Numerical Analysis of A Mixed Mode Crack Behavior in Elastic-Plastic Materials

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Abstract

The generalized HRR singularity, scaled by the *J*-integral and the plastic mixity parameter M^p, is considered as a basic solution of the near-tip asymptotic stress field of an elastic-plastic crack under mixed mode loading. A numerical method was recently developed to determine the plastic mixity parameter M^p for general yielding plasticity. The purpose of this paper is to apply this method to evaluate the validity of the HRR solution in the representation of the near-tip field of a mixed mode loaded crack. Different specimens, loading methods and material hardening behaviors have been studied by using the finite-element modeling. The results obtained show that the HRR solution has quite a good accuracy for the most mixed-mode-loaded cracks. Only for some near-mode I cracks its accuracy is poor. This observation supports the argument to use the J-M^p locus to characterize the fracture behavior of a plastic mixed mode crack, especially when the crack is not predominated by mode I loading.

1. Introduction

The fracture behavior of a mixed mode crack in linear elastic materials was well investigated in the literature. In an elastic plastic material, the pure mode I crack was also well studied. However, for mixed mode I-II cracks in elastic-plastic materials, even though many numerical and experimental studies have been published, it seems that the characterizations of their fracture behaviors have not been perfectly fixed.

The basic work in this topic was carried out by Shih [1] who showed the HRR singularity around the crack tip in a plane strain structure under mixed mode loading conditions. Two parameters, the *J*-integral and the plastic mixity M^p, were defined in order to characterize the amplitude and the mixity composition of the near-tip fields. The *J*-integral, having been defined by Rice [2], can readily be calculated by several methods. However, the mixed parameter M^p is not easy to be evaluated. So experimental results (for examples, [3], [4], [5], and [6] among others) are being reported using the ratio of the elastic stress intensity factors (K_I/K_{II}). If this approximation is rather good for small plastic yielding calculations, however, it contains some errors in the case of large plastic yielding. Recently, Li *et al*. ([7], [8]) have found a numerical method allowing the evaluation of the mixity parameter M^p for general cases of plastic yielding. This enables us to use the J- M^p locus (which is a more reasonable approach in the physical sense) rather than the presently used K_I - K_{II} locus as fracture criteria in the elastic-plastic mixed mode fracture studies.

Then the problem posed for mixed mode loaded cracks is the same as for pure mode I loaded cracks: are the two parameters, J and M^p, sufficient to characterize the fracture behavior of a mixed mode crack in elastic-plastic materials? In pure mode I crack (M^p=1), the validity of the J-integral as a single parameter in the characterization of the near-tip fields has been well studied. It seems that this kind of investigations has not been well developed for mixed mode cracks.

In this paper, our attention is focused on studying the effectiveness of J- M^p two parameter characterization of the near-tip fields for a mixed mode loaded crack from small scale yielding to large scale yielding. We carry out the finite element calculations for single-edge-cracked beams (SECB) under four kinds of loading modes, varying from the pure mode I loading to the pure mode II loading. The numerical results are compared with the HRR solution scaled by J and M^p. The results show that the J-M^p locus characterize better the near mode II loaded cracks than the near mode I loaded cracks. Further numerical studies will be necessary in order to confirm this preliminary conclusion.

2. Method of Evaluation of the Parameters *J* and *M^p*

Shih [1] showed that, for a mixed mode crack lying in a power-law hardening material, the stresses, strains and displacements fields near the crack tip are dominated by the HRR singularity, and can be characterized by two parameters, the J-integral and a mixty parameter M^p. The later is defined as follows:

$$M^p = \lim_{r\to 0} \frac{2}{\pi} \tan^{-1} \left| \frac{\sigma_{\theta\theta}(\theta = 0)}{\sigma_{r\theta}(\theta = 0)} \right| \qquad (1)$$

The method to evaluate the parameter M^p has been reported in [7] [8] that we resume briefly. First, one defines an associated J-integral, the J*-integral as follows:

$$J^* = \int_{\Gamma} \left(w^* n_1 - \sigma_{ij} n_j \frac{\partial u^*_i}{\partial x} \right) ds \qquad (2)$$

where Γ is an arbitrary path around the crack tip; σ_{ij} are the stress components of the actual field; u^*_i are the displacement components of an auxiliary field; w^* is the associated energy density defined as:

$$dw^* = \sigma_{ij} d\varepsilon^*_{ij} \qquad (3)$$

The auxiliary field can be constructed in terms of the actual field. Following the approach of Ishikawa *et al*. [8], one can decompose the actual field into symmetrical and anti-symmetrical parts with respect to the crack axis:

$$u^{*M}_i(x,y) = \frac{1}{2}\left[u_i(x,y) + (-1)^{i+M} u_i(x,-y) \right] \quad i = 1,2;\ M = I, II \qquad (4)$$

With these two auxiliary fields, we obtain two associated integrals J^{*I} and J^{*II}. It is clear that J^{*I} and J^{*II} are path independent. An equivalent elastic mixity parameter M^{*e} can be defined from J^{*I} and J^{*II}, namely:

$$M^{*e} = \frac{2}{\pi} \tan^{-1} \sqrt{\frac{J^{*I}}{J^{*II}}} \qquad (5)$$

By carrying out an asymptotic analysis near the crack tip, one can find the relationship between the M^{*e} and M^p. This relationship was given in [8]. Moreover, one can calculate the J-integral from J^{*I} and J^{*II}:

$$J = J^{*I} + J^{*II} \tag{6}$$

3. Finite Element Analysis

In order to evaluate the validity of the generalized HRR solution in representation of the near-tip fields, we have carried out a numerical study by using the finite-element modeling. A general-purpose finite-element program, named CASTEM 2000, is used in the present study. The finite element analysis is based on small strain theory and employs the flow theory of plasticity. The specimens used in the numerical calculations are the single-edge-cracked beams (SECB) under four kinds of loading modes, by three-point bending and four-point shear loading. Therefore, different mixed modes are locally obtained near the crack tip, especially the pure mode I (M^p = 1) and the pure mode II (M^p = 0). Their geometry and the loading methods are illustrated in figure1.

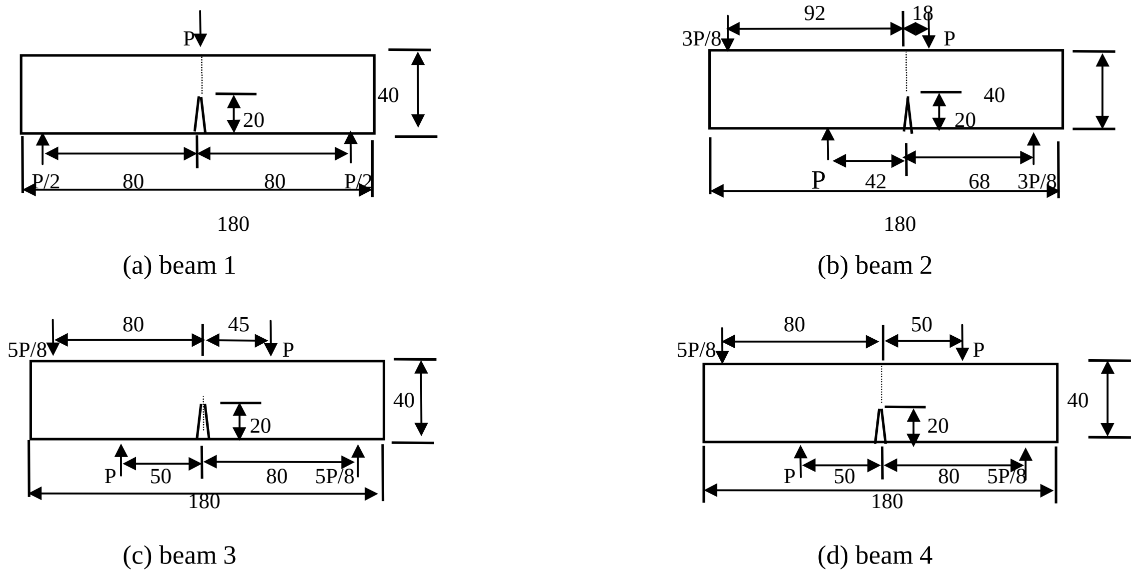


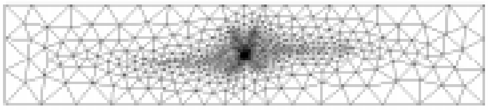
Figure 1: Specimen geometries and loading methods

The specimens are divided by triangular finite-elements of six nodes and by rectangular finite-elements of eight nodes as shown in figure 2. The smallest dimension of the elements near the crack tip is 0.00025a, with a being the crack length. The results presented in this work are obtained by using the following behavior law:

$$\varepsilon = \frac{1}{E}\left[\sigma + \alpha\sigma_0\left(\frac{\sigma}{\sigma_0}\right)^n\right] \tag{7}$$

with the following material constants: n=3 and n=9, α=0.5, E=200000MPa, σ_0=400MPa, ν=0.3. Results obtained by using different values of α and ν do not vary significantly and are not presented.

Using the definitions in (2) and (5), the associated J-integrals J^{*I}, J^{*II} and the equivalent elastic mixity parameter M^{*e} are computed. In order to easily carry out the decomposition of the real field, circular integration paths around the crack tip are chosen. Thus the auxiliary fields $\boldsymbol{u}^{I*}$ and $\boldsymbol{u}^{II*}$ can be calculated at the integration points on the integration paths. The calculation of J^*'s does not present any numerical difficulty. Nevertheless, in order to insure the accuracy and the path-independence of the J^*-integral, attentions are required in the construction of w^* from (3). Suitable steps, as numerous as possible in the capacity of the computer, are chosen in the increment of the loading. The convergence and the accuracy can be verified by comparing, according to equation (6), the sum of J^{*I} and J^{*II} with the J-integral calculated by using other conventional methods. This comparison shows that equation (6) can always be satisfied in the limit of the accuracy of the finite-element model.



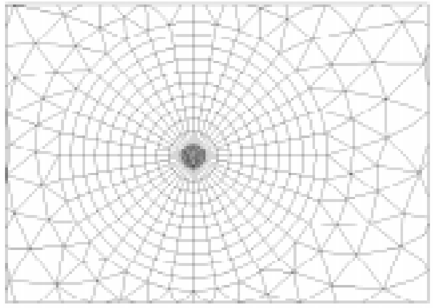


Figure 2: Finite element mesh

4. Numerical Results and Discussions

<u>*The path-independence*</u>

The path-independence of J^*'s is verified for different specimens by using several integrating paths with different radius (5 integrating paths with radius from r=0.0025a to r=0.01a, a being the crack length). Tab. 1 shows some values of J and M^p obtained by using different integrating paths. One can confirm from Tab.1 that J^{*I} and J^{*II} are path-independent, the small perturbations of the values are essentially due to the errors of the finite element modeling.

Table 1: Evaluation of J and M^p on the different integrating paths, beam 2, n=9

r/a	J^{*I}	J^{*II}	J	M^p
0.0025	20.78	3.23	24.01	0.811
0.0045	20.77	3.17	23.94	0.813
0.0055	20.72	3.20	23.92	0.812
0.0075	20.72	3.30	24.02	0.808
0.01	20.75	3.35	24.10	0.807

<u>*The representation of the near-tip field by the generalized HRR solution*</u>

Knowing the parameters J and M^p obtained by using the method proposed in this work, the near-tip asymptotic field can be represented by the generalized HRR solution. These asymptotic solutions are then compared with the results of the finite element analysis. Figure 3 plots the stress distributions near the crack tip ($r/(J/\sigma_0) \approx 2$) for a mode I loaded SECB (three-point beam) with n=3. Results with n=9 are not presented in order to save space, since similar curves can be drawn. One can observe from Figure3 that the deviation of the HRR solution from the FEM results is quite large for this configuration. These results agree with those obtained in the literature ([9] for example). Figures 4 and 5 show the comparison between the generalized HRR solution and the FE solution for two mixed mode loaded SECB's (beams 2 and 3, four-point beams) with n=3 and n=9. The mixity parameters for beam 2 are approximately 0.75 with n=3 and 0.81 with n=9. The mixity parameters for beam 3 are approximately 0.5 with n=3 and 0.6 with n=9. It can be seen that the HRR solution agrees well the FE analysis for these two beams, under small-scale yielding as under large-scale yielding, for high hardening materials as for low hardening materials. Figure6 illustrated the comparison between the HRR solution and the FE results for a pure mode II loaded SECB (four-point shear beam, beam 4). Here again good agreement between the numerical and asymptotic solutions is observed.

From these numerical calculations, one can find that a part from the near-mode I loaded crack, the generalized HRR solution scaled by the two parameters J and M^p represent better the near-tip stress fields for mixed mode cracks than for pure mode I cracks. Even though the generalization of this remark requires more numerical and experimental works, one can say that the results showed in this paper are significant at least for this kind of structures which are often used in experimental studies. One possible explication of this phenomenon may be the triaxial stress level near the crack tip. First, for mixed mode crack, the tensile stresses σ_θ and σ_r decrease as the mixty parameter M^p decreases. This means that the hydrostatic stresses decrease as M^p decreases. As a consequence, the influences of the triaxiality on the stress distribution decrease too. Second, in small-scale yielding, if the far elastic stress field can be developed into the Williams expansion [10], one can remark that the famous T-stress, which corresponds to a r-independent stress and influences significantly the plastic near-tip field [11], exists only for mode I loading. The terms of higher order in the Williams expansion provide less impact on the near-tip plastic-field.

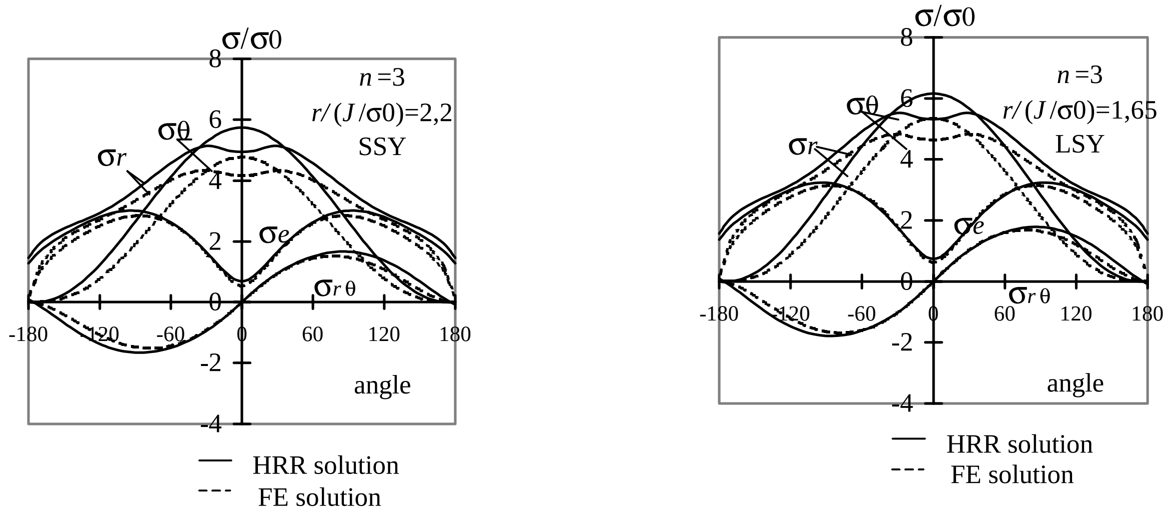
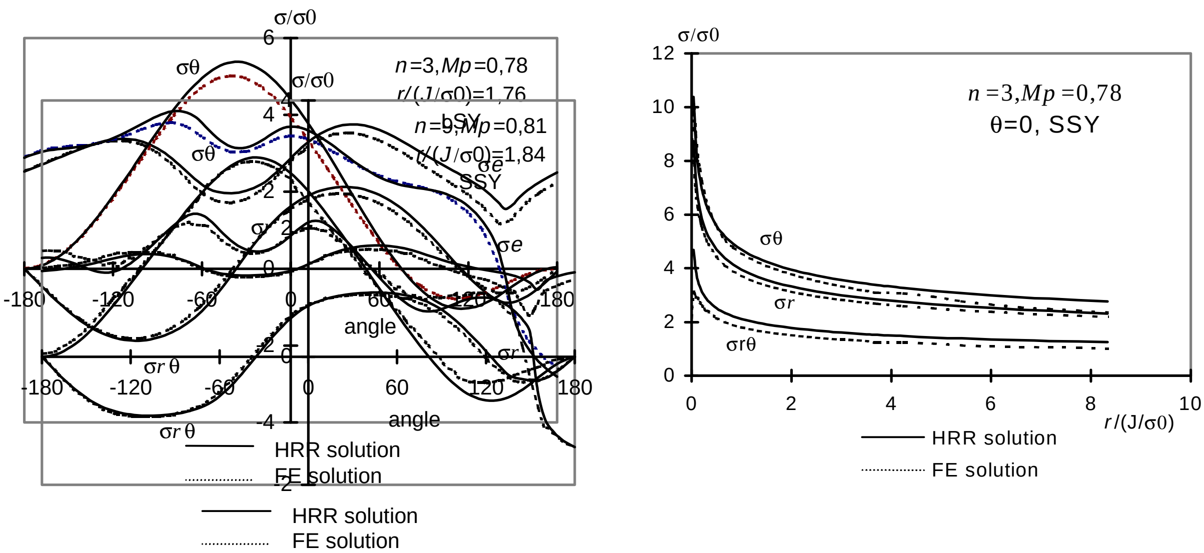


Figure 3: Stress distributions near the crack tip of beam 1, n=3



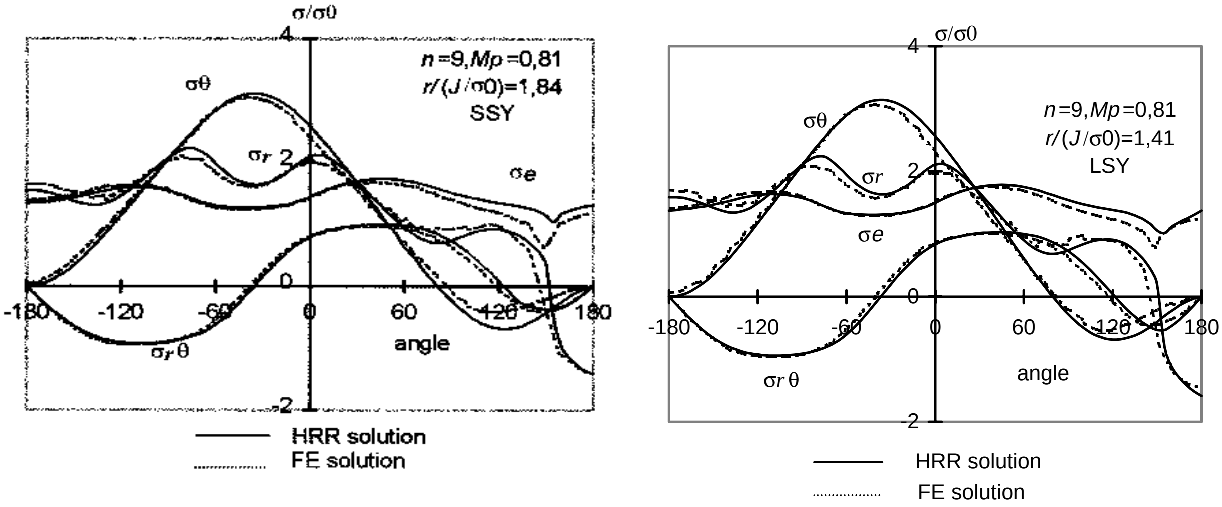


Figure 4: Stress distributions near the crack tip of beam 2, *n*=3 and 9

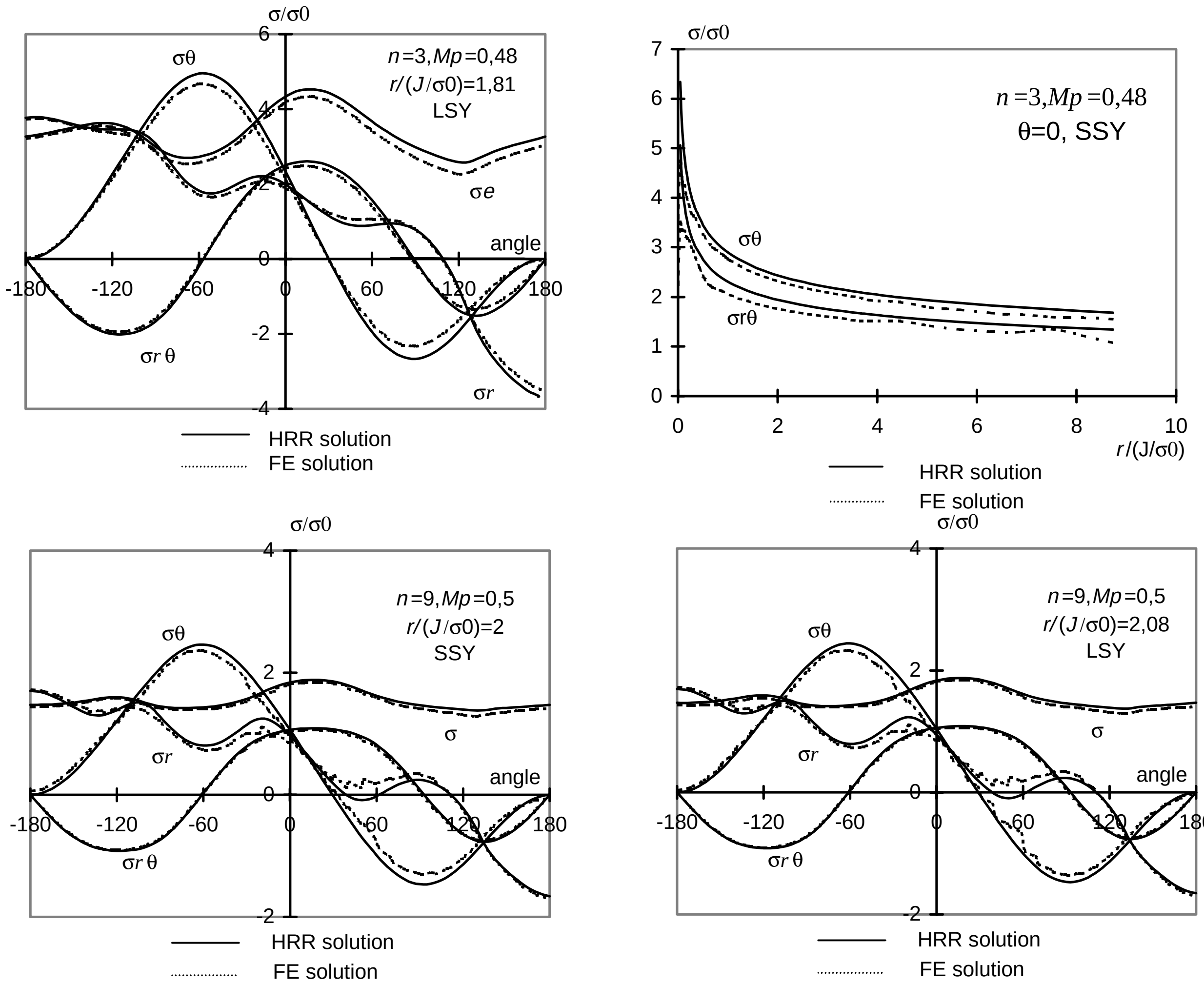


Figure 5: Stress distributions near the crack tip of beam 3, *n*=3 and 9

Numerous experimental studies show that the crack growth under mixed mode loading is more difficult to predict in elastic-plastic material than in linear elastic material. The experimental results of Pawliska *et al*. [3] on the Compact-Tension-Shear specimens, made of the aluminum alloy 2017-T4, showed that cracks propagate perpendicularly to the local mode I load. The experimental works of Tohgo and Ishii [4], using four-point beams made of the aluminum alloy 6061-T651, showed that the conclusion of Pawliska *et al*. [3] is valid only for near mode I loaded cracks. For mode II predominated cracks, the growth direction is on the plane of the largest shear stress. Moreover, the toughness for shear-type crack initiation is higher than for tensile-type crack initiation. Other works ([5], [6] etc.) showed also the competition between these two kinds of crack growth under mixed loading. All these experimental works support the argument that it exists, at least for small-scale yielding, a critical mixity parameter M^p dominating the transition between these two kinds of crack propagation. More experimental and numerical studies will be necessary to confirm this assumption.

5. Concluding Remarks

In this paper, we have presented some numerical results of the mixed mode crack in elastic-plastic materials. The bend-shear-cracked beams have been considered by using the finite element modeling. The near-tip stress fields have been compared with the generalized HRR solution scaled by two parameters: the J-integral and the plastic mixity M^p. The use of the parameters J-M^p as fracture criteria is a more reasonable approach in the physical sense than the presently used parameters K_I-K_{II}. It was showed that the HRR solution can effectively represent the stress field in mixed mode cracks in elastic-plastic materials. This observation suggests the existence of a J-M^p locus dominating the crack growth in elastic-plastic materials under mixed mode loading.

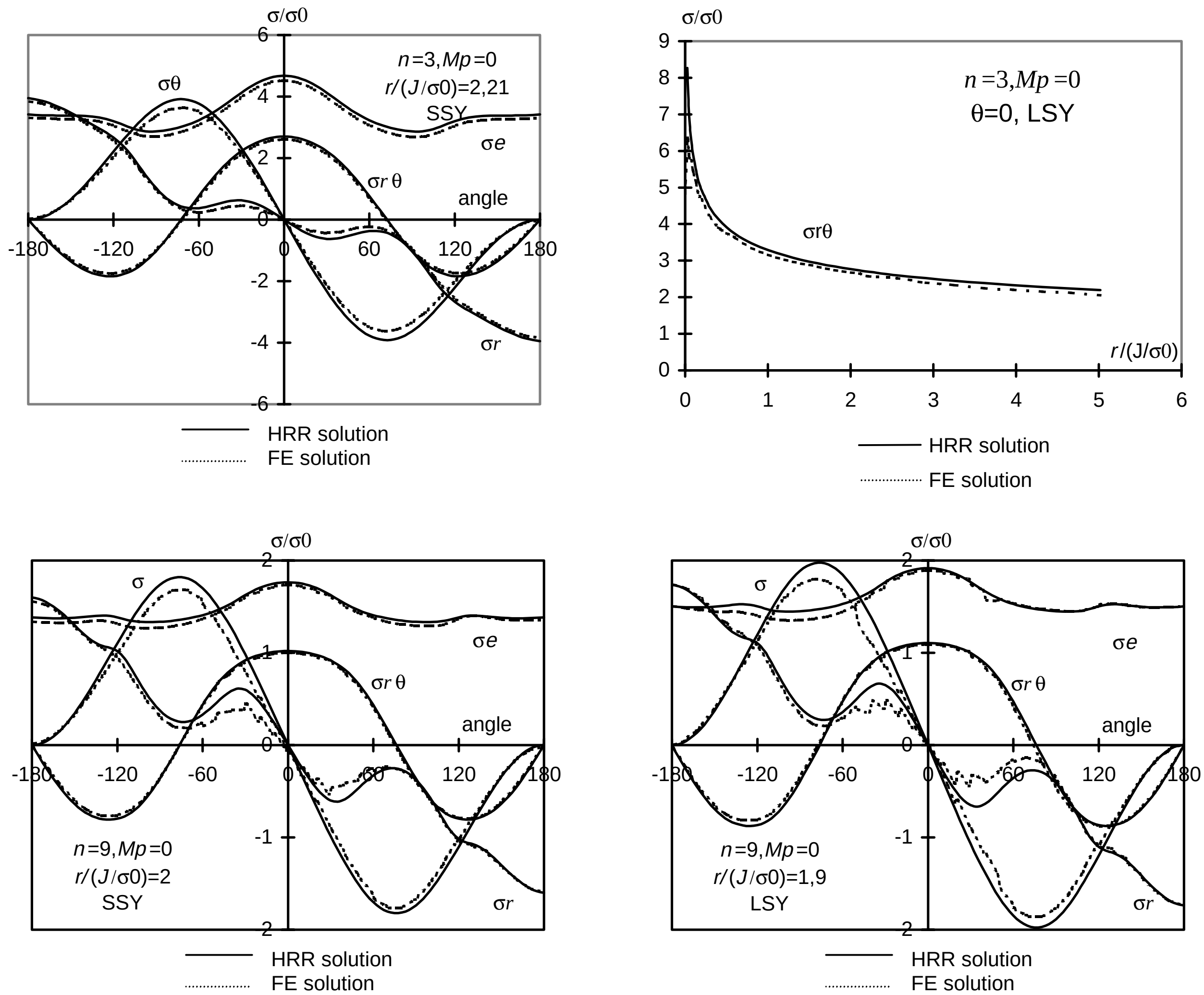


Figure 6: Stress distributions near the crack tip of beam 4, n=3 and 9

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