Mixed mode I/II fracture path simulation in a typical jointed rock slope

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ABSTRACT. *The fracture behaviour of a jointed rock slope is investigated numerically using finite element simulations. A typical rock slope containing an inclined edge crack and subjected to the weight of slope and the pore pressure applied to the crack flanks is modeled using the ABAQUS code. Fracture parameters including stress intensity factor* (K_I and K_{II}) *and the T-stress are obtained from the numerical analyses. A mixed mode fracture criterion called the generalized maximum tangential stress-GMTS (which uses three fracture parameters* K_I, K_{II} *and T) is introduced for evaluating the onset of slope failure. Furthermore, an incremental crack growth method which involves a large number of small crack extensions in appropriate directions is also used for simulating the trajectory of fracture growth in the considered rock slope. It is shown that in addition to the conventional singular terms* (K_I and K_{II}), *the non-singular stress term (T-stress) can also affect significantly the fracture initiation angle and the trajectory of fracture growth in the jointed rock slopes.*

INTRODUCTION

A variety of engineering activities (e.g. open pit mines, transportation systems such as highways and railways and industrial and urban development) require excavation of rocks. Failures of rock slopes and open pit mines include rock falls, overall slope instability and landslides. The consequence of such failures can range from direct costs of removing the failed rock and stabilizing the slope to possibly a wide variety of indirect costs such as damage to vehicles and injury or death to passengers on highways and railways, traffic delays, etc [1]. Hence, the stability analysis of rock slopes and open pit mines is among the interesting and important subjects for civil and mining engineers and it is necessary to study the failure behaviour of such structures using suitable methods. In rock mechanics, the failure characteristics of rock masses are traditionally evaluated by means of some criteria such as limit equilibrium method [2]. In such criteria, it is assumed that the loaded rock mass is primarily intact and the failure occurs along weak surfaces. Accordingly, some researchers [3-5] have used the limit equilibrium method in the past for predicting the stability of rock slopes using a factor of safety computed based on the Mohr-Coulomb failure criterion. However, rock masses (like rock slopes and open pit mines) very often contain a large number of joints, cracks, bedding planes, natural fractures, faults and other inherent geological discontinuities. Under mechanical loads (such as weight of rock slope, external live loads, earthquake and pore pressure) or environmental and thermal cyclic loads, the probability of crack propagation from these discontinuities is increased. These cracks act as stress concentrators and hence can govern the failure process of the slope. Thus, for evaluating the failure behaviour of real rock slopes, the use of more mechanistic methods like fracture mechanics approaches is expected to give more reliable results than traditional techniques. In fracture mechanics based methods, it is assumed that the overall failure of a cracked body initiates from the tip of pre-existing cracks. Hence, a number of researchers have employed the linear elastic fracture mechanic (LEFM) principles to study the crack growth behaviour of jointed rock slopes [6-8]. Because of the arbitrary orientation of pre-existing cracks inside the rock slopes or open pit mines, these structures are usually subjected to complex loading conditions and their fracture may occur under a combination of tension-shear (or mixed mode) loading.

There are some criteria for predicting the onset of mixed mode fracture [9-11]. These fracture criteria are usually developed based on the state of stress, strain, energy etc. in front of the crack tip and can evaluate both the onset of fracture and the direction of crack growth under mixed mode loading conditions. For example, based on the well-known maximum tangential stress (MTS) criterion the direction of fracture initiation (θ_0) and the onset of mixed mode fracture are determined from [9]:

$$\theta_0 = \tan^{-1}\left(\frac{-3\left(\sqrt{8K_{II}^2+K_I^2}+K_I\right)K_{II}}{3K_{II}^2+K_I\sqrt{8K_{II}^2+K_I^2}}\right) \tag{1}$$

$$K_{\rm I}\cos^3\frac{\theta_0}{2}-\frac{3}{2}K_{\rm II}\sin\theta_0\cos\frac{\theta_0}{2}=K_{\rm Ic} \tag{2}$$

where, $K_{\rm I}$ and $K_{\rm II}$ are the mode I and mode II stress intensity factors, respectively and $K_{\rm Ic}$ is the fracture toughness of material. For complex problems like jointed rock slopes subjected to multiaxial loads, the finite element method can be employed as a powerful technique for obtaining the fracture parameters including $K_{\rm I}$ and $K_{\rm II}$. Hence in this research, fracture behaviour of a jointed rock slope is investigated numerically using finite element simulations and its mixed mode fracture parameters are calculated. The obtained numerical results are then used for evaluating the onset of fracture and the path of crack growth for the considered rock slope via a generalized form of the MTS criterion.

ROCK SLOPE FINITE ELEMENT MODEL

Fig. 1 shows the geometry and dimensions of a typical rock slope containing a pre-existing inclined edge notch of length 10 m and mouth width of 1 m which is considered in this research for the fracture analysis. The crack makes an angle of 30°

relative to the vertical direction. It is assumed that the weight of rock slope and the pore pressure (P_w) applied to the fracture planes are major loads affecting the crack deformation of the considered slope. Under the combined weight-pressure loading conditions and due to the inclined geometry of the crack, a mixed mode opening-sliding deformation mechanism characterizes the fracture behaviour of the rock slope. The finite element code ABAQUS was employed for investigating the fracture behavior of the described rock slope. Fig. 2 shows the mesh pattern generated for simulating the given slope. A constant opening pore pressure of P_w = 0.3 MPa was applied to the crack flanks and also the gravity body force of the slope was applied in the model. The bottom edge of model was fixed and the elastic material properties of a typical rock as E = 70 GPa, ν = 0.25 and ρ = 2.61 g/cm^3 [1] were considered in the finite element models. A total number of 6902 eight-noded plane strain elements were used for creating the model. The singular elements were considered in the first ring of elements surrounding the crack tip for producing the square root singularity of stress/strain field. A J-integral based method built in ABAQUS was used for obtaining the stress intensity factors directly from software.

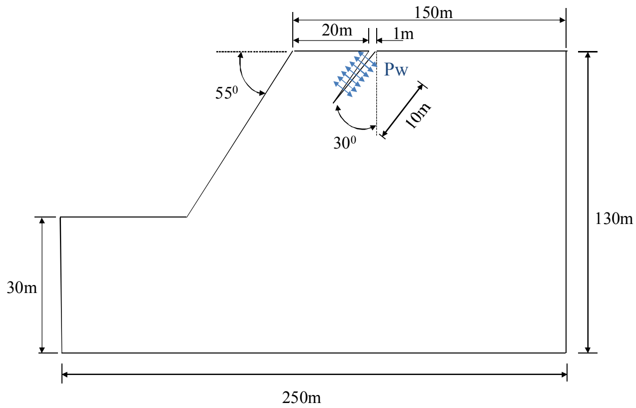

Figure1. Geometry of the considered rock slope containing an inclined edge crack and subjected to mixed mode loading conditions.

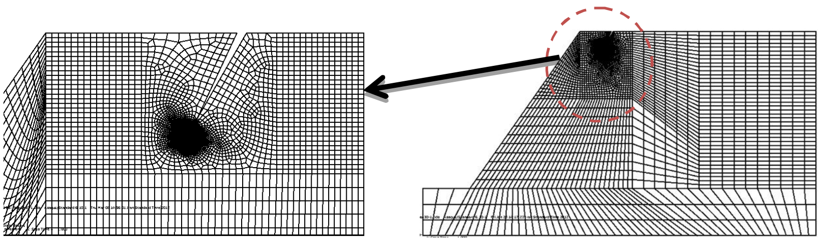

Figure 2. Finite element mesh pattern generated for numerical simulation of the cracked rock slope.

For the given geometry and loading conditions, the corresponding values of K_I and K_{II} at the tip of initial crack were about 1.95 and 0.4 MPa m$^{0.5}$, respectively. This implies that both K_I and K_{II} have noticeable values and the combined opening – sliding deformations may cause catastrophic failure in the rock slope. Furthermore, in addition to the stress intensity factors, the finite element results showed that the first non-singular stress term also has a significant value of 0.18 MPa at the tip of initial crack. This value was also obtained directly from the ABAQUS software using the contour-integral method. It is now well established that the sign and magnitude of the *T*-stress can affect noticeably the onset of fracture as well as the direction of mixed mode fracture [12-15]. Accordingly, the conventional fracture criteria that are based only on the stress intensity factors and ignore the effect of *T*-stress may not provide accurate estimations for the fracture behaviour of the cracked structures. Thus, a modified mixed mode fracture criterion called the generalized maximum tangential stress (GMTS) is employed in the next section for evaluating the onset of rock slope failure. This criterion uses a three-parameter (K_I, K_{II} and T) model for predicting the load bearing capacity and the direction of fracture initiation [12].

MIXED MODE CRACK GROWTH CRITERION

The GMTS criterion takes into account the influence of *T*-stress in addition to the mode I and mode II stress intensity factors. Hence in comparison with the conventional MTS criterion, this criterion uses a more accurate description for the tangential stress $\sigma_{\theta\theta}$ in front of the crack tip under mixed mode I/II loading as [16]:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + T\sin^2\theta + O\left(r^{1/2}\right) \qquad (3)$$

where r and θ are the crack tip co-ordinates, T is a non-singular and constant stress term which is independent of the distance from the crack tip, usually called the *T*-stress. It depends on the geometry and loading conditions and its magnitude may vary in a wide range for different cracked bodies and structures. $O(r^{1/2})$ represents the remaining terms of the series expansion which are negligible near the crack tip. The GMTS criterion proposes that the crack growth initiates radially from the crack tip along the direction of maximum tangential stress θ_o. Also the crack extension takes place when the tangential stress $\sigma_{\theta\theta}$ along θ_o and at a critical distance r_c from the crack tip attains a critical value $\sigma_{\theta\theta c}$. Both r_c and $\sigma_{\theta\theta c}$ are assumed to be material constants. According to the GMTS criterion [12], the direction of fracture initiation angle θ_o and the onset of mixed mode I/II brittle fracture can be found from:

$$\left.\frac{\partial\sigma_{\theta\theta}}{\partial\theta}\right|_{\theta=\theta_0} = 0 \quad \Rightarrow \quad [K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1)] - \frac{16T}{3}\sqrt{2\pi r_c}\cos\theta_0 \sin\frac{\theta_0}{2} = 0 \qquad (4)$$

$$K_{Ic} = \cos\frac{\theta_0}{2}\left[K_I\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{II}\sin\theta_0\right] + \sqrt{2\pi r_c}\,T\sin^2\theta_0 \qquad (5)$$

More details about how Eqs. (4) and (5) are derived can be found in Smith et al. [12]. If the effect of T in Eqs. (4) and (5) is ignored, the GMTS criterion will be identical to the conventional MTS criterion. Based on the GMTS criterion, a negative *T*-stress in a cracked geometry increases the mixed mode I/II fracture resistance and conversely a positive *T*-stress decreases it [12]. Similarly, the angle of mixed mode fracture initiation decreases when the *T*-stress is negative and increases for those situations where the *T*-

stress is positive [12]. Accordingly the onset of unstable fracture growth from the tip of pre-existing crack in the considered rock slope can be estimated by means of the GMTS criterion if the values of three fracture parameters (i.e. K_I, K_{II} and T) are know.

ROCK SLOPE FRACTURE PATH SIMULATION

Fracture growrth trajectory is also another important issue in the rock slope related problems. The GMTS criterion can be also used for simulating the path of crack growth in the jointed rock slopes. In order to predict the crack growth path of the investigated rock slope under mixed mode I/II conditions, it is necessary to determine the fracture initiation angle at the crack tip. The direction of fracture initiation can be calculated based on the GMTS criterion (i.e. Eq. (4)). For example, using the calculated values of K_I, K_{II} and T along the boundary of damage zone (r_c) in front of the crack tip (taken arbitrarily a typical value of 5 mm), the direction of initial fracture growth is determined about 22^o relative to the crack line. This angle is used as the first input data for estimating the fracture path. On the other hands, there are also several methods for predicting the fracture path after the initiation stage (e.g. [17]). The incremental crack growth method which was used in this research involves a large number of small crack extensions in appropriate directions. For simulating the crack growth path of the investigated rock slope, the direction of crack growth (θ_0) for each increment was determined by means of the GMTS criterion and using the fracture parameters at the tip of growing crack. For each stage of incremental crack growth, the fracture parameters K_I, K_{II} and T were obtained from finite element analysis and then the direction (θ_0) for the next increment was determined from Eq. (4). After calculating the direction of maximum tangential stress, the crack was remodeled with a small extension along the calculated direction and the same procedure was repeated for the next increment till the whole fracture path was predicted.

Table 1. Numerical values of fracture parameters and the calculated values of θ_0 for the tip of growing crack in the analyzed rock slope.

step	K_I (MPa m$^{0.5}$)	K_{II} (MPa m$^{0.5}$)	T-stress (MPa)	Fracture angle (degree)
1	1.952	0.401	0.180	22.36^o
2	2.303	0.171	0.153	7.79^o
3	2.613	0.164	0.173	7.36^o
4	3.078	0.168	0.201	6.41^o
5	3.757	0.206	0.248	6.45^o
6	4.756	0.269	0.331	6.63^o
7	6.400	0.373	0.484	6.88^o
8	9.259	0.551	0.794	7.04^o
9	15.20	0.909	1.54	7.13^o
10	32.25	1.912	4.19	7.17^o

Table 1, summarises the calculated values of K_I, K_{II}, T and θ_0 for each step of fracture path simulation in the considered rock slope, in which the stepe 1 coresponds to the initial crack but the other steps present the fracture parameters of the growing crack. Fig. 3 shows different steps of crack growth in the considered slope which was obtained from the finite element analyses and by using the GMTS criterion.

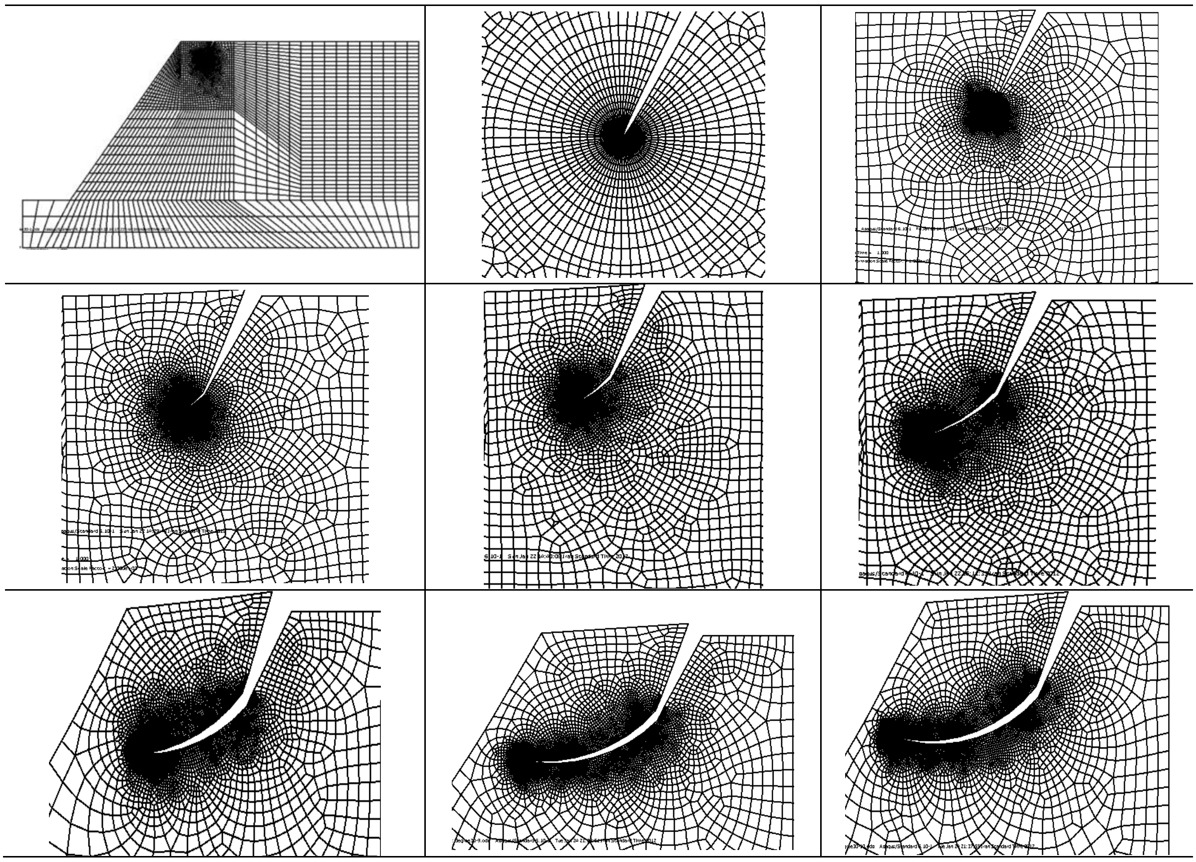

Figure 3. Some of the steps of the simulated crack growth path for the considered rock slope.

DISCUSSION AND CONCLOUSIONS

As described earlier the fracture of jointed rock slope can occur due to mixed mode loading conditions. The GMTS criterion which was described in this paper provides a theoretical framework for predicting the onset of unstable crack growth for the rock slopes containing cracks. According to the GMTS criterion, when the right hand side of Eq. (5) which is related to K_I, K_{II} and T reaches the value of critical fracture toughness of material (K_{Ic}), brittle crack growth starts from the tip. K_{Ic} is a material property which describes the resistance of a material against the crack growth. The value of K_{Ic} for different rock materials varies in the typical range of 0.5 to 5 MPam$^{0.5}$ [13-15,18].

Accordingly, if the noticeable values of K_I, K_{II} and T presented in Table 1 for the tip of initial crack are replaced into Eq. (5), the crack growth is expected to occur. Since the fracture parameters are directly related to the values of applied loads, it can be concluded that the risk of failure for the considered rock slope becomes more when the magnitude of P_w are increased in the winter. Furthermore , according to the GMTS criterion, a positive T-stress decreases the fracture toughness and the load bearing capacity of a cracked body. It is seen from Table 1 that the T-stress is positive for the investigated jointed rock slope. Therefore, a reduction in the crack growth resistance is expected for the considered crack geometry in this research. After the fracture initiation, the crack will grow suddenly in the rock slope due to the brittle behaviour of rock. However, because of mixed mode I/II loading conditions, fracture of rock slope may grow in a curvilinear path and not necessarily along the direction of the original crack. Thus, the GMTS criterion was used for simulating the trajectory of fracture path in the investigated rock slope and the whole trajectory was evaluated using the incremental crack growth method. Again the T-stress had a significant influence on the path of fracture due to its noticeable value in all of the incremental stages. As a conclusion, the effect of T-stress should be taken into account in addition to the conventional stress intensity factors to provide more precise estimations for the onset of instability and fracture trajectory in the jointed rock slopes.
The numerical finite element results demonstrated also that the fracture process for the growing crack is governed mainly by the mode I stress intensity factor. During the crack path simulations, it was always observed that the magnitude of K_I increased dramatically in comparison with the shear mode component K_{II} (as presented in Table 1). Therefore, a dominantly tensile type fracture controls the fracture behaviour of the jointed rock slope after the initiation stage. The concluding remarks of this paper are:

- Fracture behaviour of a typical rock slope containing an inclined edge crack and subjected to mixed mode loading was investigated numerically using finite element code ABAQUS.
- Three fracture parameters (K_I, K_{II} and T) were calculated for the tip of initial and growing crack in the rock slope.
- The generalized maximum tangential stress (GMTS) criterion was used for estimating both the onset of crack growth and the direction of fracture using the obtained K_I, K_{II} and T for the investigated rock slope.
- A noticeable positive T-stress that was observed for the crack tip in the rock slope has a significant effect on the failure behaviour of slope.
- The failure trajectory in the considered rock slope was simulated using an incremental crack growth method by means of several finite element analyses.

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