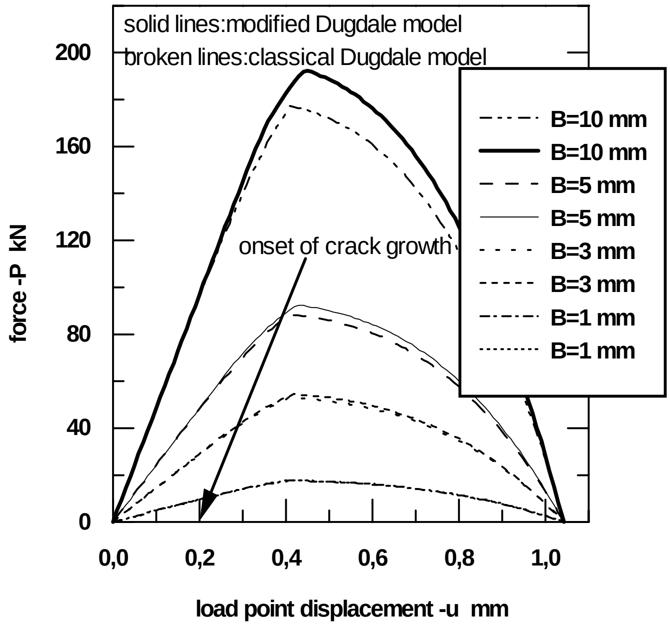

Figure 7. *P=f(u)* diagrams for different specimen thickness' both for classical and modified Dugdale model, *L=240mm, 2W=60mm,* $2a_0$*=30mm,* u_i*=0.2mm,* σ_y*=600MPa,*α*=3*

the model is correct and gives a good *qualitative* picture of a fracture process. Quantitative differences come from the oversimplification of a real physical situation that in turn is due to a very simple model of specimen. In most of real specimens the shape and extent of a plastic zone are much different than those computed with the help of Dugdale model. However, if other, more realistic solutions of elastic–plastic boundary value problems are known they can be easily implemented into a model.

Acknowledgements – The financial support from KBN (Polish National Research Committee) under the contract: 7 T07C 008 15 is gratefully acknowledged.

REFERENCES

1. Broberg, K.B. (1999) *Cracks and Fracture.* Academic Press,
2. Neimitz A. and R. Molasy. (1998) "Investigations towards J-R curves independence of the specimen size" in "*Fracture from Defects"*, Eds M.W.Brown, E.R. de los Rios, K.J.Miller, Vol.2, pp.625-630,
3. Turner, C.E. (1990) „A re-assesment of ductile tearing resistance" in *Fracture Behaviour and Design of Materials and Structures (ECF 8),* Ed. By Firrao, Emas Ltd. Warley, Vol. 1,part 1, pp. 933-940, part 2, pp. 951-968.
4. Dugdale, D.S., (1960). „Yielding of steel sheets containing slits", *J. Mech. Phys. Solids,* **8**, pp. 100-108.
5. Neimitz, A., "Dugdale model modification due to the geometry induced plastic constraints", accepted for publication in *Engng Fract. Mech.*
6. M. Isida, (1971), "Stress intensity factors and Dugdale types plastic zones for a finite plate with an internal crack" in *Proceedings of Symposium on Applications and Extensions of Fracture Mechanics,* Japanese Society of Mechanical Engineers, No. 710-7 p.119 (in Japanese) also in *The Stress Analysis of Cracks – Handbook*, Del Research Corporation, Hallertown, Pa. (1973),
7. Neimitz A. "Phenomenological model of the elastic-plastic specimen containing growing crack" submitted for publication in *Engng Fracture Mechanics*