A CRACK AND A DEBONDING INITIATION FROM A CIRCULAR RIGID INCLUSION UNDER UNIFORM TENSION OR COMPRESSION

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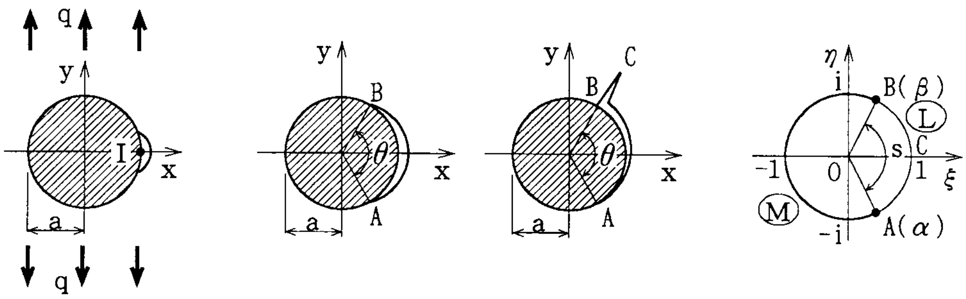
ABSTRACT

A model of debonding or debonding and crack at the interface of a circular rigid inclusion existing in an infinite elastic body is analyzed under uniaxial uniform loading in the y directions. It is investigated how the debonding develops along the interface of the inclusion under applied loading and when a crack occurs from the tip of debonding. The angle at which the debonding develops and the position of crack occurrence are determined. As the criterion for fracture, the strain energy release rates of debonding and crack are used. Moreover, the normal stress to the circular inclusion at the tip of debonding and the stress intensity factor of mode I of the crack are used as the restricting condition. The analysis is carried out as a mixed boundary value problem of plane elasticity. The rational mapping function of the sum of fractional expressions and complex stress functions are used for the analysis.

INTRODUCTION

It is well known that the fracture of the material containing inclusion occurs as debonding and crack due to the local stress concentration at the interface of the inclusion. According to the results of the experimental observation of materials, in which the rigidity of inclusion is larger than that of the base material and the adhesion of the base material with the inclusion is weak such as concrete and high strength steel, it was reported that first, a debonding occurs at the interface of inclusion and develops. It is also found that the debonding develops from initial defects at the interface. And it has been investigated that at the tip of debonding which is the singular point of stress, large stress concentration occurs, a crack arises, and thus fracture advances further. Thereupon, when the fracture originating at the inclusion is investigated, it is necessary to consider the problem, in which the debonding and crack are coupled. Already a number of problems on the inclusion accompanied by debonding on the interface of inclusion and crack existing near the inclusion have been analyzed [1], but the analytical solutions on the inclusion problem accompanied by both a debonding and a crack on the inclusion seem not many [3].

In this present paper, at the interface near Point I at a circular rigid inclusion existing in an infinite elastic body as shown in Fig.1a, models of the development of debonding (Fig. 1b), and of a debonding and a crack (Fig.1c) are analyzed under uniaxial tension or compression. Under the applied load, the conditions under which a debonding develops and under which a crack arises at a certain size of the debonding are investigated. Particularly when there are both possibilities of the debonding development and of the crack occurrence from the tip of the debonding, it can be decided which phenomenon actually occurs. Moreover, when the acting load is increased gradually from zero, the phenomena of fracture are also investigated. The



(a) z-plane (b) z-plane (c) z-plane (d) ζ –plane

Figure 1 : Analytical region (z-plane) and a unit circle(ζ -plane)

strain energy release rate obtained by this analysis and the fracture toughness value are used as the fracture criterion. Also the load at which a debonding develops or a crack arises can be determined. Cracking is analyzed as the case when it occurs at one tip of the debonding (Point B) shown in Fig.1b, but it is also the condition when cracking occurs simultaneously at another debonding tip (Point A).

The analysis under tension in the x direction has been reported by [2].

The rational mapping function of the sum of fractional expressions and the complex stress functions are used for the analysis, which is carried out as a mixed boundary value problem of plane elasticity. The inclusion is a rigid body, and then the analysis of stress and stress singular value in the state that both a debonding and a crack exist is feasible [3, 4, 5]. For the shape that the rational mapping function represents, the exact solution is obtained. The stress intensity of debonding expressing the magnitude of the stress singularity at the tip of the debonding and the stress intensity factor immediately after a crack initiation at the tip of the debonding are calculated. Using these stress singular values, the strain energy release rate of the debonding development and that of the crack occurrence are obtained. And by using these strain energy release rates as the fracture criteria, the phenomena of fracture due to the debonding and crack at a circular rigid inclusion are elucidated.

ANALYTICAL METHOD, STRESS INTENSITY FACTOR AND STRESS INTENSITY OF DEBONDING

As the stress analytic method, complex stress functions and a conformal mapping function are used. The mapping function which maps the infinite region of the outside of the circular hole with a crack as shown in Fig.1c into the outside region of the unit circle (Fig.1d) is formed as the rational function. And the mixed boundary value problem of the plane elasticity is solved, where the displacements on the rigid inclusion are zero and the stress is free on the debonding and crack surface. The solution was reported [3, 4].

The stress intensity factors K_I and K_{II} are obtained from the stress function and the following nondimensional stress intensity factor are defined,

$$F_I + iF_{II} = \frac{K_I + iK_{II}}{q\sqrt{\pi a}} \quad (q > 0) \tag{1}$$

“q” is a uniform tension and ”*a*” is a radius of the circular rigid inclusion. Also the strain energy release rate G_c of the crack is expressed by using stress intensity factors as follows:

$$G_c = \frac{(1+\kappa)}{8G}\left(K_I^2 + K_{II}^2\right) = \frac{\pi a(1+\kappa)}{8G}\left(F_I^2 + F_{II}^2\right)q^2 \tag{2}$$

where G is the shear modulus. κ is 3-4ν for plane strain state and (3-ν)/(1+ν) for plane stress state, and ν is Poisson’s ratio.
In this paper, the very short crack length of b/*a*=0.0005 and 0.001 normal to the boundary is used, and its

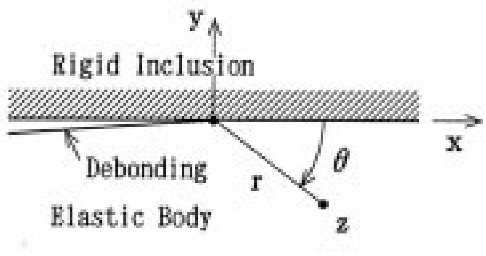


Figure 2 : Coordinates at the debonding tip

length is identified as the crack length just after the crack occurrence. G_c obtained by using these stress intensity factors is adopted as the strain energy release rate in crack occurrence.

As shown in Fig.2, when the origin is set at the tip of debonding, and the x-axis is taken as the direction of the debonding, and its normal direction is taken as the y-axis, the stress components on the bonded surface of the distance *r* away from the tip of the debonding are expressed as follows [6]:

$$\begin{aligned} \sigma_y &= \frac{1+\kappa}{r^{0.5}}\left|\tilde{\beta}_0\right|\cos\left[\theta_0+\gamma\cdot\ln(r)\right] \\ \sigma_x &= \frac{3+\kappa}{r^{0.5}}\left|\tilde{\beta}_0\right|\cos\left[\theta_0+\gamma\cdot\ln(r)\right] \\ \tau_{xy} &= -\frac{1+\kappa}{r^{0.5}}\left|\tilde{\beta}_0\right|\sin\left[\theta_0+\gamma\cdot\ln(r)\right] \end{aligned} \tag{3}$$

where θ_0=arg $\tilde{\beta}_0$ and $\gamma=(\log\kappa)/(2\pi)$. Similarly to the stress field near the crack tip, these stress components possess the singularity of –0.5 power in relation to the distance *r* from the tip of the debonding. $\left|\tilde{\beta}_0\right|$ in Eqn. 3 represents the magnitude of singularity at the tip of debonding. In the present case, in order to distinguish it from the stress intensity factor of the crack in homogeneous case, $\left|\tilde{\beta}_0\right|$ is named “stress intensity of debonding”. $\tilde{\beta}_0$ is calculated by the stress function.

In this paper, the dimensionless stress intensity of debonding defined by the following expression is used:.

$$F_d = \frac{\left|\tilde{\beta}_0\right|}{q\sqrt{a}} \qquad (q>0) \tag{4}$$

The strain energy release rate in debonding development G_d is expressed by using the stress intensity of debonding as follows [7]:

$$G_d = \frac{\pi\kappa(1+\kappa)}{2G}\left|\tilde{\beta}_0\right|^2 = \frac{\pi a\kappa(1+\kappa)}{2G}F_d^2 q^2 \tag{5}$$

CRITERIA FOR FRACTURE

As shown in Fig.1, the cases that (a) the debonding exists at Point I at the circular rigid inclusion under loading q and (b) the debonding develops to a certain position expressed by angle θ, or the initial debonding expressed by angle θ is considered. (c) At this time, which behavior arises, the further debonding development or a crack occurrence at the tip of the debonding, is investigated. By this means, how the fracture phenomena due to the debonding and the crack occur can be determined. In order to examine under what condition debonding develops or cracking occurs, the strain energy release rate in the debonding development G_d and that in the crack occurrence G_c are used as fracture criteria. The fracture toughness

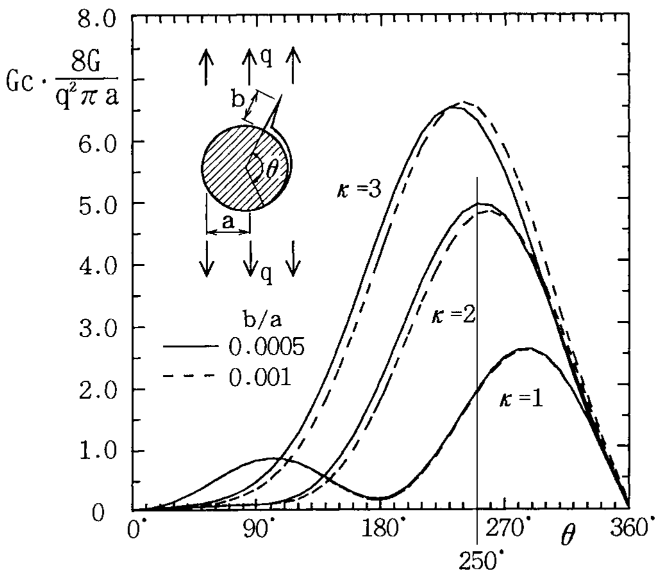


Figure 3 : Strain energy release rate of crack occurrence under uniaxial tension in the y direction

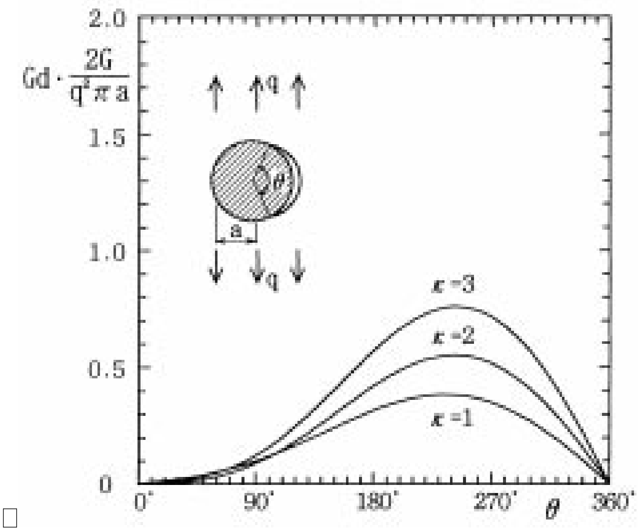


Figure 4 : Strain energy release rate of debonding development under uniaxial tension in the y direction

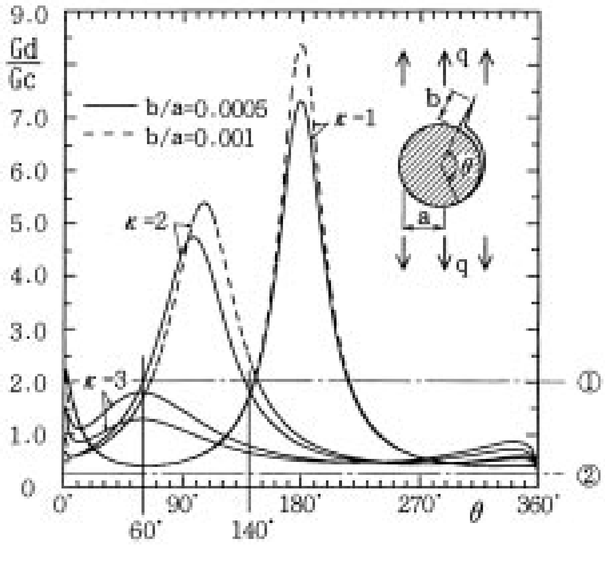


Figure 5 : Ratio of strain energy release rates of debonding development and crack occurrence under uniaxial tension in the y direction

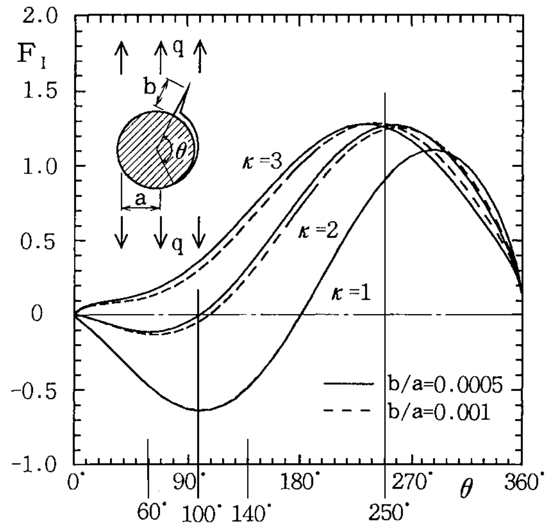


Figure 6 : Nondimensional Mode I stress intensity factor under uniaxial tension in the y direction

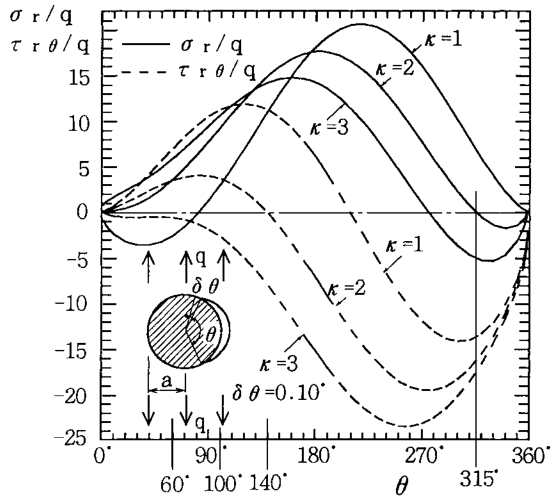


Figure 7 : Normal stress σ_r and tangential stress $\tau_{r\theta}$ near the debonding tip under uniaxial tension in the y direction

value related to the strength of adhesion of base material and inclusion generally is different from the fracture toughness value for the crack occurrence in the base material. The fracture toughness in the debonding development expressed by the strain energy release rate is denoted by G_{d0} and that in the crack occurrence is denoted by G_{c0}.

When the restricting condition mentioned later is satisfied, according to the relative magnitude of the values of G_d and G_{d0} and the values of G_c, and G_{c0}, the following matters for the fracture behavior can be stated:

(A) Case of $G_d < G_{d0}$ and G_c. $< G_{c0}$: neither debonding develops nor cracking occurs.
(B) Case of $G_d < G_{d0}$ and $G_c > G_{c0}$: debonding does not develop but cracking occurs.
(C) Case of $G_d > G_{d0}$ and $G_c < G_{c0}$: debonding develops but cracking dose not occur.
(D) Case of $G_d > G_{d0}$ and $G_c > G_{c0}$: there are both possibilities of debonding development and crack occurrence.

In (A), (B) and (C) mentioned above, respective fracture behaviors can be specified, but in the case of (D), it becomes necessary to judge further which actually occurs, debonding or cracking. For this purpose, the ratio of the strain energy release rates of debonding and cracking G_d / G_c and the ratio of the fracture toughness values of debonding and cracking G_{d0} / G_{c0} are considered. According to the relation of the magnitude of the values of G_d / G_c and G_{d0} / G_{c0}, the following matters can be stated:

(a) Case of $(G_d / G_c) < (G_{d0} / G_{c0})$, i.e. $G_d / G_{d0} < G_c / G_{c0}$:cracking occurs.
(b) Case of $(G_d / G_c) > (G_{d0} / G_{c0})$, i.e. $G_d / G_{d0} > G_c / G_{c0}$:debonding develops.

The analysis is carried out with the model that a crack occurs from one tip of debonding (see Fig.1c).

However, this is also the model that cracks occur simultaneously from both tips of debonding, because the cracks are minute just after they occur, and their mutual effect can be neglected and the inclusion is rigid. However, the state of crack development after the crack has occurred is not referred to in this paper.

In this paper, as the restricting condition of crack occurrence, the condition of the stress intensity factor of mode I just after crack occurrence being positive, namely the condition of $F_1 > 0$ is considered. However, strictly speaking, the condition to which also the stress intensity factor of mode II is related may be necessary. Moreover, the restricting condition of the debonding development is also used, and in this case, the condition that the normal stress σ_r at the interface near the tip of debonding is positive, namely $\sigma_r > 0$, is used. Strictly speaking, the restricting condition may be the function of σ_r and $\tau_{r\theta}$, which is the tangential stress near the tip of the debonding.

RESULTS OF ANALYSIS AND INVESTIGATION

The strain energy release rate in the crack occurrence G_c is shown in Fig.3 for uniform tensile load in the y direction, and κ=1, 2, 3. On the ordinate, G_c and on the abscissa, θ, the angle of circumference representing the size of debonding are taken. The value of G_c is shown for the micro crack lengths b/a=0.0005 and 0.001. There are some differences between them. The strain energy release rate for debonding development G_d is shown in Fig. 4, and the ratio G_d / G_c is shown in Fig.5. The mode I stress intensity factor F_1 of dimensionless form is shown in Fig.6. In this paper, $F_1 > 0$ is used as the restricting condition of crack occurrence. The normal stress σ_r / q and the shearing stress $\tau_{r\theta} / q$ at the interface of the location $\delta\theta = (0.1/180)\pi a$ apart from the tip of debonding are shown in Fig.7 with solid lines and broken lines, respectively. The sign of σ_r is positive in tension and negative in compression to the normal. Shearing stress is positive when $\tau_{r\theta}$ tends to turn in counterclockwise direction. In this paper, for simplicity, σ_r>0 is used as the restricting condition of the debonding development.

The restricting condition for the case of κ=2 is used for the account. According to Fig.6, F_1 is positive in $100^\circ < \theta < 360^\circ$, therefore, a cracking can occur in this range of θ. However in the range of $0^\circ < \theta < 100^\circ$ in which F_1 is negative, a cracking cannot occur. According to Fig.7, σ_r is positive in $0^\circ < \theta < 315^\circ$, Therefore, debonding can develop in this range of θ. However, in the range of $315^\circ < \theta < 360^\circ$ in which σ_r is negative, the debonding cannot develop.

In relation to an arbitrary initial debonding angle θ, and under the condition (A) of the criteria for fracture mentioned in previous section, neither debonding development nor crack occurrence arises, Namely if the respective values of G_{d0} and G_{c0} are larger than the respective values of G_d and G_c in Figs.3 and 4, neither debonding nor cracking occurs. If G_{d0} and G_{c0} are larger than the maximum values of G_d and G_c, respectively, debonding and cracking never occur for arbitrary values of θ.

Under the condition (B) of the criteria for fracture, the debonding does not develop. Accordingly, the value of G_{d0} is larger than G_d in Fig.4 for the initial debonding angle θ being considered. A crack occurs at the debonding tip of the angle θ with the restricting condition $F_1 > 0$ and $G_c \geq G_{c0}$. It is known that a crack is most apt to occur when the initial debonding is around $\theta = 260^\circ$ for κ=2, at which the curve of G_c for κ=2 takes the maximum value. Namely, a crack is occured by the smallest q.

Under the condition (C) of the criteria for fracture, a cracking does not occur. Accordingly, the value of G_{c0} is larger than that of G_c in Fig.3 for the debonding angle θ being considered. The debonding of the angle θ that satisfies the restricting condition $\sigma_r > 0$ and $G_d \geq G_{d0}$ begins to develop, and develops in the range in which $\sigma_r > 0$ and $G_d \geq G_{d0}$ are satisfied. On the other hand, when the strain energy release rate in the cracking G_c also becomes larger together with debonding development, the case of G_c exceeding G_{c0} during debonding development is conceivable. Then the fracture condition changes from the condition (C) to (D). In this way, the phonomenon of the fracture is not constant, but sometimes changes together with debonding development.

Under the condition (D) of the criteria for fracture, there are possibilities of debonding development and crack occurrence. Therefore, the judgment is made by using the ratio of the strain energy release rates in the debonding and cracking G_d / G_c in Fig.5. According to the ratio of fracture toughness value G_{d0} / G_{c0}, the following three cases can be classified. As example by taking some values of G_{d0} / G_{c0} and using the case of κ=2, the concrete explanation is made.

(i) Case of the value of G_{d0} / G_{c0} being larger than the maximum value of G_d / G_c, for κ=2 in Fig.5: the condition (a) in the previous section is always satisfied. Accordingly, a crack occurs at the debonding tip of the initial angle θ satisfying the restricting condition $F_I > 0$ which is known from Fig.6.

(ii) Case of the value of G_{d0} / G_{c0} being between the maximum value and the minimum value for κ=2: Explanation is made, for example, by assuming G_{d0} / G_{c0} =2.0 (Line ① in Fig.5). According to Fig.5, in the case that there is the initial debonding for 0°<θ<60° and 140°<θ< 360° (60° and 140° are at the intersection of Line ① and the curve G_d /G_c), the condition (a) in the previous section is satisfied, and $F_I > 0$ for 100°<θ<360°. Accordingly, a crack occurs at the debonding tip with the initial angle θ for 100°<θ<140°. In the case of 60°<θ< 140° in Fig.5, it comes under the condition (b). Since $\sigma_r > 0$ between 0° and 315°, the debonding develops till 140°. Therefore, a crack arises at θ=140° if $G_c > G_{c0}$ is still satisfied.

(iii) Case of the value of G_{d0} / G_{c0} being smaller than the minimum value for κ=2: Explanation is made, for example, by assuming G_{d0} / G_{c0} =0.25 (Line ② in Fig.5). In this case, the condition (b) is always satisfied. Accordingly, in the case that the initial debonding angle is 0°<θ<315° where σ_r>0, the debonding develops up to around θ=315° at which σ_r<0, and stops there. And if $G_c \geq G_{c0}$ is still satisfied there (G_c changes in the range of θ), a crack occurs.

In the above description, the fracture phenomena in relation to the criteria (A) ~ (D) were investigated when an arbitrary debonding angle θ is given, and a constant load *q* is applied. Next, the case of an applied load *q* gradually increasing from zero is considered.

As known from Eqn. 2 and Eqn. 5, G_c and G_d increase in proportion to q^2. The fracture phenomenon occurs by either G_c or G_d has reached first to the fracture toughness value G_{c0} or G_{d0} when the load *q* increases. Which reaches first to the fracture toughness value can be known by the relation of the magnitude in the ratio G_d / G_c and the ratio G_{d0} / G_{c0}. The value of G_d / G_c is not dependent on the magnitude of load *q*(see Eqns. 2 and 5). On the other hand, also the value of G_{d0} / G_{c0} does not change due to the variance in the magnitude of load *q*. From these facts, when the debonding tip is at the position of angle θ, the following matters can be said under the increase of load:

Case of (G_d / G_c) < (G_{d0} / G_{c0}) i.e. $\frac{G_d}{G_{d0}} < \frac{G_c}{G_{c0}}$: When load *q* increases, and $G_c = G_{c0}$ is attained, G_d is still satisfying $G_d < G_{d0}$. For example, in the case of κ=2, Line ① in 0°<θ<60° and 140°<θ<360° in Fig.5 comes under the present case. For example, under this condition and when an initial debonding angle θ is in 140°<θ<360°, in which F_1>0 is satisfied, a crack occurs at the tip of the debonding. The magnitude of *q* at this time is determined by $G_c = G_{c0}$. When the initial debonding is θ=260° at which the G_c curve for κ=2 takes the maximum value, a crack occurs at the smallest value of *q*.

Case of (G_d / G_c) > (G_{d0} / G_{c0}) : When load *q* increases, and $G_d = G_{d0}$ is attained, G_c is still satisfying $G_c < G_{c0}$. Accordingly, G_d first reaches G_{d0}. For example, in the case of κ=2, Line ② in 0 °<θ<360° or Line ① in 60°<θ< 140° in Fig.5 comes under the present case. Under this condition and when an initial debonding angle θ is in 0°<θ< 315°, in which σ_r > 0 is satisfied, the debonding starts to develop. The magnitude of *q* at this time is determined by $G_d = G_{d0}$. The debonding develops till θ=315° for Line ② and till θ=140° for Line ① while $G_d > G_{d0}$ is satisfied.

When $G_c = G_{c0}$ is attained with increasing load, a crack occurs at those angles, respectively.

CONCLUSIONS

The phenomena regarding the development of debonding at the interface of a circular rigid inclusion and the occurrence of a crack from the tip of the debonding were investigated. The considered load is the uniaxial uniform tension or compression of arbitrary magnitude in the y direction. The results of compressive load must be necessary for investigation of the compressive fracture. When q < 0, the sign of F_I, σ_r and $\tau_{r\theta}$ are changed. Then the range of F_I > 0, and σ_r > 0 has meaning. The strain energy release rate in the debonding development G_d determined by Eqn. 5 corresponds to the square of stress intensity of debonding.

As the criteria for fracture in debonding development and crack occurrence, the strain energy release rates G_d and G_c were used. By the relation of the magnitude of the fracture toughness value with that of the strain energy release rate, how the fracture phenomena occur due to debonding and cracking is determined. In particular, in the case of the condition (D) in the criteria for fracture, it is determined by the relation of the magnitude in the ratio G_d / G_c and G_{d0} / G_{c0} in the debonding and the cracking. Moreover, the load when a debonding develops or a crack occurs can be determined. This way can be applied to not only the case of constant load but also a load becoming gradually larger from zero. Besides, when a cyclic load acts, this way may be also applicable.

In this paper, as the restricting condition of debonding development, the condition of the normal stress σ_r, being positive at the interface near the debonding tip was used. As the restricting condition of the crack occurrence, the condition of the stress intensity factor F_I being positive immediately after crack occurrence was used. When other criteria for fracture and restricting conditions must be used, the investigation of the fracture may be carried out by using them in a way as well as that has been followed in this paper.

Since the strain energy release rates in the debonding development and the crack occurrence possess the maximum values, the debonding development and the crack occurrence can be prevented by using fracture toughness values larger than these values.

The strain energy release rates, stress intensity factors and stresses depend on the value of κ, which depends on Poisson's ratio, and κ affects the characteristics of debonding development and the position of crack occurrence. The direction of the crack initiation has been assumed as the normal direction to the boundary for calculating G_c and F_I. Accutually the direction of the crack initiation must be determined by using the proper criterion for calculating G_c and F_I.

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