Propagation of small cracks under RCF: a challenge to Multiaxial Fatigue Criteria

S. Beretta, S. Foletti

Politecnico di Milano, Dipartimento di Meccanica, Via la Masa 1, 20156 Milano
stefano.beretta@polimi.it, stefano.foletti@polimi.it

ABSTRACT. *RCF is traditionally a very critical load condition for fatigue and, moreover, material defects (inclusions, inghomogeinities) play a significant role in determining the service life of materials exposed to the out-of-phase stresses which typically occur at the interface and below the surface of contacting bodies.*
In this paper we summarize the fatigue test results obtained for three different steels (a bearing, a gear and a railway wheel steels) that have been subjected to out-of-phase multiaxial fatigue tests simulationg RCF conditions in presence of small shallow pre-cracks. The results show that the fatigue resistance domain is characterized by the presence of two different phenomena. In the region of $\sigma_h < 0$, tests simulating RCF for deep defects show a peculiar co-planar propagation driven by shear while in torsional tests the fatigue strength appears to be controlled by the onset of Mode I propagation. The experimental results have then been discussed adopting the Dang Van criterion showing that the dependence on crack size can be easily predicted in terms of Mode I and Mode III thresholds at the tip of the micro-cracks.

INTRODUCTION

Multiaxial fatigue has been the subject for the proposal of many criteria intended to predict fatigue strength (or fatigue life) under multiaxial conditions from a limited number of tests under uniaxial or torsional conditions. Among the different multiaxial loads, the out-of-phase (OOP) conditions that are typical of *rolling contact fatigue* (RCF), both for subsurface and surface failures, are in general the most detrimental for mechanical applications, with a severe reduction of the allowable fatigue shear strength respect to simple torsion.

RCF is traditionally treated in terms of an allowable hertzian pressure [1], while the Dang Van criterion [2] has been the theory widely adopted for application to RCF [3, 4] because of its treatment of out-of-phase histories for the stress components and its simple definition of allowable shear stress as a linear function of the hydrostatic stress σ_h. However, the most demanding applications such as bearings are made of high strength steels, which are very sensitive to the presence of small defects and inclusions. Therefore it is important to consider the presence of defects for a significant strength prediction under RCF conditions.

In the literature two typical treatments of RCF in presence of defects are adopted: (i) a fatigue criterion such as Dang Van's one in which the fatigue limit depends on defect size (according to Murakami's concept defects can be treated as small cracks [5]); (ii) calculation of SIF's at the tip of defects and comparison with ΔK_{th} obtained under Mode II/III [6].

The first approach does not appear to be fully correct for two different reasons: (i) RCF stress field is characterized by negative hydrostratic stresses - σ_h - and the analysis of fatigue limit condition under RCF suggests that the allowable shear stress should not be increasing in the region where $\sigma_h < 0$, as predicted by the Dang van's criterion [7, 8]; (ii) it is very difficult for a fatigue criterion to correctly describe experimental results obtained in presence of defects [9]. The second approach, even if more correct since it refers to the threshold condition for shear propagation (while fatigue strength models for tension/torsion refer to Mode I threshold condition), is usually based on $\Delta K_{II,th}$ (or $\Delta K_{III,th}$) values obtained in shear/torsion tests which overestimate the real threshold under RCF conditions.

Recently the authors have presented a novel series of experiments on microcracked specimens subjected to out-of-phase loads which have clearly shown that the threshold $\Delta K_{III,th}$ in RCF conditions (both for a gear and a bearing steel) are much lower than the ones under simple torsion [10, 11]. This appear to be due to the crack opening caused by the severe plastic deforamation and rubbing of crack lips [11].

In this paper we present a summary of experimental results on very small defects under RCF conditions for a three different types of steel (respectively bearing, gear and railway wheel steels) showing how these experiments depict the *failure locus* for the Dang Van criterion. The results are then discussed in terms of *local* stress singularities at the crack tip.

EXPERIMENTS FOR MODE III THRESHOLDS IN TORSION AND RCF

Materials

The materials tested for investigating crack thresholds under shear for pure torsion and RCF conditions are a bearing steel, a Q&T steel for gears and R7T, a steel widely used for manufacturing railway wheels. Properties of the three steels are listed in Tab. 1.

Table 1. Tensile properties of the three steels

MATERIAL	UTS [MPa]	Monotonic yield stress S_y [MPa]	Cyclic yield stress $S_{y,cyclic\,(0.2\%)}$ [MPa]
Bearing steel	2360	1980	2070
Gear steel	2150	1395	1735
R7T	875	545	480

Specimens

All the fatigue tests were carried onto micronotched hourglass specimens. After machining the specimens were hand polished and then electro-polished (surface removal 30-40 μm) in order to reduce the residual stresses.

After surface finish, artificial micronotches were then introduced onto the surface of the specimens by EDM machining: three different defects, characterized by a size (expressed in terms of Murakami's √area parameter) of 220, 315 and 630 μm. Defect sizes are shown in Fig. 1.

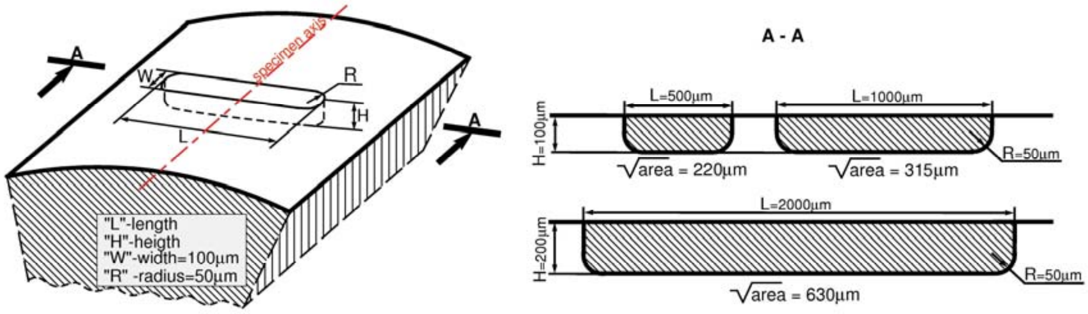


Figure 1. Micronotches adopted for fatigue tests.

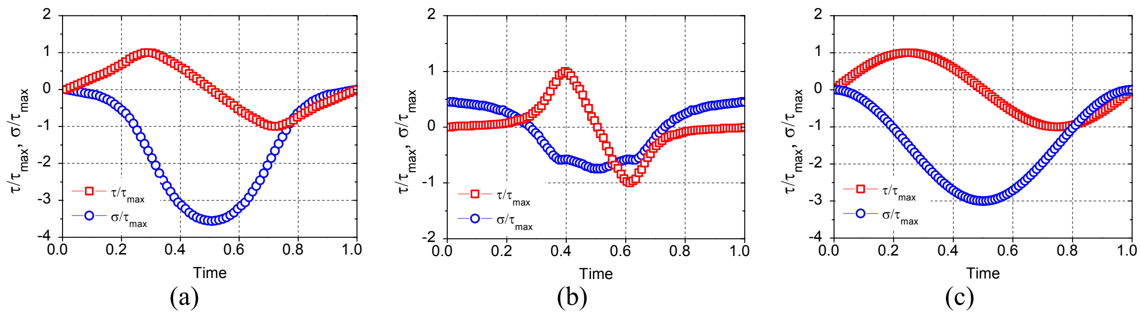


Figure 2. Load patterns. (a) Load Path 1 (LP1) – Bearing steel, (b) Load Path 2 (LP2) – Bearing steel, (c) Load Path 3 (LP3) – Gear steel and R7T steel.

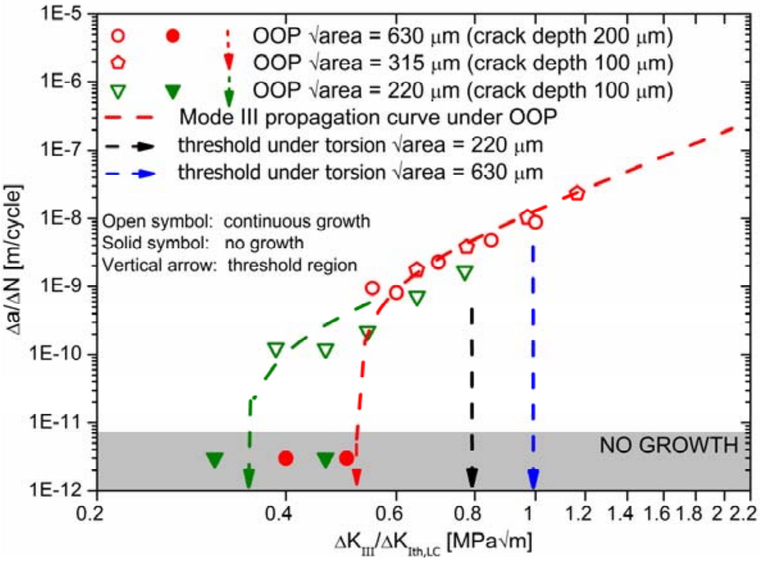
Precracking & fatigue test details

Specimens made of the gear and bearing steel were firstly tested under tension at R=-1 in order to determine Mode I thresholds. Then, for the sake of torsional and out-of-phase tests, the specimens were precracked at R=-2 at stress levels very close to fatigue limits at R=-1 for 10^7 cycles, for inducing the formation of small non-propagating cracks at the bottom of the notches. All the specimens were observed under SEM to verify the success of pre-cracking procedure (if not successfull the Mode I loading was repeated).

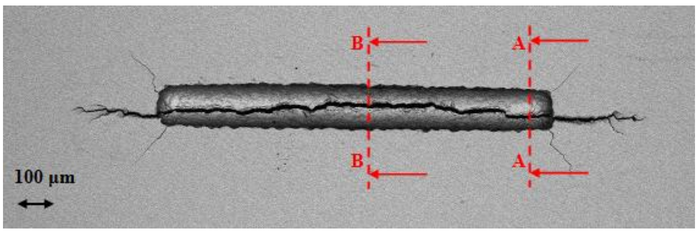
After the pre-cracking procedure, the specimens were subjected to torsional and out-of-phase tests at different ΔK_{III} levels. The OOP tests were carried out according to three different load paths: in the case of the bearing steel two load paths suggested by SKF were adopted (namely LP1 for deep defects and LP2 for sub-surface ones), while the gear and the railway steel were subject to a load path LP3 very similar to LP1. The three load patterns are shown in Fig. 2.

Fatigue test results – bearing steel

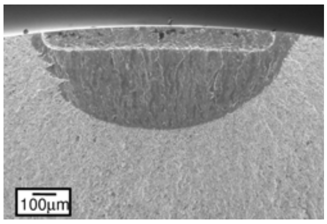
The crack advance was monitored interrupting fatigue tests and breaking the specimens under liquid nitrogen: crack growth rate was then estimated as $\Delta a/\Delta N$. The resulting crack growth diagram is shown in Fig. 3.



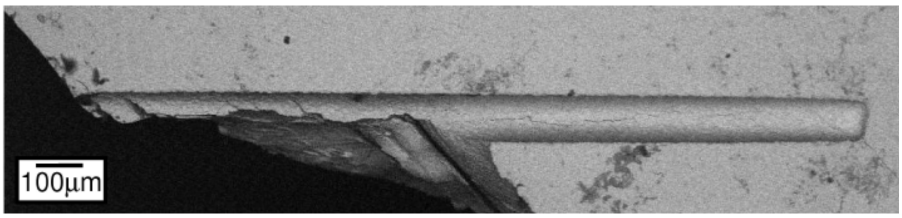
(a)



(b)



(c)



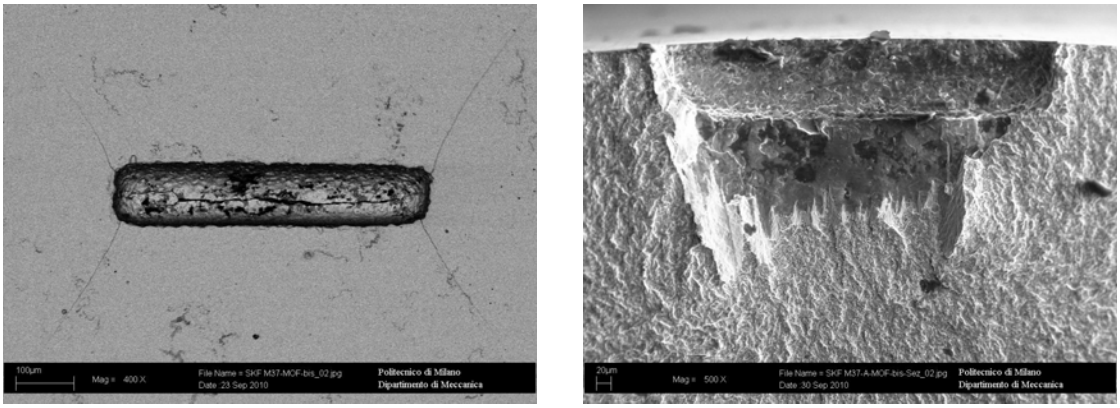
(d)

Figure 3. Fatigue test results onto the bearing steel: (a) prospective Da/DN diagram; (b) and (c) aspect of the co-planar crack growth; (d) frature under torsion onto tilted planes.

Crack growth under LP1 tests is characterized by a co-planar shear growth (in Mode II at the surface point and in Mode III at the defect tip) accompanied by the formation of debris and plastic deformation of the crack mouth and it occurs at ΔK_{III} levels much lower than the ones corresponding to Mode I threshold.

Tests under simple torsion show a small co-planar crack advance (with a growth rate comparable to threshold region) at ΔK_{III} levels similar to $\Delta K_{I,th}$ together with the formation of pure Mode I cracks on planes tilted at 45° respect to crack plane: these Mode I cracks are the ones responsible for fatigue failure at stress levels higher than the fatigue limit and, correspondingly, the fatigue limit is the Mode I threshold onto the tilted cracks.

Out-of-phase tests under LP2 show a behaviour similar to torsion. The failure is controlled by Mode I propagation: the observation of fracture surfaces for interrupted tests reveals a small co-planar growth with th early development of Mode I on tilted planes, see Fig. 4.

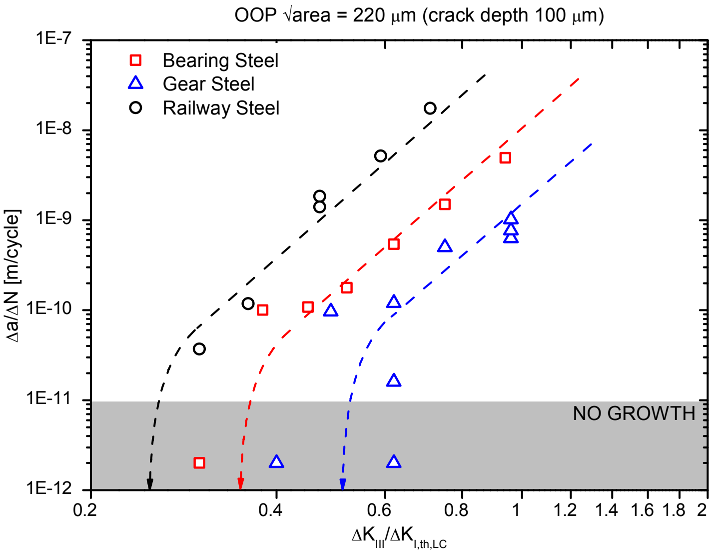


(a) (b)

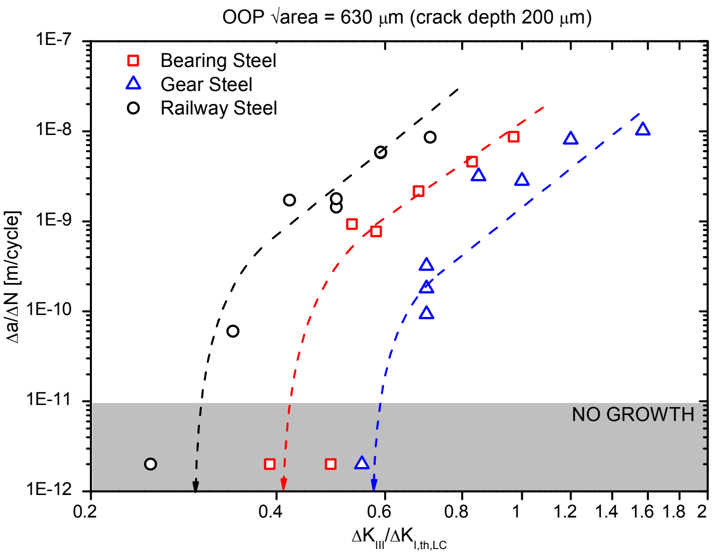
Figure 4. Fatigue test results onto the bearing steel under load path 2. (a) Mode I kinks, (b) Coplanar propagation

Fatigue test results – two other steels

The results obtained by the gear and railway wheel steels under torsion and LP3 are very similar to the ones of the bearing steel. In particular, torsional fatigue limit corresponds to the onset of Mode I propagation onto tilted planes, while OOP tests show a threshold ΔK_{III} much lower than $\Delta K_{I,th}$. It appears a clear tendency: higher tensile properties correspond to a much higher $\Delta K_{III,th,OOP}$ presumably because of the higher material resistance against the rubbing and plastic deformation of crack lips (see Fig. 5).



(a)



(b)

Figure 5. Coplanar crack growth rate under out-of-phase fatigue tests. a) defect size √area = 220 μm, b) defect size √area = 630 μm.

APPLICATION OF MULTIAXIAL FATIGUE CRITERIA

Dang Van fatigue criterion

The basis of the Dang Van's criterion is the application of the elastic shakedown principles at the mesoscopic scale (more details can be found in [12, 13]). The Dang Van's criterion can be expressed by:

$$\tau_{DV}\left(t\right)+\alpha\sigma_h\left(t\right)\leq\tau_W \quad (1)$$

with α_{DV} being a constant to be determined, τ_W the fatigue limit in reversed torsion, $\sigma_h(t)$ the instantaneous hydrostatic component of the stress tensor and $\tau_{DV}(t)$ the instantaneous value of the Tresca shear stress, i.e.,

$$\tau_{DV}\left(t\right)=\frac{\hat{s}_I(t)-\hat{s}_{III}(t)}{2} \quad (2)$$

evaluated over a symmetrized stress deviator found at the mesoscopic scale, which is obtained by subtracting from the stress deviato $s_{ij}(t)$ a constant tensor, $s_{ij,m}$, i.e.,

$$\hat{s}_{ij}(t)=s_{ij}(t)-s_{ij,m} \quad (3)$$

where $s_{ij,m}$ is a residual stress deviator able to fullfill the condition of an elastic shakedown state at the mesoscopic scale.

The constant α_{DV} appearing in the expression of the Dang Van's criterion is usually calibrated with two fatigue tests, tension–compression, σ_W and pure torsion, τ_W [1 on the application]:

$$\alpha_{DV}=3\left(\frac{\tau_W}{\sigma_W}-\frac{1}{2}\right) \quad (4)$$

Application of Dang Van fatigue criterion to experimental results

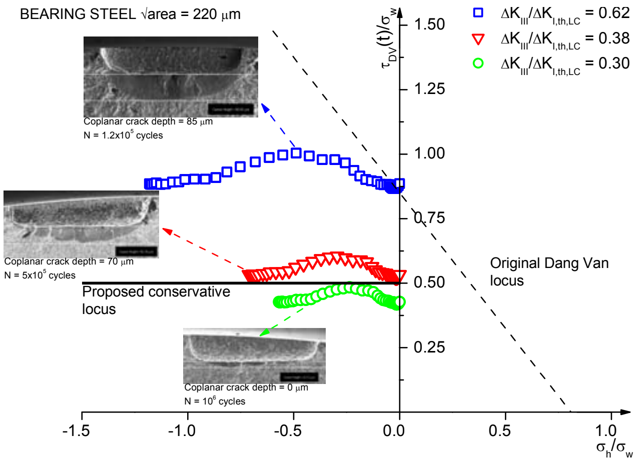
The Dang Van's criterion has been applied to the experimental results in order to evaluate the predictive capabilities of a multiaxial fatigue criterion in presence of defects. The ratio between the torsional fatigue limit and the axial fatigue limit, τ_W/σ_W, has been experimentally obtained. In pure torsional fatigue tests, the axial fatigue limit has been calculated reffering to the √area projected onto planes at 45°, obtaining a value of the ratio $\tau_W/\sigma_W = 0.85$.

Bearing steel – load path 1

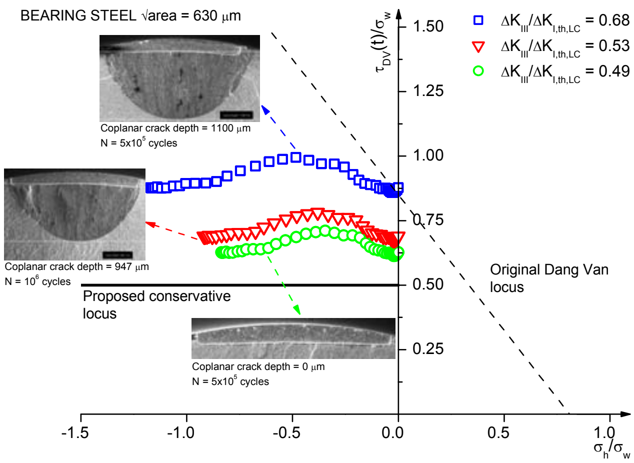
The application of the Dang Van's criterion to the bearing steel for the load path 1 is reported in Fig. 6. The results are normalized in respect to the axial fatigue limit σ_W. For each defect size three different fatigue tests are reported in the Dang Van's plane (τ_{DV} vs. σ_h): the first, characterized by the lower value of the ratio $\Delta K_{III}/\Delta K_{Ith,LC}$, is below the out of phase fatigue limit with the presence of non propagating cracks; the second is a middle value with a initial coplanar crack growth; finally the last one is characterized by a evident coplanar crack growth. Despite the evident coplanar crack growth, all the fatigue tests are below the original locus proposed by Dang Van. Hence, it is possible to conclude that, at least for a coplanar/Mode III failure, the original Dang Van's criterion it is not able to correctly predict the experimental results, leading to unsafe predictions.

The introduction of a new conservative locus, as proposed by Desimone et al in [7], seems to better reproduce the experimental results. For the smaller defect size, see Fig. 6a, the fatigue limit in out of phase is correctly predicted. This tendence is confirmed by the second defect size, also if the prediction seems to be a little more conservative.

The introduction of the conservative locus seems to be a necessary condition for the prevision of mode III failure: the out-of-phase tests have clearly shown that in fact this is failure mode completely different from the 'usual' Mode I that defines the Dang-Van's locus (which is defined by fatigue limits under tension-compression and torsion), so the superposition of a more conservative limit appears to be correct.



(a)



(b)

Figure 6. Application of Dang Van's criterion. Bearing steel- Load Path 1. a) defect size √area = 220 μm, b) defect size √area = 630 μm.

This idea apparently looks to be supported by the analysis of LP2 and torsional tests. Since failure under these tests is controlled by Mode I, drawing all these load paths in the Dang Van's plane, it is possible to observed that in all cases the mode I failure is characterized by the crossing of the original locus (see Fig. 7). However, the faigue limit under torsion would be significantly underestimated.

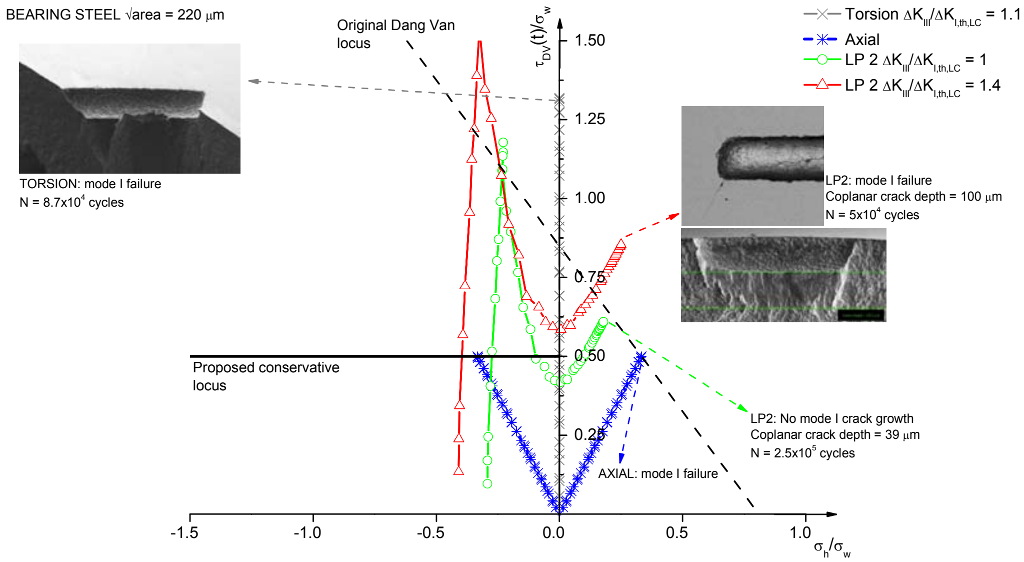


Figure 7. Application of Dang Van's Criterion. Bearing steel – Tension-compression, torsion and LP2. Defect size √area = 220 μm.

Other steels - load path 3

The predictive capabilities of the Dang Van's criterion have been checked for two other steels. In Fig. 8 the experimental results obtained on gear steel are reported in the Dang Van's plane. The results confirm the same trend previously discussed for the bearing steel. The original Dang Van's locus fails in the prediction of a Mode III crack growth, while the proposed conservative locus is able to correctly predict the multiaxial fatigue limit in out of phase. Similar plots can be also drawn for the railway steel and they confirm that the conservative locus is close to experimental results.

ANALYSIS IN TERMS OF SIF

A more detailed analysis for the onset of propagation for the small defects could be done in terms of SIF at the tips of the small co-planar precracks ahead of the defects. In particular [10] the SIF's at the tip of the crack could be expressed as:

$$\begin{cases} \sigma_{\varphi'} = \dfrac{1}{\sqrt{2\pi r}}\left(\dfrac{K_I}{2}(1+2\nu)+\dfrac{K_I}{2}\cos(2\theta)(1-2\nu)-K_{III}\sin(2\theta)\right) \\ \tau_{\varphi'z'} = \dfrac{1}{\sqrt{2\pi r}}\left(\dfrac{K_I}{2}\sin(2\theta)(1-2\nu)+K_{III}\cos(2\theta)\right) \end{cases} \qquad (1)$$

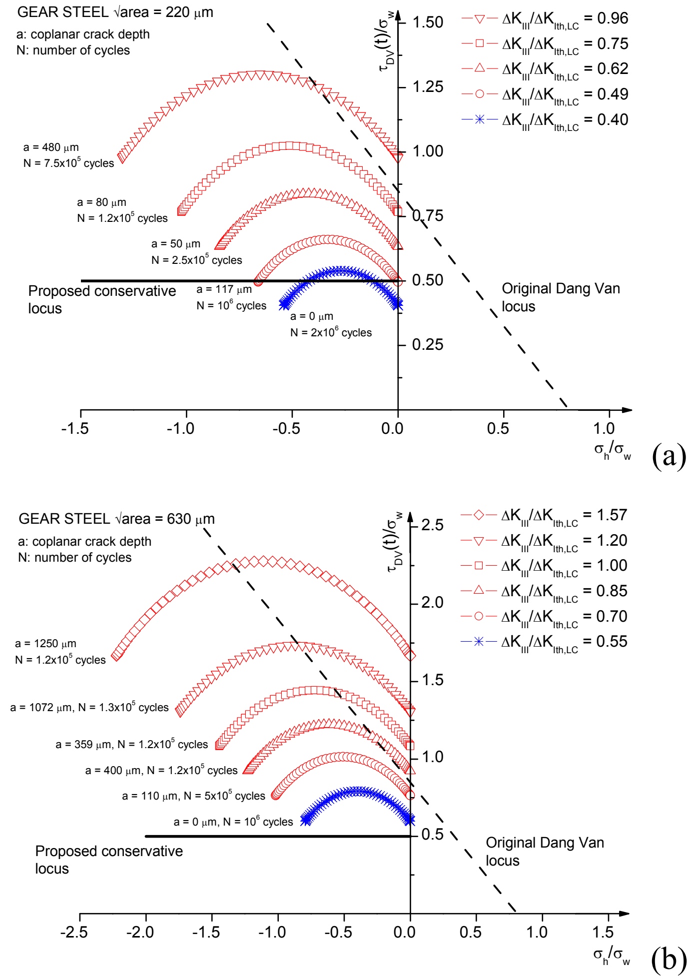


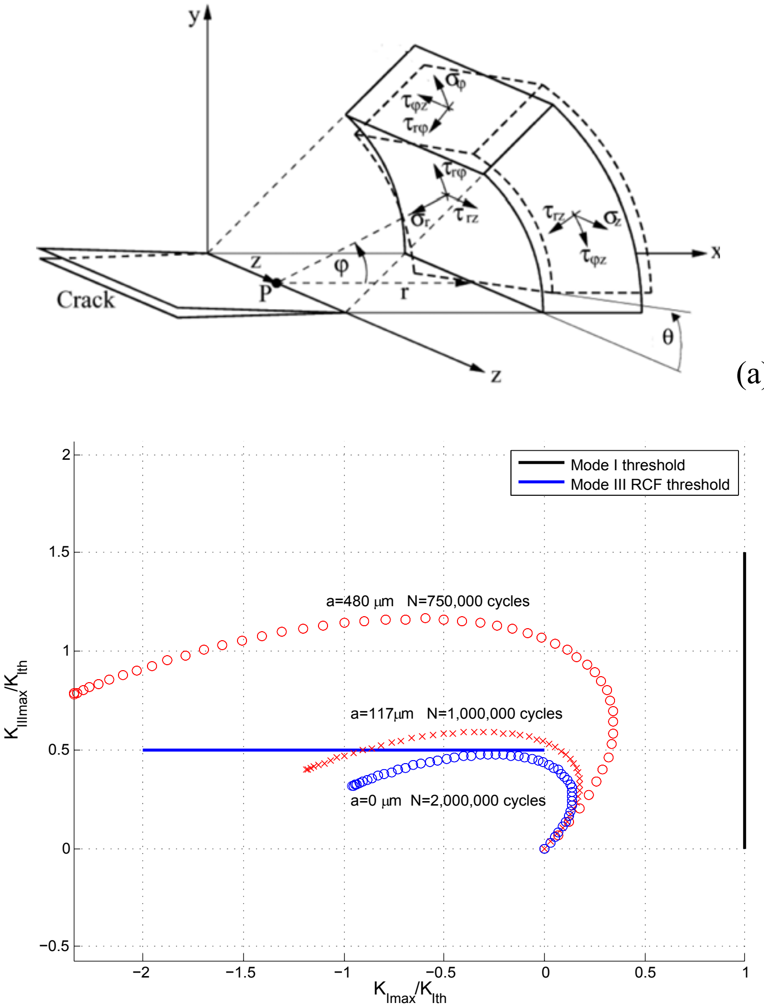
Figure 8. Application of Dang Van's Criterion. Gear steel – Load Path 3. a) Defect size √area = 220 μm, b) Defect size √area = 630 μm.

The SIF's onto a prospective propagation plane identified by the propagation angle θ, see Fig. 9.a, are expressed by:

$$\begin{cases} k_I = \dfrac{K_I}{2}(1+2\nu) + \dfrac{K_I}{2}\cos(2\theta)(1-2\nu) - K_{III}\sin(2\theta) \\ k_{III} = \dfrac{K_I}{2}\sin(2\theta)(1-2\nu) + K_{III}\cos(2\theta) \end{cases} \tag{2}$$

where K_I, K_{III} are the SIF's applied to the crack. By applying these formulas to the different load cycles, $k_{I,max}$ and $k_{III,\,max}$ could be calculated by maximizing k_I and k_{III} at each point of the load cycle. The results shown in Fig. 8.a could be re-plotted as shown in Fig. 9.b.

It can be clearly seen that for the test with the longest coplanar propagation, $k_{I,max}$ does not reach the Mode I threshold, even if it exceeds the Dang Van's locus in Fig. 8.a.



(a)

(b)

Figure 9. Analysis of OOP tests in terms of SIF's: a) cylindrical coordinate system along the crack; b) $k_{I,max}$-$k_{III,max}$ plots for the the out-of-phase tests onto the gear steel.

So it appears that for $\sigma_h < 0$ the fatigue limit criterion is not able to catch the physics of the phenomena, which is on the other hand very clear from Fig. 9.b. It can be seen that there are two different limits: a Mode I limit described by $\Delta K_{I,TH}$ and a Mode III limit that for RCF approximately corresponds to 0.5 $\Delta K_{I,th}$ for the gear and bearing steel.

By adopting this approach it is possible to identify the fatigue limit as the superposition of two conditions (corresponding to the two failure mechanisms): $\Delta k_{I,max} < \Delta K_{I,th}$ for Mode I and $\Delta k_{III,max} < \Delta K_{III,th}$ for co-planar propagation is then also possible to correctly catch the dependence of fatigue limits on crack size (see Fig. 10).

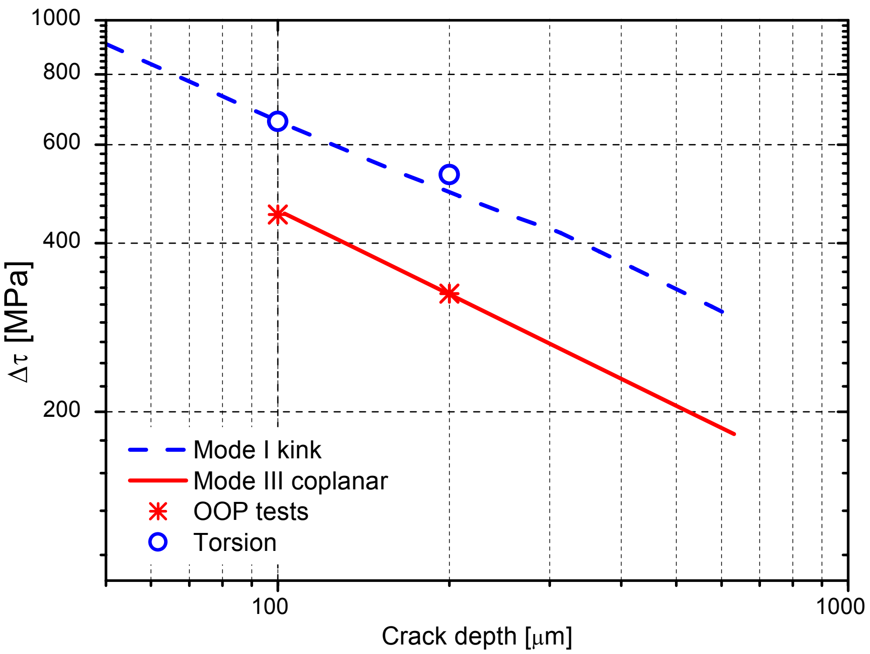


Figure 10. Dependence of fatigue limits on crack/defect depth for the gear steel.

CONCLUSIONS

In this paper we summarize the fatigue test results obtained for three different steels (a bearing, a gear and a railway wheel steels) that have been subjected to out-of-phase multiaxial fatigue tests simulationg RCF conditions in presence of small shallow pre-cracks. The experimental results have then been discussed adopting the Dang Van criterion, that is widely adopted for fatigue analysis in contacting bodies.

The results show that the fatigue resistance domain is characterized by the presence of two different phenomena. In the region of $\sigma_h < 0$, tests simulating RCF for deep defects show a peculiar co-planar propagation driven by shear, while in torsional tests and out-of-phase tests with $\sigma_h < 0$ the fatigue strength appears to be controlled by the onset of Mode I propagation.

The recognition of the two limits leads to a simple analysis of the effect of defects under RCF and torsional tests.

REFERENCES

1. T. A. Harris and M. N. Kotzalas. Rolling Bearing Analysis. CRC Press, Boca Raton (FL), 2007.
2. K. Dang Van et al. Criterion for high cycle fatigue failure under multiaxial loading. In Biaxial and Multiaxial Fatigue, pages 459–478. Mechanical Engineering Publications, London, 1989.

3. A. Ekberg, E. Kabo, and H. Andersson. An engineering model for of rolling contact fatigue for railway wheels. Fatigue Fract. Engng. Mater. Struct., 25(899-909), 2002.
4. A. Bernasconi and P. Davoli and M. Filippini and S. Foletti. An integrated approach to rolling contact sub-surface fatigue assessment of railway wheels. Wear, 258:973–980, 2005.
5. Y. Murakami. Metal Fatigue: Effects of Small Defects and Nonmetallic Inclusions. Elsevier, Oxford, 2002.
6. G. Donzella and C. Petrogalli. A failure assessment diagram for components subjected to rolling contact loading. Int. J. Fatigue, 32:256–268, 2010.
7. H. Desimone, A. Bernasconi, and S. Beretta. On the application of Dang Van criterion to rolling contact fatigue. Wear, 260:567–572, 2006.
8. L. Houpert, F. Chevalier. Rolling Bearing Stress Based Life – Part 1: Calculation Model. Journal of Tribology, 134, 2012.
9. H. Desimone, A. Bernasconi, and S. Beretta. Are multiaxial fatigue criteria appropriate when steels with surface defects are subjected to RCF ? Proc. Bearing 2005 Conference, Reno.
10. S. Beretta, S. Foletti, and K. Valiullin. Fatigue crack propagation and threshold under out- of-phase multiaxial loading in a gear steel. Engng. Fract. Mech., 77:1835–1848, 2010.
11. M.G. Tarantino, S. Beretta, S. Foletti, and J. Lai. A comparison of Mode III threshold under simple shear and RCF conditions. Engng. Fract. Mech., 78:1742–1755, 2011.
12. K. Dang Van, Macro–micro approach in high-cycle multiaxial fa- tigue, in: D.L. McDowell, R. Ellis (Eds.), Advances in Multiaxial Fatigue, ASTM STP 1191, Philadelphia, 1993, pp. 120–130.
13. K. Dang Van, B. Griveau, O. Message, On a new multiaxial fatigue limit criterion: theory and applications, in: M.W. Brown, K.J. Miller (Eds.), EGF 3, Mechanical Engineering Publications, London, 1989, pp. 479–496.