Modelling of Crack Path Evolution in Round Bars under Cyclic Tension and Bending

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ABSTRACT. *This paper shows the evolution of the surface crack front in round bars constituted of different materials (determined by the exponent m of the Paris law), subjected to fatigue tension loading (with free ends) or fatigue bending loading. To this end, a numerical modeling was developed on the basis of a discretization of the crack front (characterized as an ellipse) and the crack advance at each point perpendicular to such a front, according to a Paris-Erdogan law, using a three-parameter stress intensity factor (SIF). Each analyzed case was characterized by the evolution of the semielliptical crack front, studying the progress with the relative crack depth a/D of the following three key variables: (i) crack aspect ratio a/b (relation between the semiaxes of the ellipse which defines the crack front); (ii) maximum dimensionless SIF; (iii) minimum dimensionless SIF.*

INTRODUCTION

One of the most relevant geometries in the field of fatigue and fracture mechanics applied to the structural engineering is a cracked cylinder under tension loading or bending moment. As a matter of fact, many structural elements, mainly in civil engineering consist of wires, bolts, shafts, cables or other components of cylinder shapes under constant or cyclic loading, so that the risk of surface cracking by mechanical or environmental actions is not negligible.

Growth of surface cracks in round bars due to fatigue can be modeled using different criteria. Prediction of the 90º intersecting angle of the crack with the surface or the iso-*K* criterion along the crack front exhibit small differences in their aspect ratio but both lead to a unique fitting [1]. Another criterion is based on the crack growth according to the Paris Erdogan law considering the crack advance perpendicular to the crack front, assuming elliptic geometry of the crack [2-4], avoiding the shape hypothesis [5,6] or using the modified Forman model [7].

Characterization of fatigue crack growth, whose crack front has been commonly represented as straight, circular or elliptical with centre on the wire surface, necessarily implies knowing the dimensionless stress intensity factor (SIF), *Y*, which makes it essential to discern how it changes along the crack front. The dimensionless SIF has been obtained by several authors under different loading conditions (tension, bending and torsion) and deducted from different procedures: flexibility method, finite element method, contour integral analysis, experimental techniques, etc. [2-3,8-12].

Fatigue crack growth in round bars with different initial geometry leads to a preferential crack path, with an aspect ratio between 0.6 and 0.7 for a relative crack depth close to 0.6 for tension [2,5], since the geometry of the crack front must be defined with, at least, two independent parameters [9]. Growth patterns are closer for a higher value of the Paris coefficient m. The crack always tries to propagate towards an iso-*K* configuration; however, it can not be maintained due to the existence of the surface, where the stress has a two-dimensional state and the singularity of the square root can be lost at the crack tip [5].

NUMERICAL MODELLING

In order to study how a crack propagates on the cross section of a round bar under tension or bending cyclic loading (Fig. 1), a computer program in Java programming language was developed to determine the geometrical evolution of the crack front.

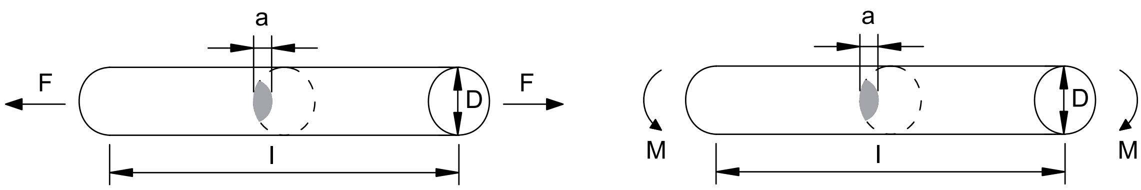

Figure 1. Cracked bar under tension loading (left) and bending moment (right).

The basic hypothesis of the modeling consisted of assuming that the crack front can be modeled as an ellipse with centre on the bar surface [13] and the fatigue propagation takes place in a direction perpendicular to this crack front, following a Paris Erdogan law [14],

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^{\mathrm{m}} \tag{1}$$

Every elliptical arc of the crack was divided in *z* segments with exactly the same length using the Simpson method to discretize the front. The point on the wire edge was not taken into account, since it presents some difficulties regarding the computation of the dimensionless SIF (there is a plane stress state on the crack edge). After that, every single point was shifted according to Paris Erdogan law perpendicular to the front, so as to keep constant the maximum crack depth increment, $\Delta a(\max) \equiv \max \Delta a_i$. The advance of every front point, Δa_i, can be obtained from the maximum crack increment and the ratio of the dimensionless SIF,

$$\Delta a_{\mathrm{i}} = \Delta a(\max)\left[\frac{Y_{\mathrm{i}}}{Y(\max)}\right]^{\mathrm{m}} \tag{2}$$

The newly obtained points, fitted by the least squares method [13], generate a new ellipse with which the process is repeated iteratively until the desired crack depth is reached. Due to the existing symmetry, only half of the problem was used for the computations (Fig. 2).

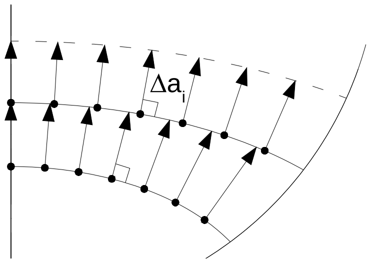

Figure 2. Process followed to compute the fatigue crack growth.

The dimensionless SIF used in the computations is that proposed by Shin and Cai [4] obtained by the finite element method together with a virtual crack extension technique, which depends on the crack geometry *a*/*b*, the crack depth *a*/*D* and the position of the point considered on its front *x*/*h* (Fig. 3).

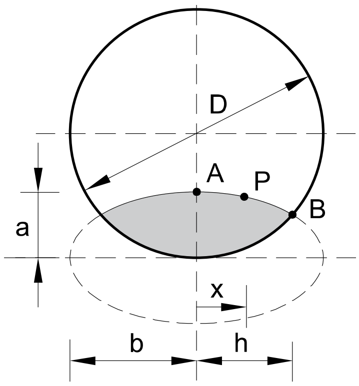

Figure 3. Elliptical crack model used by Shin and Cai.

The fitting of the results provides three-parametrical expressions which are defined as a function of the coefficients M_{ijk} for tension with free ends [4],

$$Y = \sum_{i=0}^{2} \sum_{j=0}^{7} \sum_{k=0}^{2} M_{ijk} \left(\frac{a}{b} \right)^{i} \left(\frac{a}{D} \right)^{j} \left(\frac{x}{h} \right)^{k} \tag{3}$$

and of coefficients N_{ijk} for bending [4].

$$Y = \sum_{i=0}^{2} \sum_{j=0}^{6} \sum_{k=0}^{2} N_{ijk} \left(\frac{a}{b} \right)^{i} \left(\frac{a}{D} \right)^{j} \left(\frac{x}{h} \right)^{k} \tag{4}$$