

# ON THE PREFERABLE INITIAL CRACK LENGTH IN FRACTURE TOUGHNESS TESTING USING SUB-SIZED BENDING SPECIMENS

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## ABSTRACT

In order to guarantee maximum crack-tip constraints, fracture toughness testing standards require a minimum specimen size and initial crack lengths of about  $a_0=W/2$ . When performing impact tests on smaller specimens the situation is somewhat different insofar as size and constraint requirements are not fulfilled anyway, so it is questionable whether the initial crack length should still be about  $a_0=W/2$ . On impact testing of small specimens, shorter cracks exhibit several experimental advantages, the main ones being, first, the lower influence of dynamic oscillations and second, the extended validity range of the measured toughness properties. For these reasons, we suggest to use shorter cracks and account for the insufficient initial crack-length by correcting the measured (apparent) fracture toughness values, using the theoretical correction formula presented in this paper. Based on the standard size requirements one obtains a lower-bound fracture toughness that can be used as a conservative design value. The latter turns out to be maximum for an initial crack length of about  $a_0/W=0.25 - 0.3$ . Thus, using specimens with crack lengths in this range is not only advantageous experimentally but also beneficial with respect to the obtained fracture toughness data.

## INTRODUCTION

If the available testing material is limited, or to determine local toughness values, or even to reduce testing costs, using specimens smaller than those required by the current fracture toughness testing standards like ASTM E1820 [1] or ISO/DIS 12135 [2] may be suitable or even the only possibility. Such specimens, which do not fulfill the size requirements for “valid” fracture toughness, are called sub-sized. Still they can be tested and evaluated like full-size specimens, but one has to be aware of the fact that the resulting fracture toughness values are not transferable to larger structures without restrictions. For other purposes like qualitative comparison of different types of materials they still can be useful. Often impact loading of sub-sized specimens is advantageous, since the increased loading rate and the correspondingly increased local strain rates tend to rise the plastic flow stress and, correspondingly, to extend the size criteria. A typical sub-sized specimen suitable for many purposes is the pre-cracked Charpy specimen [3-5]. Its main advantage is that it can be easily tested dynamically at various temperatures by means of a standard instrumented Charpy pendulum hammer [6]. At present time a guideline for testing pre-cracked Charpy specimens is in preparation by the ESIS Technical Committee TC5 [7]

When dealing with sub-sized specimens the question arises to which degree full-size static testing standards like [1, 2] have to be adopted or whether some modifications concerning size- and validity requirements are

justified. The size requirements ensure that the constraints at the crack-tip are essentially "plane strain", which is necessary to enable the evaluated fracture toughness values to be used in failure assessment of larger components. For the same reason, these standards include requirements concerning crack length, since the latter is known to affect the in-plane constraints. For example, edge cracked bending and CT- specimens reach the maximum in-plane constraints at crack-depths-to-width ratio of about  $a_0/W=0.5$ . However, when dealing with sub-sized specimens, further aspects have to be taken into account as well. Introducing a fatigue crack of about  $a_0/W=0.5$  into a sub-sized specimen leaves only a very small ligament of testing material, which may be regarded – if the crack is too deep – as a waste of the limited testing material, since the material in the rear of a too deep crack is not loaded, but serves only to impose constraints. In sub-sized specimens the size requirements are not fulfilled anyway, which means that the out-of-plane constraints are not sufficient to provide perfect plane strain conditions, so the question is justified whether or not it is reasonable to strive for maximum in-plane constraints. As discussed in [8] and below, shorter initial cracks exhibit several advantageous features concerning accuracy of test evaluation, range of validity and transferability, particularly in dynamic testing. Thus, the choice of the most favourable initial crack length is a matter of optimising several effects and depends on the purpose of testing. For example, the specimens exposed to irradiation within the Swiss surveillance program of nuclear power-plants contain fatigue cracks of only about  $a_0/W = 0.3$  [9, 10]. An experimental investigation [11] did not exhibit a significant effect of the initial crack-length in the range  $0.3 < a_0/W < 0.5$  on the J-R-curve beyond the limit of valid J.

In the present paper, the question of the most advantageous length of the pre-crack in a sub-sized bending specimen is dealt with regarding mainly the validity and transferability of the evaluated fracture toughness values. The analysis is based on the simplified theory to account for constraints as suggested by the author in [12]. It enables experimental data that are determined on specimens with relatively short pre-cracks to be corrected theoretically for maximum constraints. Therewith in principle specimens with any crack-lengths can be used. From additional conditions concerning validity and transferability the optimum initial crack-length is determined.

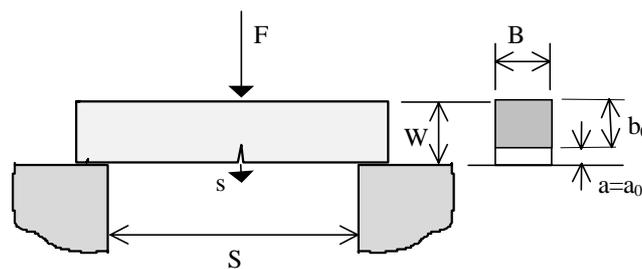
### VALIDITY OF FRACTURE TOUGHNESS FROM SUB-SIZED BENDING SPECIMENS

Sub-sized 3PB specimens (Fig. 1) can be evaluated in the same way as full sized specimens [4, 5]. If the fracture behaviour is completely ductile, then fracture toughness is characterised by a near initiation J-value denoted by  $J_{0.2Bl}$  (using the terminology of [2]); if an unstable cleavage fracture takes place, the characteristic fracture toughness parameter is denoted by  $J_u$  (using the terminology of [2]). According to [1] these properties are “valid”, (i.e. transferable to an analysis of cracks in a larger structural part) if the specimen size meets the criteria

$$J_{0.2Bl} \leq \sigma_f b_0 / 25 \quad \text{and} \quad J_{0.2Bl} \leq \sigma_f B / 25 \tag{1a}$$

$$J_u \leq \sigma_f b_0 / 200 \quad \text{and} \quad J_u \leq \sigma_f B / 200 \tag{1b}$$

where  $\sigma_f = (R_p + R_m) / 2$  represents the flow stress, with  $R_p$  and  $R_m$  denoting the yield stress (e.g.  $R_{p0.2}$ ) and the ultimate tensile strength, respectively.



**Fig. 1:** Mechanical system of an impact test in three-point-bending (3PB)

If these requirements are violated, then the measured fracture toughness value is called to be “not valid”, and the specimen “sub-sized”. Nevertheless, testing a sub-sized specimen can be useful. During the loading process of the specimens J-values that fulfil the criteria have been past without crack instability or substantial extension, so it is obvious that the highest J-value that fulfils criteria (1a) or (1b) (denoted in the following by  $J_{0.2Bl, LB}$  and  $J_{u, LB}$ , respectively) represents a lower bound of the corresponding fracture toughness, i.e.

$$J_{0.2Bl, LB} = \text{Min}(\sigma_f \cdot b_0 / 25, \sigma_f \cdot B / 25) \quad (\text{for } b_0 < 0.8W)^1 \quad (2a)$$

$$J_{u, LB} = \text{Min}(\sigma_f \cdot b_0 / 200, \sigma_f \cdot B / 200) \quad (\text{for } b_0 < 0.8W)^1 \quad (2b)$$

For PCC-specimens  $b_0 < B$ , so  $b_0$  determines the lower bound according to (2a) or (2b). Accordingly, the lower bound fracture toughness is increased if  $b_0 = W - a_0$  is increased, i.e. if the initial crack length  $a_0$  is decreased (for  $b_0 < 0.8W$ ). Further experimental advantages of short cracks<sup>1</sup> are discussed in the next section.

## ADVANTAGES OF SHORT CRACKS

### *Range of Validity*

As shown above, the range of validity of the evaluated fracture toughness values in terms of J is extended according to (1). According to (2) the lower bound fracture toughness values is increased, which is beneficial in any case. Furthermore, the range of validity of R-curves evaluated from such tests is increased, since according to [2] the fracture process can be regarded as sufficiently J-controlled for  $\Delta a < b_0 / 10$ .

### *Maximum Force*

When testing small specimens, the maximum force reaches in general the plastic limit load, which is given by

$$F_{\max} = \frac{c_1 \cdot \sigma_f \cdot B \cdot b_0^2}{S} \quad (3)$$

where  $c_1$  denotes the plastic constraint factor in bending, which is close to 1. Thus  $F_{\max}$  is increasing with increasing  $b_0$ , which makes the force measurement more accurate. Particularly this is true in impact testing, where the force signal is disturbed by superimposed inertial oscillation. The latter are proportional to the impact rate and the properties of the specimen, but not dependent on  $a_0$ .

### *Elastic Energy Release Rate*

The elastic component of J,  $J_{el}$ , represents the energy release rate. For an edge crack under bending and  $a_0 > W/3$ , it can be calculated approximately as

$$J_{el} = \frac{K_I^2 (F = F_{\max})}{E} \cong \frac{0.92 \cdot F_{\max}^2}{E \cdot B \cdot b_0^3} = \frac{0.92 \cdot (c_1 \cdot \sigma_f)^2 \cdot B \cdot b_0}{S^2 \cdot E} \quad (4)$$

thus  $J_{el}$  is increasing with increasing  $b_0$ . A higher elastic part of J is advantageous in fracture toughness testing. The higher the elastic component of a given value of J, the more likely a significant amount of unstable cleavage fracture occurs, since  $J_{el}$  promotes unstable spreading of cleavage fracture.

### *Geometrical Aspects*

The shorter  $a_0$ , the closer is the shape of the ligament to the square shape of the ligament of a standard CT-specimen as is used in quasi-static testing. Furthermore, the conditions for straightness of the fatigue crack

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<sup>1</sup> The term “short crack” is used here for cracks that are shorter than the minimum standard length, which is  $a_0 = 0.45W$ . However, in order to prevent plastic slip lines emanating from the crack-tip to the rear surface, initial crack lengths shorter than about  $0.2W$  should be avoided in any case.

front as given in the standards is also easier to be fulfilled. The deflection to span ratio at a given J-value is smaller, so the system is less affected by geometrical nonlinearity.

### Dynamic Effects

The shorter  $a_0$ , the stiffer the specimen and the higher the frequency of the predominant mode of oscillations in the force signal. Thus, the number of oscillations up to maximum load is increased. Together with the higher maximum load as shown in Eqn. 3, the load signal is less affected by the disturbing oscillations. Moreover the loading rate in terms of  $dJ/dt$  and the local strain rate at the crack-tip at a constant impact rate is higher.

## EFFECT OF CRACK LENGTH ON CONSTRAINTS AND FRACTURE TOUGHNESS

It is well known that the crack length affects the in-plane constraints in terms of T or Q [13, 14]. In [12] the author proposed a simple theoretical framework to predict the effect of constraints on fracture toughness, which in the following is briefly recapitulated. As a parameter to characterise the constraints, the ratio

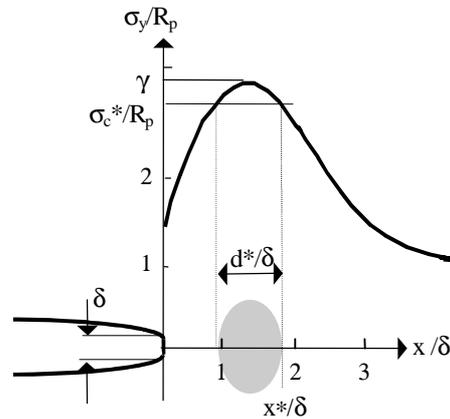
$$\gamma = \sigma_{y\max} / R_p \quad (5)$$

is considered, where  $\sigma_{y\max}$  is the maximum principal stress in the vicinity of the crack tip (Fig. 2). For the 3PB specimen  $\gamma$  can be estimated by an analytical method as suggested in [12]. Initiation of unstable cleavage is assumed to be governed by:

- i) The maximum stress in the vicinity of the crack tip must exceed the cleavage stress  $\sigma_c^*$ , i.e.

$$\sigma_{y\max} = \gamma \cdot R_p > \sigma_c^* \quad (6)$$

- ii) The elastic portion of the strain energy in a critical Volume  $V^*$  of the width  $d^*$  (Fig.2),  $W_{el} = \int U_{el} dV^*$  (with  $U_{el}$  being the elastic part of the strain energy density), must be sufficient to produce a cleavage fracture in the area  $0 < x < x^*$ .



**Fig. 2:** Non-dimensional representation of the stress distribution in the vicinity of a crack-tip

Using the proportionalities

$$U_{el} \propto (\gamma \cdot R_p)^2; V^* \propto \delta^2, d^* \propto \delta; x^* \propto \delta$$

and the general relation

$$J = m \cdot R_p \cdot \delta \quad (7)$$

in criterion ii), one finds the proportionality

$$J_c \cdot \frac{\gamma^2}{m} = const \quad \text{for } \gamma > \sigma_c^*/R_p \quad (8)$$

For initiation of ductile tearing, we assume that  $\delta$  at crack initiation is proportional to the plastic fracture strain  $\epsilon_{pf}$  in the fracture process zone,

$$\delta_i \propto \epsilon_{pf} \quad (9)$$

which is known to be constraint-dependent [15]. Based on the failure hypothesis of Gillemot [16], which states that ductile fracture occurs when the plastic strain energy density,  $U_p$ , reaches a certain critical value,  $U_{pf}$ . The former can be estimated by we simply assume that at crack initiation the "true" failure strain multiplied by  $\sigma_{y\max} = \gamma \cdot R_p$  is equal to  $U_{pf}$ , i.e.

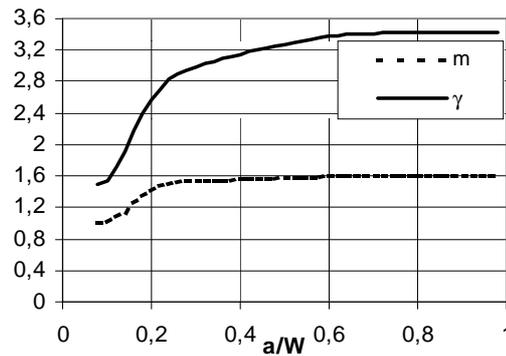
$$\gamma \cdot R_p \cdot \ln(1 + \epsilon_{pf}) = U_{pf} \quad (10)$$

The logarithmic ("true") strain is used here because  $\epsilon_{pf}$  is in general not small enough for the kinematical relations to be linearized. According to [12]  $U_{pf}$  can be roughly obtained from a uniaxial tensile test as the area under the true stress-true strain diagram in the necking area, which is approximately

$$U_{pf} \cong \frac{\sigma_f \cdot Z}{1 - Z} \quad (11)$$

where  $\sigma_f = (R_p + R_m)/2$  denotes the flow stress and  $Z = (A_0 - A)/A_0$  the reduction of area in a uniaxial tensile test (with  $A_0$  and  $A$  being the initial and final cross section, respectively). With (10), (11) and (7), (9) leads to

$$\frac{J_{0.2Bl}}{m \cdot \left\{ \exp \left[ \frac{\sigma_f \cdot Z}{R_p \cdot \gamma \cdot (1 - Z)} \right] - 1 \right\}} = const \quad (12)$$

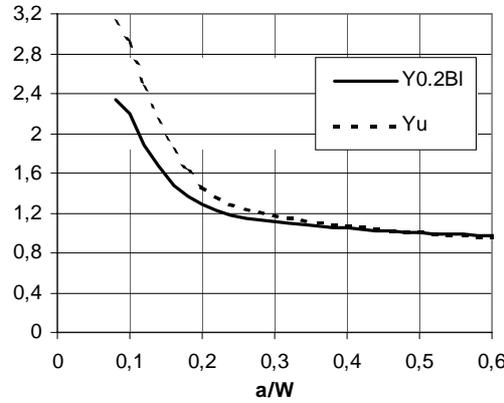


**Fig. 3:** Constraint factors  $m$  and  $\gamma$  of an edge crack in a beam in bending (from [12]) as a function of non-dimensional crack length

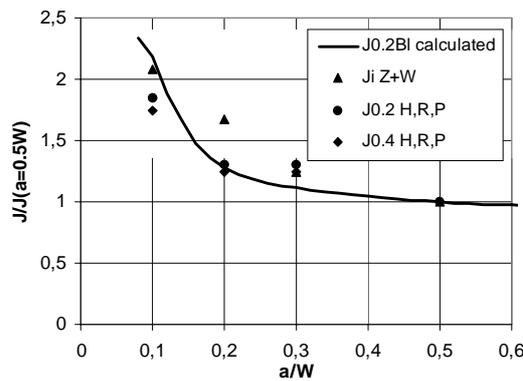
In [12] the constraint parameters  $\gamma$  and  $m$  of an edge-cracked beam in bending are estimated. The corresponding curves are shown in Fig. 3. Using these values in (8) and (12), one obtains the relation between the standard fracture toughness properties  $J_u$  and  $J_{0.2Bl}$  and the effective fracture toughness as measured on a specimen containing a short initial crack,  $J_{0.2Bl,eff}$  and  $J_{u,eff}$ . The corresponding ratios are denoted by  $Y_u$  and  $Y_{0.2Bl}$ , respectively, i.e.

$$Y_u(a/W) = \frac{J_{u,eff}(a/W)}{J_u} \quad ; \quad Y_{0.2BI}(a/W) = \frac{J_{0.2BI,eff}(a/W)}{J_{0.2BI}} \quad (13)$$

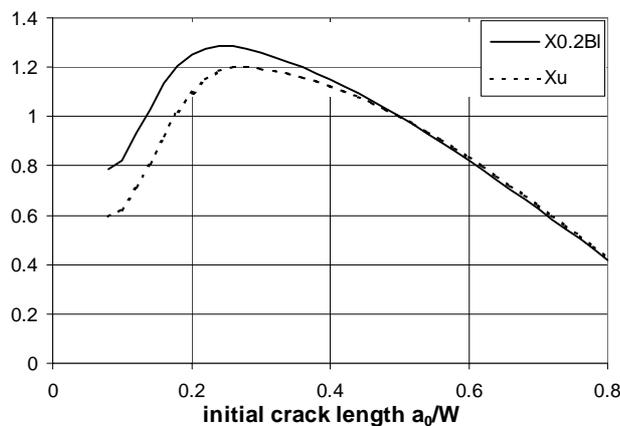
where  $J_u = J_{u,eff}(a/W=0.5)$  and  $J_{0.2BI} = J_{0.2BI,eff}(a/W=0.5)$  represent the standard fracture toughness values at saturated constraints.



**Fig. 4:** Effect of the crack length of a 3PB-specimen on effective fracture toughness (see Eqns. (13) for definition of  $Y_u$  and  $Y_{0.2BI}$ )



**Fig. 5:** Comparison of near-initiation  $J$  predicted by (12) with experimental  $J$ - values ( $Z+W$  from [17],  $H,R,P$  from [13]).



**Fig.6:**  $X_u$  and  $X_{0.2BI}$  (see eqs. (14) for definition) as ratios to the corresponding values for a standard crack length  $a_0/W=0.5$ )

The ratios  $Y_u$  and  $Y_{0.2BI}$  are shown in Fig. 4 as a function of crack length (for a typical value of  $Z$  for a medium-strength structural steel,  $Z=0.6$ ). In Fig. 5 the theoretical ratio  $Y_{0.2BI}$  as determined from (12) is

compared with some experimental data from the literature. Regarding the significant scatter of the data in the literature the agreement is reasonably good.

## LOWER BOUND FRACTURE TOUGHNESS

By (13) and using  $Y_u(a/W)$  or  $Y_{0.2BI}(a/W)$ , respectively, as given in Fig. 4, it is possible to correct effective fracture toughness values measured on specimens of arbitrary crack sizes to the standard crack-length  $a_0/W=0.5$ . Consequently, the same correction has to be made to the lower bound values according to (2), if the initial crack size does not meet the requirements of the testing standards. Thus, the transferable lower bound fracture toughness is obtained by

$$J_{0.2BI, LB} = \frac{\sigma_f \cdot W \cdot (1 - a_0/W)}{25 \cdot Y_{0.2BI}(a_0/W)} =: \frac{\sigma_f \cdot W \cdot X_{0.2BI}(a_0/W)}{25} \quad (14a)$$

$$J_{0.2BI, LB} = \frac{\sigma_f \cdot W \cdot (1 - a_0/W)}{200 \cdot Y_u(a_0/W)} =: \frac{\sigma_f \cdot W \cdot X_u(a_0/W)}{200} \quad (14b)$$

In Fig. 6 the values of the functions  $X_{0.2BI}(a_0/W)$  and  $X_u(a_0/W)$  as defined in (14a) and (14b), respectively, are shown, normalized with their values for the standard  $a_0/W=0.5$ . These curves also represent the lower bound values that can be obtained normalized with their values at the standard initial crack length of  $a_0/W=0.5$ .

When testing sub-sized specimen, one of the main aims is obtaining a lower bound fracture toughness that is as high as possible, because the higher the lower bound the closer it is to the standard fracture toughness. According to Fig. 6 the lower bound fracture toughness is maximum if an initial crack length of about  $0.25W$  is used.

## DISCUSSION AND CONCLUSIONS

Fracture toughness values determined from bending specimens depend on the crack-length, if the latter is shorter than about  $0.5W$ , which is due to the fact that the constraints are decreasing with decreasing crack length. That is why the current testing standards require initial crack lengths in the range  $0.45 < a_0/W < 0.55$ . In the present paper theoretically derived formulas for the effect of constraints on fracture toughness values are presented. Therewith it is possible to use initial crack-length in the range  $0.2 < a_0/W < 0.45$  and correct the measured data such that they correspond to the standard initial crack-length of  $a_0/W=0.5$ .

If sub-sized specimens are used, one does not obtain standard, transferable fracture toughness values, but only lower bounds thereof. Using the proposed correction formulas, the calculated lower bound fracture toughness turned out to be maximum for an initial crack length of about  $a_0/W = 0.25 - 0.3$ . Therewith it is shown that such short cracks not only are advantageous experimentally, but also deliver higher lower bounds of fracture toughness, which means less conservative, more realistic values. Thus, if one strives for maximum transferable fracture toughness values of sub-sized specimens, it is recommended to use shorter cracks. Beyond the boundary of valid J, an experimental investigation also revealed no significant effect of the initial crack length on the J-R-curve [11].

Of course the accuracy of the underlying theoretical models is limited: Concerning the location of the maximum of the curves shown in Fig. 6 on the  $a/W$  - axis the accuracy is estimated to be about  $\pm 0.05$ . One probably could improve the accuracy of the underlying functions  $Y_u(a/W)$  semi-empirically by including experimental data such as the ones shown in Fig. 5.

It has to be kept in mind that specimen size and crack length not only affect the validity of the fracture toughness and the magnitude of their transferable lower limits, but also the brittle-to-ductile transition temperature. When transferring fracture toughness obtained from sub-sized specimens to larger structural parts, one has to be aware of this fact and to look for possibilities to account for it, which in most cases require additional experimental data obtained on larger specimens. Generally speaking, the size effect on the temperature shift is partly compensated by the increased loading rate of impact testing. Concerning the crack length one can assume that there is not much additional effect, because at a given impact velocity the crack tip loading rate in terms of  $dJ/dt$  is increasing with decreasing crack length. However, additional experimental data are required to establish semi-empirical formulas to estimate the temperature shift

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