ESTIMATION OF STRESS INTENSITY FACTORS WITH CONSIDERING CRACK SURFACE CONTACT

X. B. Lin

nCode International Limited,
230 Woodbourn Road, Sheffield S9 3LQ, UK

ABSTRACT

Negative stress intensity factors are often determined for cracks within a compressive stress field. However, it should be aware that negative solutions are, in a physical sense, incorrect. The faces of a practical crack that lies in a stress field containing partial or full compression are anticipated to be partially or completely closed, and impossibly to interpenetrate anywhere. Such crack surface contact will alter the stress intensity factor or its distribution along the crack front if the crack is a surface or embedded one, and this, therefore, should be taken into account in the estimate of both stress intensity factors and fatigue lives. This paper describes a method of estimating the stress intensity factors by using the well-known finite element based displacement method with considering the possible closure of surface or embedded cracks with an arbitrary stress field. Both physically incorrect and correct models are analysed; the former allowing the crack faces to be freely deformed, the latter preventing them from inter-penetrating each other anywhere. A kind of interface element is used to simulate such possible crack surface contact for the latter case. The method is demonstrated through several calculation examples, including semi-elliptical surface and elliptical embedded cracks in a finite thickness plate with a residual stress field, and semi-elliptical surface cracks in an autofrettaged cylinder. The stress intensity factors are compared with the results available in the literature.

INTRODUCTION

It has been realized that residual stresses due to an undesired consequence of the manufacturing and joining technologies (welding and cold forming) or an intentional process for improving the fatigue resistance (shot peening and autofrettage) have an influence on fatigue crack propagation for practical components and structures. They could either speed up or slow down the fatigue crack propagation, depending on the nature of both residual stresses and cracked geometries. In order to assess the influence of residual stresses on the integrity of these components and structures, solutions of stress intensity factors for cracks within a residual stress field are required because the prediction of fatigue crack propagation life is based on them.

In the past two decades, a great effort has been devoted to the development of techniques that can be used to estimate the stress intensity factors for planar cracks subjected to a residual stress field. The weight function method has often been used to deal with a complex residual stress distribution with good computing efficiency, but the finite element based method has also been regarded as a powerful tools for the evaluation of the stress intensity factors for a practical cracked component or structure. In addition, the superposition method has been widely used because of its simplicity. However, it has been noticed that negative stress intensity factors are often determined by the above methods. Obviously, negative solutions are, in a physical sense, incorrect. The faces of a practical crack that lies in a residual stress field containing partial compression are anticipated to be partially or completely closed, and impossibly to interpenetrate anywhere. Such partial crack closure, therefore, should be taken into account during the calculation of stress intensity factors if any negative stress intensity factor results have been predicted by the superposition method or other methods.

This paper aims to investigate the possibility of using a finite element based one-quarter point displacement method to evaluate the stress intensity factors. Both physically incorrect and correct models are analysed; the former allowing the crack faces to be freely deformed, the latter preventing them from inter-penetrating each other anywhere. A kind of interface element is used to simulate such possible crack surface contact for the latter case. The method is demonstrated through several calculation examples, including semi-elliptical surface and elliptical embedded cracks in a finite thickness plate with a residual stress field, and semi-elliptical surface cracks in an autofrettaged cylinder. In particular, the results obtained by Cordes and Joseph [1], using the line spring model, are compared with the present estimates by the one-quarter point displacement in order to demonstrate the numerical accuracy of the finite element based method.

NUMERICAL PROCEDURE

Estimation of stress intensity factors by the finite element method

There are several methods available for the estimation of stress intensity factors for planar cracks, such as alternating, weight function, body force, boundary element and finite element methods. However, the finite element method has been a popular one, because of its versatility of generality for complex cracked geometries. One of the finite element based methods is the well-known one-quarter point displacement method, which was suggested by Henshell and Shaw [2] and Barsoum [3]. This method simulates the square-root theoretical singularity of stresses and strains adjacent to the crack tip by relocating the mid-side nodes of 20-node isoparametric elements to one-quarter positions. The displacements at the one-quarter nodes behind the crack tip, obtained from a finite element analysis, are used to evaluate the stress intensity factors along a crack front as follows, see Figure 1(a):

$$K = \sqrt{\frac{2\pi}{r_{(1/4)}}}\frac{Eu_{z(1/4)}}{4\left(1-\nu^2\right)} \qquad (1)$$

where E is Young’s modulus; ν is Poisson's ratio; and $r_{(1/4)}$ and $u_{z(1/4)}$ are the distance of the one-quarter point away from the crack tip and the outer-of-plane displacement at the one-quarter points. This method has been employed by the author to evaluate the stress intensity factors for a variety of cracked geometries, and found to be able to achieve good numerical accuracy generally [4-8].

Consideration of crack surface contact

In order to evaluate the practical stress intensity factors in a residual stress field, interface elements have been introduced into the cracked surface, as shown in Figure 1(b). With zero thickness and quadratic shape functions, the interface element has a capability that the crack faces are not allowed to overlap but can be open freely as usual. One surface of the interface elements is constrained in the direction normal to the crack plane if the geometry is symmetrical. The interface element is of 18 nodes, so the common 20-node mesh shown in Fig. 1(a) has been modified to that shown in Fig. 1(b), where some 27-node elements are arranged to match the 18-node interface elements. Such match of elements permits the crack faces to be closed partially within a single interface element, so that the calculation accuracy could be increased. More details about the interface element and mesh configuration around the crack front can be found in the author’s previous work [6].

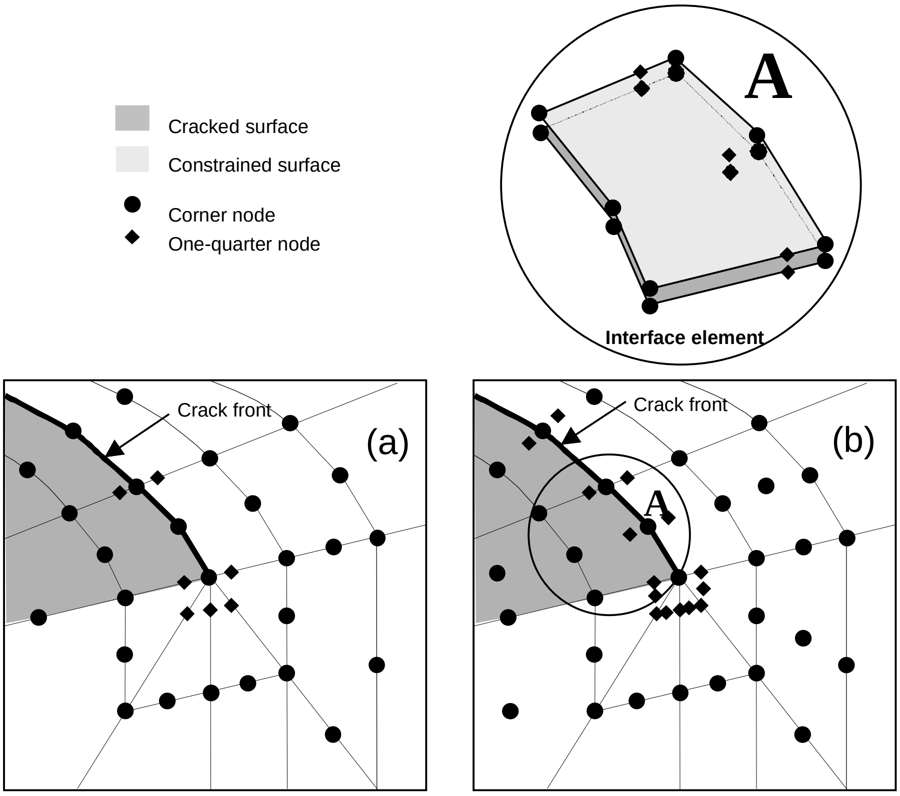

Figure 1: Illustration of mesh configuration around the crack front

CALCULATION EXAMPLES

Several calculation examples are given below. Comparison of stress intensity factor results for surface and embedded cracks in a residually stressed plate is first performed with those obtained by Cordes and Joseph [1]. The application of the present numerical method to a practical engineering problem is then reported. This is related to the evaluation of stress intensity factors for internal semi-elliptical surface cracks in a cylinder that has received an autofrettage treatment prior to its use.

Surface and embedded cracks in a finite thickness plate with a residual stress field

Cordes and Joseph [1] have published the stress intensity factor results for several surface and embedded cracks in a finite thickness plate with a residual stress distribution existing on the crack plane, see Figure 2. The residual stress distribution can be expressed in the following function,

$$\sigma(x) = \sigma_0\left(1 - 3\left(1 - \frac{2x}{t}\right)^2\right) \tag{2}$$

It can be seen from Figure 2 that a large compressive stress exists near the free surfaces. The stress intensity factors for the same cracked geometry subjected to the above residual stress distribution have been estimated in this study by using the one-quarter point displacement method described above.

Figure 3 shows the comparison of stress intensity factor results for a semi-elliptical surface crack of a/c = 0.3 and a/t = 0.6 with the results of Cordes and Joseph, which are directly obtained from their paper [1] and re-plotted using a different normalization factor, $\sigma_0\sqrt{\pi a}$. The results for both inclusion and exclusion of crack surface contact are presented together. It can be found that there is good agreement between the present results with Cordes and Joseph's estimates. For the case of excluding crack surface contact, positive and negative stress intensity factors are obtained at the crack depth and free surface positions, respectively. This means that the overlapping of crack surfaces occurs near the free crack surface due to the large compressive

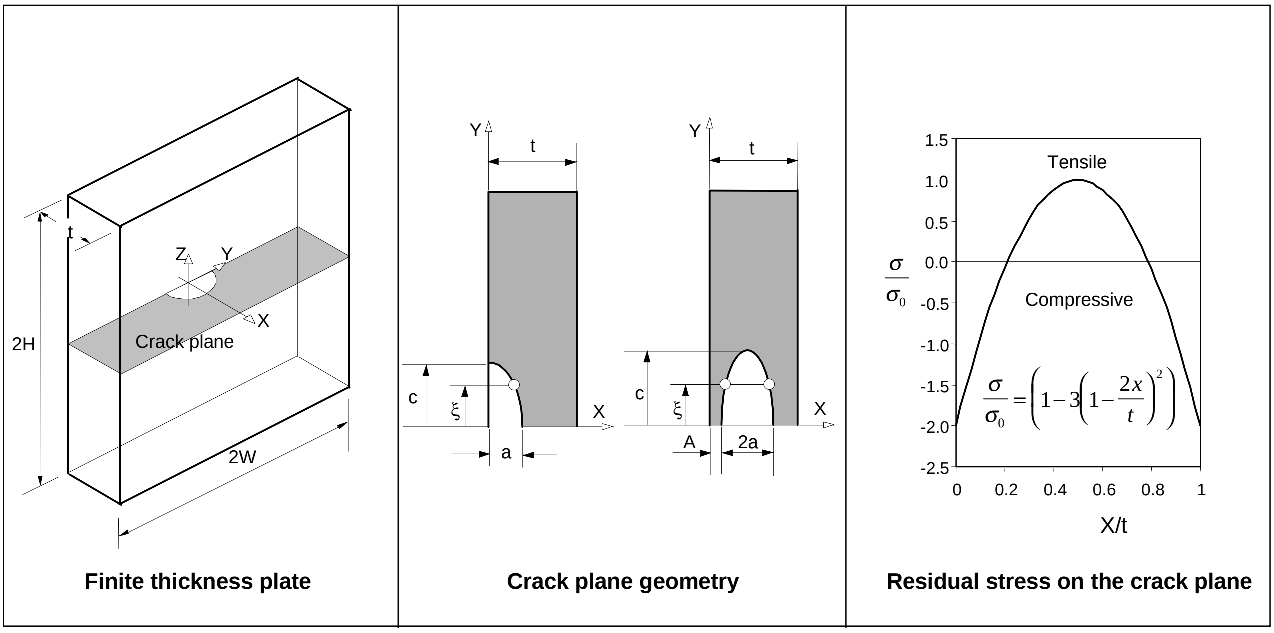

Figure 2: A residually stressed finite thickness plate with a surface or embedded crack

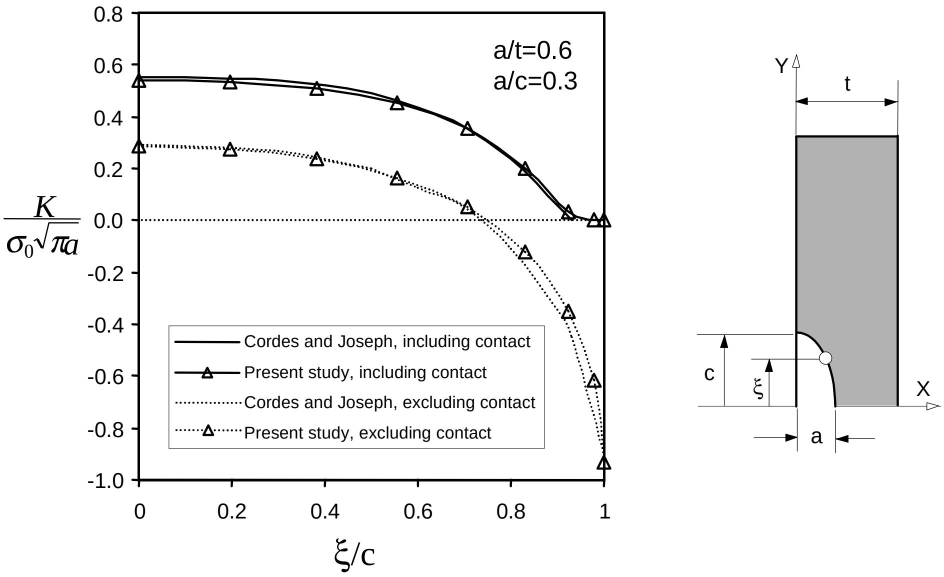

Figure 3: Comparison of stress intensity factor results with Cordes and Joseph's estimates for the surface semi-elliptical crack of a/c = 0.3 and a/t = 0.6, with inclusion and exclusion of crack surface contact

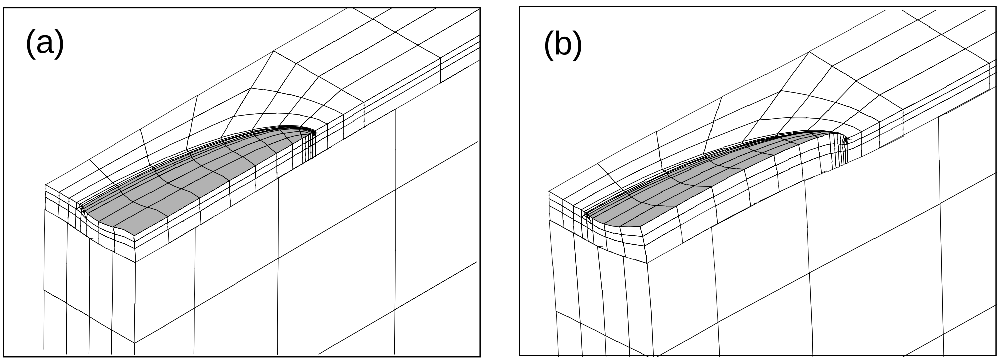

Figure 4: Meshes after deformation with inclusion (a) and exclusion (b) of crack surface contact for the semi-elliptical surface crack of a/c = 0.3 and a/t = 0.6

residual stress, but the crack is still open at the crack depth position. After introducing the interface elements, the stress intensity factor in the overlapping region is found to becomes zero, i.e. the crack surface is closed near the free surface. Such crack surface contact makes the stress intensity factors elsewhere increase. The normalized stress intensity factor at the crack depth point increases from about 0.29 to 0.55. The partial meshes after deformation are shown in Figure 4 for both inclusion (a) and exclusion (b) of crack surface contact. It is clear that the crack surface overlapping occurs if no interface elements are introduced into the cracked area, but such overlapping can be correctly prevented by the interface elements.

Figure 5 shows the comparison of the stress intensity factors obtained from the one-quarter point displacement method with Cordes and Joseph's estimates for three other semi-elliptical surface cracks. Crack surface contact has been considered for these three cracks. The comparison shows that the present results agree well with Cordes and Joseph's estimates, with the maximum difference being less than 10%. It can be found that the surfaces of these three cracks are all closed near the free surface, because the stress intensity factors become zero as the value of ξ/c approaches 1.

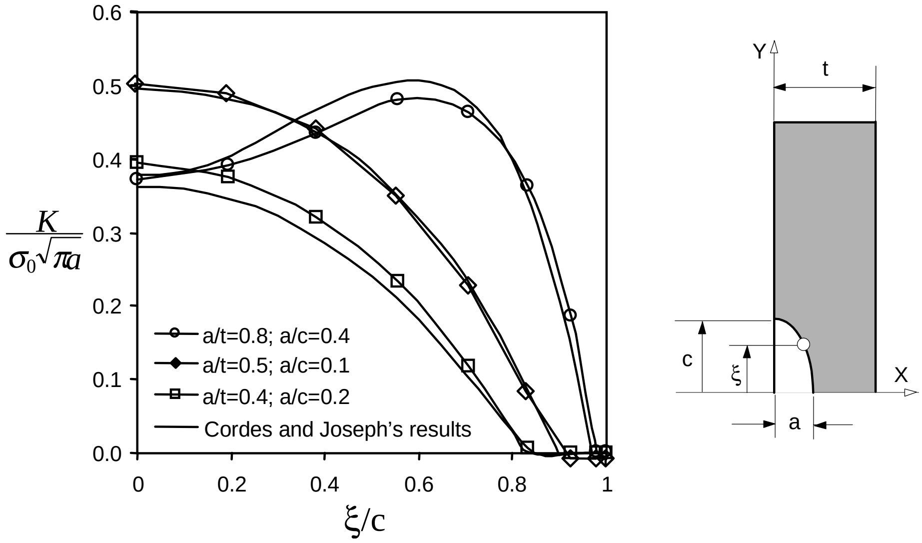

Figure 5: Comparison of stress intensity factor results with Cordes and Joseph's estimates for three semi-elliptical surface cracks with including crack surface contact

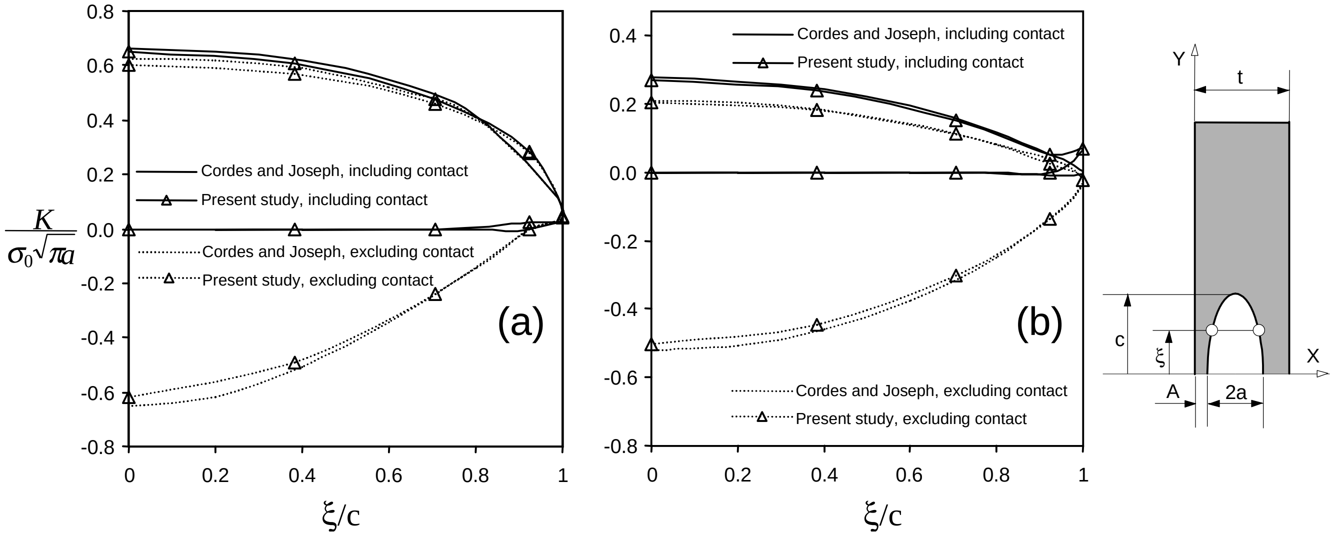

Figure 6: Comparison of stress intensity factor results with Cordes and Joseph's estimates for two embedded elliptical surface cracks with including crack surface contact,
(a) A/t=0.05, a/t=0.2, a/c=0.1; (b) A/t =0.1, a/t=0.1, a/c=0.05

Semi-elliptical surface cracks in an autofrettaged cylinder

Autofrettage is a common process of producing residual stresses in the wall of a thick-walled cylinder prior to use. A proper pressure, large enough to cause the material yielding within the wall, is applied to the inner wall of the cylinder and then removed. As a result, the inner wall is left in a state of circumferential compression whilst the outer wall is in tension. This process has been experimentally verified to be effective in delaying fatigue crack growth for the cracks initiated at the inner core under subsequent alternating internal pressures [9-11], and, therefore, has often been used for enhancing the fatigue behaviour of, for example, gun barrels and high pressure pipelines.

The stress intensity factor variations along the crack front for two internal elliptical cracks are shown in Figure 6. Both the present and Cordes and Joseph's results are included for comparison. Clearly, they agree very well to each other for both the cases of including and excluding the crack surface contact. Partial crack closure occurs at the crack front position close to the free front surface.

An internal surface crack (a/t=0.6 and a/c=0.6) in a cylinder (t/R_i=1) that has received a 100% autofrettage treatment prior to use is investigated in this study. The geometry of the cylinder is shown in Figure 7. The analytical residual hoop stress across the cylinder wall due to the 100% autofrettage treatment has been derived by the author [6] as follows,

$$\sigma_{\theta}^{\mathrm{R}} = -p^{*} + \sigma_{\mathrm{s}}\left(1+\ln(r/R_{\mathrm{i}})\right) - \frac{p^{*}R_{\mathrm{i}}^{2}}{\left(R_{\mathrm{o}}^{2}-R_{\mathrm{i}}^{2}\right)}\left(1+\frac{R_{\mathrm{o}}^{2}}{r^{2}}\right) \tag{3}$$

where $p^{*}(=\sigma_{\mathrm{s}}\ln\left(R_{\mathrm{o}}/R_{\mathrm{i}}\right))$ is the autofrettage pressure that causes the complete (100%) yielding of the cylinder, and σ_{s} is the yield stress. Its distribution is also illustrated in Figure 7.

Figure 8 shows the stress intensity factor results (normalized by $\sigma_{\mathrm{s}}\sqrt{\pi a}$) for the crack of a/t=0.6, a/c=0.6 and δ=100%, obtained by the finite element analyses including and excluding the interface elements respectively. The crack is found actually not to be completely closed; a portion of the crack near its depth is still in an open state. The practical stress intensity factor as calculated by introducing the interface elements are larger than zero for this case, although the stress intensity factors obtained along the entire crack front are negative when the overlapping of the crack surfaces is allowed. Therefore, care should be taken when such a case is dealt with in the use of the superposition scheme. Figure 8(b,c) shows the deformed meshes predicted by the two finite element analyses; from Figure 8(b) an open region close to the crack deepest position can be clearly seen, whilst in Figure 8(c) the penetration across the crack faces is evident.

It can be anticipated that the crack may behave from initially being in a state of either partial or complete contact to being possibly fully open as the internal pressure applied increases; and, more importantly, the change in stress intensity factor is non-linear during the loading process if the contact exists. In order to reveal such a non-linear behaviour, five internal pressures, p=0, $\sigma_{\mathrm{s}}/12$, $\sigma_{\mathrm{s}}/6$, $\sigma_{\mathrm{s}}/4$, and $\sigma_{\mathrm{s}}/3$, have been analysed for the crack with a/c=1 and a/t=0.6 in a cylinder that has the residual stress distribution corresponding to the 100% autofrettage treatment. The results are shown in Figure 9, in which the stress intensity variations against the internal pressure at both the crack depth and surface points with and without using the interface elements are included for comparison. It can be seen that after the internal pressure increases to about $\sigma_{\mathrm{s}}/4$, the crack begins to become fully open; before $p<\sigma_{\mathrm{s}}/4$, the practical stress intensity factor at the surface point has been zero whilst the stress intensity factor increases non-linearly at the crack depth point. The non-linear relation between the internal pressure and stress intensity factor is obviously not seen if the contact is ignored (see the dashed lines in Figure 9). After the internal pressure becomes larger than $\sigma_{\mathrm{s}}/4$, the linear change in stress intensity factor is returned because of the disappearance of crack contact. From Figure 9, it can also be found that the contact of crack faces not only makes the stress intensity factor rise compared with the result from the superposition method, but also alters the loading form if fatigue calculations are involved.

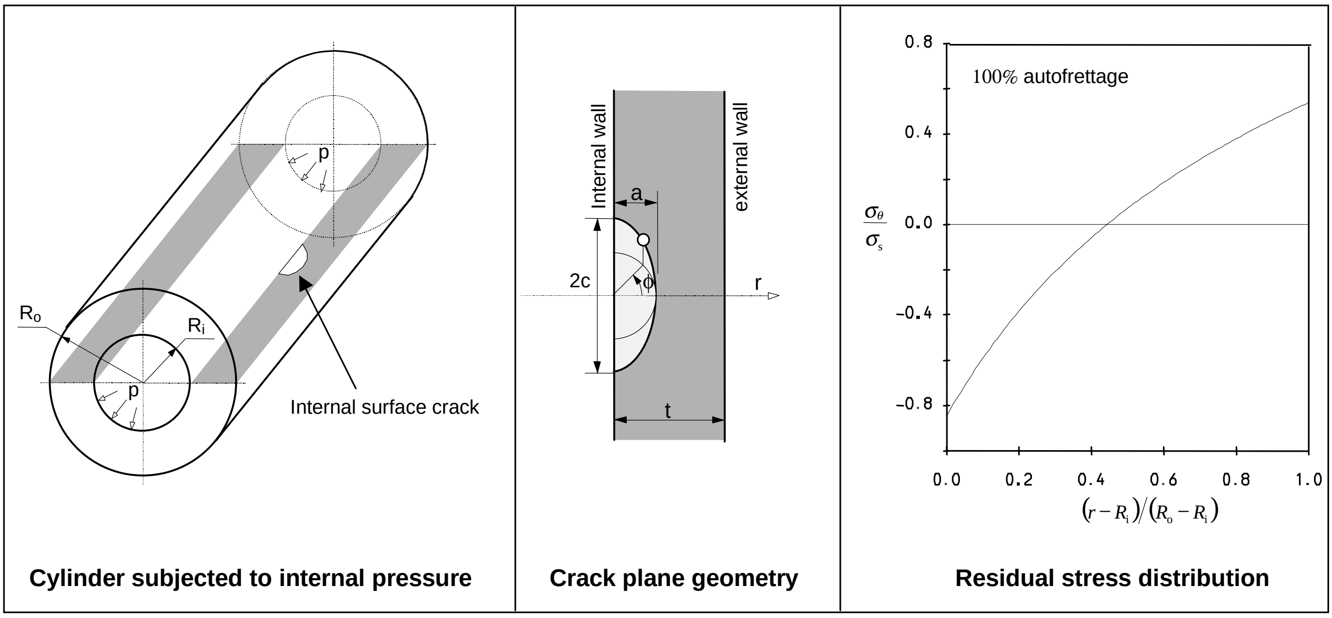

Figure 7: An autofrettaged cylinder with an internal semi-elliptical surface crack

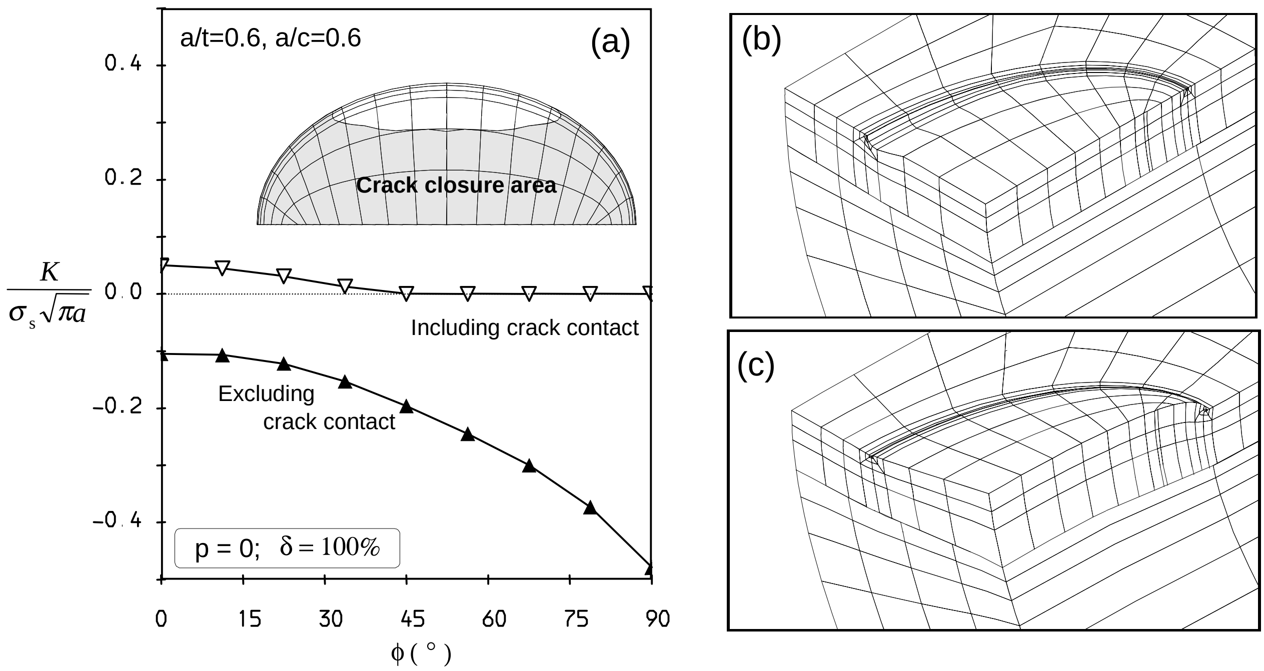

Figure 8: Stress intensity factor solutions including and excluding crack surface contact

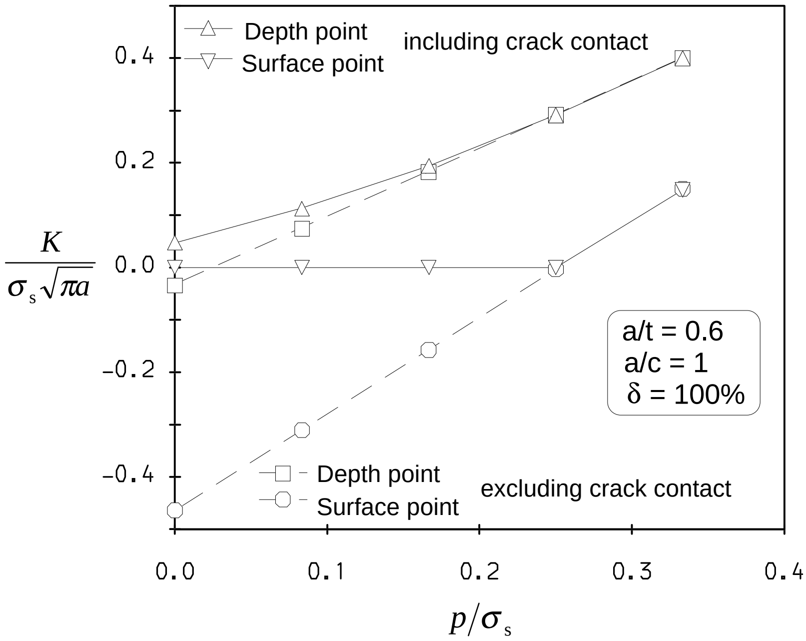

Figure 9: Stress intensity factor changes with increasing internal pressure

CONCLUSIONS

A method of estimating stress intensity factors for planar cracks with considering crack surface contact has been proposed and investigated. This is based on the finite element based one-quarter point displacement method. Crack surface contact is simulated by introducing the interface elements into the actual cracked area. Several calculation examples are given to demonstrate the numerical accuracy and applicability of this method. It has been shown that the crack surface contact can be simulated correctly, and that the results estimated by the present method are generally in good agreement with Cordes and Joseph's estimates from the line spring model for both surface and embedded cracks in a residually stressed plate.

REFERENCES

1. Cordes, R.D. and Joseph, P.F. (1994) *Int. J. Fracture* **66**, 1.
2. Henshell, R.D. and Shaw, K.G. (1975) *Int. J. Num. Meth. Engng* **9**, 495.
3. Barsoum, R.S. (1976) *Int. J. Num. Meth. Engng* **10**, 25.
4. Lin, X.B. (1994) *Numerical Simulation of Fatigue Crack Growth*, PhD Thesis, The University of Sheffield, UK
5. Lin, X.B. and Smith, R.A. (1997) *J. Strain Analysis* **32**, 229.
6. Lin, X.B. and Smith, R.A. (1997) *J. Strain Analysis* **32**, 351.
7. Lin, X.B. and Smith, R.A. (1999) *Engng Fracture Mech.* **62**, 535.
8. Lin, X.B. and Smith, R.A. (1999) *Engng Fracture Mech.* **63**, 523.
9. Throop, J. F. and Fujczak, R.R. (1982) *Experimental Mechanics* **22**, 277.
10. Stacey, A. and Webster, G.A. (1988) In: *Analytical and Experimental Methods for Residual Stress Effects in Fatigue*, *ASTM STP 1004*, pp. 107-121, Champoux, R.L., Underwood, J.H. and Kapp, J.A. (eds) American Society for Testing and Materials, Philadelphia.
11. Liu, C., Zhu, L., Wang, Y. and Tao, X. (1992) In: *Proceedings of the Seventh International Conference on Pressure Vessel Technology*, pp. 947-959, Germany.