# INVERSE ANALYSIS OF A BOX-SECTION COMPOSITE BEAM WITH IMPACT DAMAGE

P.J. Sherratt, D.C. Panni, and A.D. Nurse

Department of Mechanical Engineering, Loughborough University,
Loughborough, U.K., LE11 3TU, a.d.nurse@lboro.ac.uk

## ABSTRACT

Box-section composite beams are widely used in industry but comparatively little is known on their resistance to low-velocity impact loads. This paper provides some results on impact testing of GFRP box-section beams and also considers the possibility of using inverse analysis to detect the observed damage. Focus is given on the need for static testing of the beams to measure the reduction in apparent stiffness to reveal the damage in-situ.

## INTRODUCTION

Following a review by Richardson & Wisheart [1] it is seen that the majority of research into low-velocity impact damage sustained by fibre-matrix composite materials has focussed on flat plates or coupons. This is understandable since the experiments are simplified and can be used to predict, for example, the behaviour of aircraft skin panels under impact from foreign objects. To date, there has been relatively little investigation into the behaviour of more complex specimen geometries. To an extent this has been rectified by work on panels under different boundary conditions [2,3].

Box-section beams are widely used in a variety of load-bearing applications yet their behaviour under impact is not fully understood. Research is being performed in an attempt to understand the effects of varying the geometry of the composite box section and that of the loading/impactor in response to low velocity impacts. This information can then be used to develop analytical tools capable of accurately predicting the post-impact behaviour of structural components in service. The paper sets out the experimental work to date, describing and characterising the damage mechanisms. The problem of detecting and characterising the damage is also presented as an inverse problem. In theory, experimentally determined data from the response of the structure under static load is used to modify a theoretical or analytical model to characterise the damage by degrading the stiffness of certain elements of the model. In this instance, the analytical model is a finite element (FE) model with sufficient elements to characterise the geometry of the box-section and its likely damage under impact. To detect damage using inverse techniques is a demanding exercise, however, it offers the means to perform inspections in-situ and at low-cost. One of the stages in implementing inspections by an inverse technique is to apply static tests to provide sufficient information to enable damage to be detected as accurately as possible with minimum effort. The latter part of this paper focuses on the issue of what form the tests should take with reference to damage in box-sectioned beams by low velocity impact.

## LOW-VELOCITY IMPACT EXPERIMENTS

The test samples used were glass-fibre reinforced plastic (GFRP) pultruded box-sections of dimensions 600 by 50mm and a wall thickness of 3.5mm. They consisted of alternate layers of unidirectional fibres and discontinuous chopped strand mat bound in an epoxy resin. Each sample was subjected to quasi-static low-velocity impact equidistant between two simple supports (Fig.1a). The impactor was a $90^o$ knife-edge aligned perpendicular to the longitudinal axis of the beam with the contact edge parallel to the impacted face of the beam. A low-velocity rate of impact was maintained throughout the tests; the final resting vertical displacement of the impactor was used to vary and measure the severity of the test. The samples appeared to fail in the same manner for each test. Damage observed by 'eye' takes the form of two longitudinal cracks running along the corners of the box section (Fig.1b). Other visible damage takes the form of smaller matrix cracks under the impactor due to localised crushing and shear cracks initiating at the tips. Load-deflection data was plotted on a chart recorder as shown in Fig.2. Each test is labelled by the maximum deflection of the impactor before the test was stopped. Plotted graphs show a peak force of approximately 6 KN occurring at 9-10mm deflection.

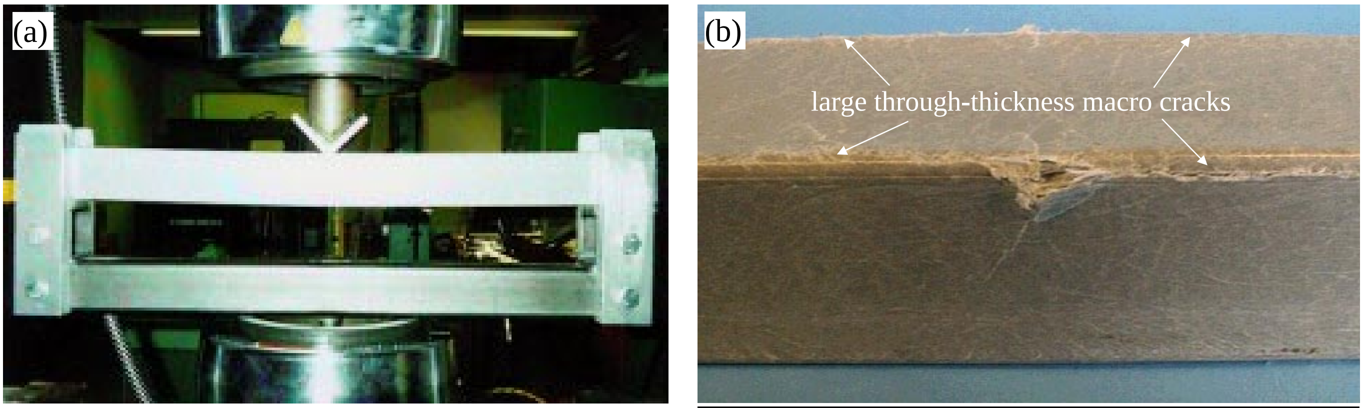



Figure 1: Longitudinal box-sectioned composite beams:
(a) Three-point bend loading arrangement and (b) Photograph of damaged surface.

In each test the load deflects the impacted surface and matrix cracks grow along the corners (Fig.1b). The upright surfaces deflected outwards under a 'buckling' load and suffered localised penetration at the impactor point with cracks more visible as the deflection increased. These were characteristically aligned at $45^o$ in the same orientation as the impactor. V-shaped notches were visible after the load was removed and fibre breakage was present around the impact site. At higher deflections matrix cracks could be seen on the upright surfaces. This behaviour can be explained by the properties of the samples tested. The layers of unidirectional fibres parallel to the beam axis give the beam good longitudinal properties and transverse stiffness is provided by layers of chopped strand mat. Hence cracks form at the corners where the high shear stresses exceed the material properties supplied largely by the resin. The bottom surfaces showed little signs of damage although small cracks at the corners were visible at maximum deflections.

The question now posed is how could the damage be detected by inverse analysis? In other words how could the response from of a series of static tests applied to the damaged beam be used to identify the damage? The longitudinal cracks and the smaller matrix cracks may be modelled using the FE method by de-equivalencing nodes along the length of the cracks. The aim of the inverse analysis is, therefore, to obtain information that enables the undamaged FE model of the beam to be adjusted to include the de-equivalenced cracks. Inverse techniques are not yet fully developed but research into examples of known damage should lead to technique development that may then be used for inspection of composite structures. The advantages of inverse techniques compared to laboratory-based passive techniques such as C-scan are that they be performed in-situ at low-cost under active loads.

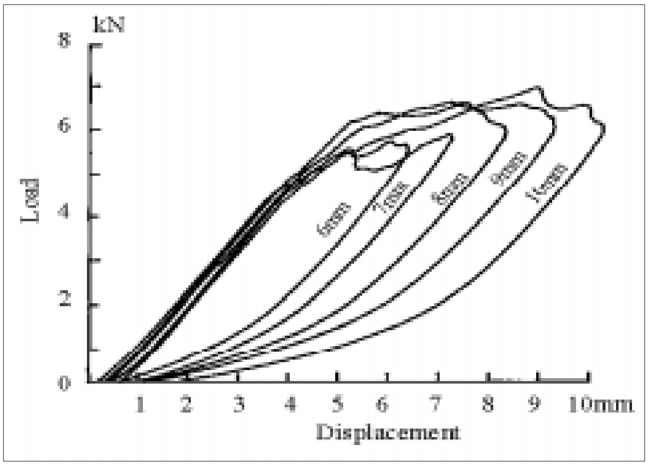



**Figure 2**: Load-Displacement Graphs.

## INVERSE ANALYSIS

In a structural analysis, the "direct" problem consists of determining the "response" of a structure, discretised into elements (of defined geometry and material properties), to external inputs. Consider the matrix relationship:

$$\{U\}=[S]\{P\} \tag{1}$$

between the unknown "response" of each element, $\{U\}$, the "causes" of this response, $\{P\}$, and the "system" matrix $[S]$ defining geometry and material(s). Generally, expression (1) above leads to a well-defined solution and represents the vast majority of structural problems solved by the Finite Element (FE) method. Now consider that either the causes $\{P\}$ or the system $[S]$ are unknown and are to be determined from the response $\{U\}$. This is termed an "inverse" problem. If $\{P\}$ contains unknowns the problem belongs to the category of inverse problems of the 1$^{st}$ kind. If internal material parameters included in the elements of $[S]$ are unknown, this belongs to the category of inverse problems of the 2$^{nd}$ kind. Determination of the Young's modulus ($E$) of a material in the standard "tensile test" is an inverse problem of the 2$^{nd}$ kind in its simplest form. Since there are normally more elements in the system $\{S\}$ than there are in the response $\{U\}$, inverse problems of the 2$^{nd}$ kind can be under-deterministic. The additional information required may be obtained from further (unequivocal) testing, or by constraining the solution (based on scientific experience).

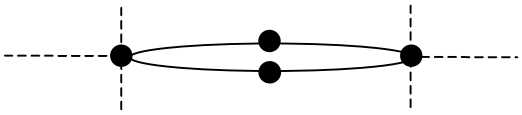
There are five main stages that to perform an inverse analysis to detect damage. These are: (I) Determination of a mathematical model for the structure; (II) Determination of a mathematical model for

the damage; (III) Static testing to obtain response data; (IV) Formulation of the objective function; (v) Minimisation of the objective function to determine damage. Issues concerning (I) and (II) have been reported by the authors elsewhere [4,5]. Stages (IV) and (V) may draw on the experience of system identification techniques used in civil engineering. This paper focuses on item (III).
The structure may be inspected in a series of unequivocal static tests in which the measured responses (displacements, strains or stresses) are due to a series of known inputs (loads or deflections). These static tests do *not* have to involve the service loads normally attributed to the structure. Inverse analysis in this form was first applied to large civil engineering frameworks where the structure is formulated as a mathematical model based on the FE method. System identification techniques are used to determine damage in framework elements characterised by degraded constants of elasticity. The resulting FE model of the damaged structure need only be sufficiently accurate to 'flag' the possible existence of dangerous flaws to the inspectors. Use of more accurate methods would then be implemented; inverse analysis was intended to prevent unnecessary use of more expensive laboratory-based methods.

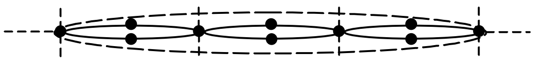
One must first produce a FE mesh of the undamaged composite structure. Experience is relied upon to determine how many elements are required and to what extent the material properties are homogenised. Experimental test(s) are then performed by applying static load(s) to the damaged structure and data is collected through strain gauges, whole-field optical techniques etc. Stiffness values in the mesh are adjusted so that results from the FE model match the measured response in a suitable manner. One way of "introducing" damage mathematically is through the stiffness of each element via the orthotropic elastic material constants ($E_{ij}$, $\mu_{ij}$, i,j=1,2,3). These may be the intended values for the material or may be degraded considerably due to the presence of defects or flaws following some damaging event. The effects of widespread porosity or microscopic flaws, for example, would be homogenised at element level, and characterised by a degraded value of the material constant(s). Alternatively, larger finite-sized defects or cracks may be determined as an actual part of the FE geometry if the element sizes used are sufficient to resolve them. Orthotropic material properties may be considered as unknowns to be solved through matching the FE response with test data. Given that with more elements the number of unknown properties rises accordingly it is anticipated that a large amount of unequivocal test data has to be collected. Damage due to porosity or high populations of micro-voids will be detected through a degraded, homogenised value of $E$ for each element representing the damaged material. Alternatively, large cracks may be resolved by elements of zero stiffness.

For damaged composite materials the FE method can also be used to model cracks or delaminations by de-equivalencing nodes between adjacent elements. The first step is to find the displacements (or stresses) in the *undamaged* FE model under the static test load(s). The response of the FE model differs from that of the damaged structure due to the under-compliance of the undamaged structure. De-equivalenced nodes forming 'cracks' are then distributed within the mesh in such a manner that any errors between the FE and measured responses are eliminated. Fig.3(a) shows how de-equivalenced mid-side node(s) form a crack between two, eight-noded, quadrilateral elements. Conceptually, a longer crack can be thought of as a series of these distributed in a line (Fig.3b) though in practice the crack would 'open' as shown by the hatched ellipse. A solution is required for the linear system defined as $g(x,y)=\Im f(x,y)$, where the output $g(x,y)$ is the cracked (damaged) FE model, and $\Im$ is the mathematical operator acting on the input $f(x,y)$ that represents the solution to each individual pair of de-equivalenced mid-side nodes in the FE model. One may determine the solution for each pair and then $\Im$ is found by finding the which combinations of $f(x,y)$ yield minimum difference between the response of the FE model and the damaged structure.

Fig.4 shows the Von-Mises stresses calculated using the FE method for a damaged box-section GFRP beam. Damage has been introduced in the form of de-equivalenced cracks at appropriate locations. This has been performed to verify that the damage model is agreement with experimental data [4]. It should be noted that this is not the result of an inverse analysis.



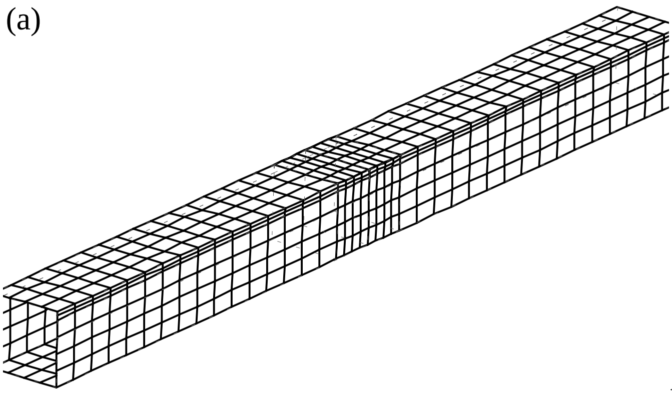
(a) Single de-equivalenced noded crack.



(b) Longer de-equivalenced crack.

Fig.3: Cracks by de-equivalencing of mid-side nodes.

A third method to detect damage is to relax strain energy between the damaged and undamaged states. The approach taken is attractive since it is mesh independent. As before, the first step is to find the displacements (or stresses) in the undamaged FE model except this time it is under the experimental boundary conditions of the damaged structure. There will be more strain energy in the FE model due to the under-compliant nature of the undamaged structure. It is necessary to make the structure more compliant by introducing numerical 'damage' through FE routines based on a pseudo 'fracture' parameter to degrade element stiffnesses. The fracture parameter is chosen such that there is no residual strain energy between the FE model and the actual damaged structure, and does not necessarily correspond to a given material property. Application of this method requires a non-linear algorithm so that the effects of 'load-shedding' may be taken into account. This approach may model forms of micro-flaw damage as well as finite-sized cracks.



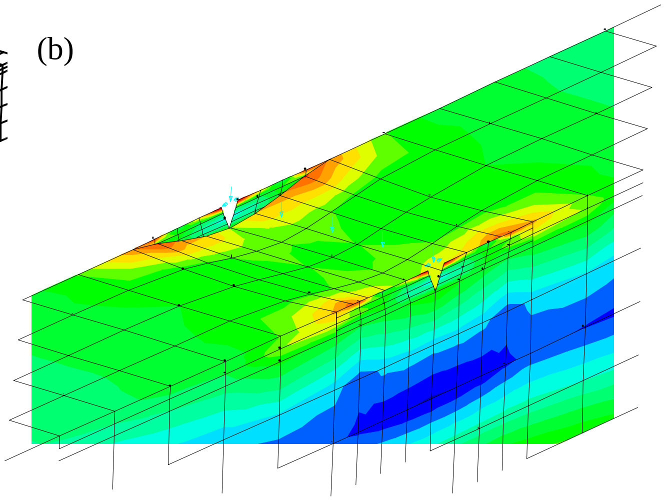




Fig.4: (a) Equivalent FE mesh and loading; (b) Von-Mises stresses with de-equivalenced cracks.

**APPLICATION OF STATIC TESTS**

A programme of static tests has been implemented to assess the response of the damaged samples to an inverse analysis. The three-point bend test has been chosen as the static test since it represents the simplest form of loading that may be applied to the beam specimen. It is important in inverse techniques to be able to produce a large quantity of unequivocal test data and the three-point bend test may be easily manipulated to produce sufficient variation in the loading. That the damage was actually caused by the three-point bend test should be seen as purely coincidental. The same static test procedure is applicable to detecting damage in the beam specimen whatever the origin of the damage.

The procedure followed is explained with reference to Fig.5 that shows a schematic of the loading arrangement. The three-point loading as applied has a reduced span to that which caused the damage. The load-deflection curve was measured for the downward force applied at different locations of the three-point bend as it traverses the length of the beam. These are denoted by (Mid), (1-5) and (End) as detailed in Fig.5. In words, the downward force and the two upward forces maintain the same separation but the damaged sample is moved to the left for increasing *D*. It may be observed that (Mid) corresponds to the downward force being applied to the original impacted location. The damaged beams analysed varied in damage severity according to the maximum impactor deflection as shown in Fig.2. The higher the maximum deflection the longer the visible cracks. Beyond a certain value of *D* these cracks (Fig.1b) will no longer be within the span *L* depending on their length. It is to be expected that the results of these static tests will not be able to 'detect' the damage. Therefore, in an inverse analysis of this form one needs to be able to perform sufficient unequivocal tests to be able to locate and resolve the size of cracks. For comparison an undamaged sample was analysed in the same manner.

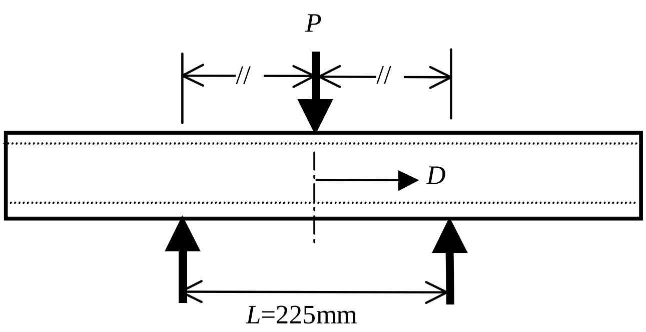



Fig.5: Static test arrangements: (Mid) *D*=0mm; (1) *D*=25mm; (2) *D*=50mm; (3) *D*=75mm; (4) *D*=100mm; (5) *D*=125mm; (End) *D*=Max.

For three damaged samples and an undamaged sample the results of the load-deflection measurements are graphed in Fig.6. Fig.6(a) shows the undamaged specimen and as expected there is no perceivable difference between the curves for different loading locations denoted by *D*. For the damaged samples it may be observed that the curves do not coincide for some of the tests. In all of them the tests denoted by (4), (5) and (End) all coincide; this depicts the situation where the static test has been applied outside the damaged region. Where the test witnesses the damage the load-deflection curve responds accordingly. For the severest case of damage, i.e. the 13mm deflection (remembering that this is the deflection of the impactor when causing the damage), the reduced stiffness of the beam is observed for tests (Mid) and (1-3) with increasing effect as more of the damaged area is exposed to the three-point bend test span. For the other two, with decreased severity of damage, fewer of the tests are able to witness the damage but nonetheless the damage is exposed for some of the static tests.

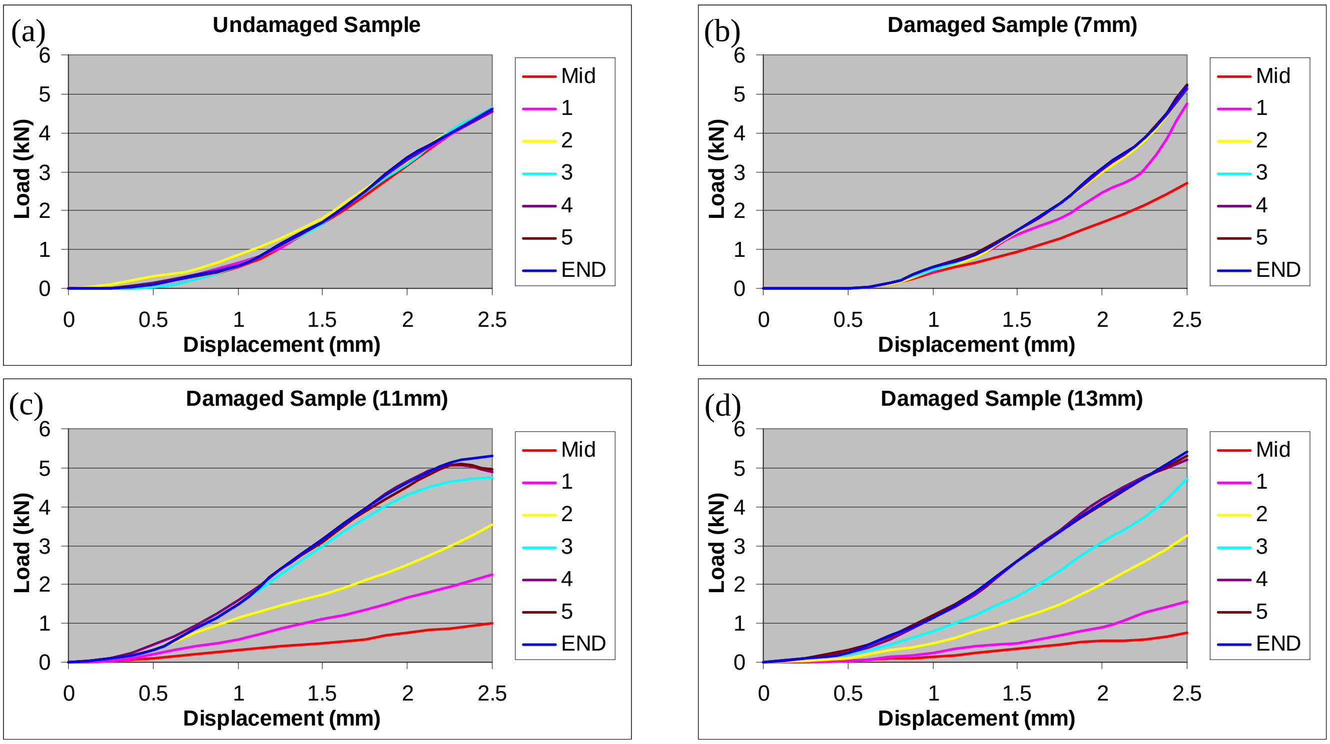



Fig.6: Load-deflection results of three-point bend static tests at different locations:
(a) Undamaged sample; (b) Damaged sample (max. defl.=7mm);
(c) Damaged sample (max. defl.=11mm); (d) Damaged sample (max. defl.=13mm).

The information contained in the graphs of Fig.6 may now be used to perform the remaining stage of the inverse analysis. This is to quantify the damage having chosen a suitable mathematical model for the structure and the damage, and formulated the objective function. The process involved in solving the objective function is still the subject of on-going research. The results presented thus far are encouraging in that the static tests have yielded a comprehensive set of accurate data for comparatively little effort. It should be mentioned that the static tests applied assumed that the damaged surface was the upper surface. In reality this would be unknown but, for this geometry at least, it is a simple task to repeat the same static tests for the four rotations of the beam about longitudinal axis.

It is perhaps obvious but nonetheless worthwhile to state that it is important that in applying the static test loading that it does not impart further damage to the structure.

An alternative to the static test arrangement used here is to use of an optical technique to determine whole-field displacement or strain data. In an accompanying paper applications for inverse techniques are described [5] in which shearing interferometry is used to obtain displacement gradient information. Considerably more data is involved for just one static test and this may prove invaluable for more complex components. In many applications only one static test would be required to obtain quantitative information on the damage.

## CONCLUDING REMARKS

Some observations on the behaviour of box-sectioned composite beams under low-velocity impact are presented. In a series of static tests for a potential inverse analysis it is clearly demonstrated that under a three-point bend the damaged sample responds with a reduced stiffness when the loading is directed through the impact damage. This may be used in subsequent calculations to characterise the damage using one of three approaches to modification of a FE model as described.

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