Probabilistic modelling of concrete cracking. First 3D results.

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ABSTRACT. *Cracks form a barrier for heat conduction and create preferential flow paths for fluid, gas and pollutants, i.e. their description is crucial for predicting the life expectancy of structures such as dams, nuclear power plants vessels, waste (nuclear or not) storage structures, tunnels, etc. In this paper, a model taking into account the heterogeneous nature of concrete is presented, i.e. a model describing scale effects, cracking nucleation and propagation as well as initial defects in the material. This modelling strategy is validated via an original experimental test (four point bending test) performed at LCPC. The comparison will be given not only in terms of the global answer but also on cracks opening and distribution. The presented model is also adequate for describing 3D cracking processes.*

INTRODUCTION

Concrete modeling is a challenging task as a pertinent model should take into account not only scale effects, but also those phenomena related to the heterogeneous nature of concrete such as initial defects in the material, cracks nucleation and propagation. In this paper, a model taking into account all the mentioned phenomena is presented. Material characteristics are defined via statistical distributions (requiring only two parameters) based on a large experimental campaign held at LCPC [1]. This kind of approach allows obtaining a pertinent, statistical global response and, simultaneously, local information (such as crack mouth opening and distribution). The objective of this paper is to provide a macroscopic model capable of bridging the gap between the local description of the mechanisms at the material level and the global response at the structural level, i.e. a model which is capable of properly describing the global structural answer and which provides information on the local response.

CONCRETE HETEROGENEITY AND PROBABILISTIC MODELING

Concrete heterogeneity can be taken into account by introducing statistical distributions of local material characteristics, in particular of the Young's modulus and the tensile strength [1]. This technique gives a first hint to the size effect problem if one assumes that there is equivalence between the finite elements of the mesh and a volume of material: the distribution function relating the material characteristics and the size has then to be experimentally determined. Rossi et al. [2] have highlighted that it is possible to establish a link between tensile strength f_t or Young's modulus E and the volume of the tensile specimen for concretes having a compressive strength between 35*MPa* and 130*MPa*. An experimental scale effect law has been then established for the mean tensile strength $m(f_t)$ and the standard deviation $\sigma(f_t)$ as functions of easily measurable quantities such as the volume of the specimen V_s and the volume of the coarsest grain of the concrete V_g (which can be related to the size of the major heterogeneity) and the compressive strength of concrete f_c (an indicator of the quality of the cement paste).

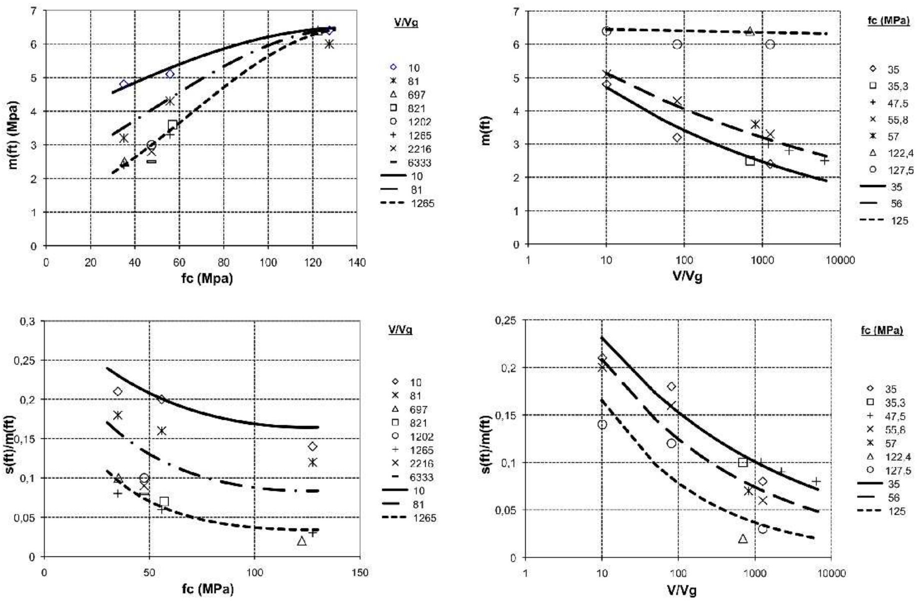


Figure 1: Tensile strength mean value/dispersion evolutions [1, 2, 3].

These scale laws have been used as input data in a numerical model based on a probabilistic approach. Two strategies are here presented: a discrete explicit model [2] and an original continuum based approach (see also [3]).

Discrete approach

Rossi [1] originally presents a probabilistic model implemented via a discrete approach in which interface elements are used to describe the discontinuities. The mechanical properties of the interface elements (Young modulus and tensile strength) are considered as randomly distributed variables. The volume of the massive elements which are adjacent to the considered interface element, acts as the reference (material) volume. Distribution characteristics (mean value and standard deviation) can be obtained from an extrapolation of the empirical formulas given in fig.1. The model is considered as probabilistic, but after the random distribution of mechanical properties over the mesh, the computation remains deterministic. It is then necessary to perform a large number of computations to statistically validate the results (following a Monte Carlo-like method). Scale effects are effectively taken into account and the model is auto-coherent in the sense that data at the local scale are coherent with results at the global scale since a generic law taking into account volume effects can define concrete mechanical properties at each scale. Although locally no energy is dissipated (the failure of the elementary volume remains elastic-perfectly brittle), the model allows to statistically representing a global dissipation of energy through inelastic residual strains, softening behaviors.

This modeling strategy has, however, some shortcomings at least in the original formulation presented in [1, 2]. According to the local and probabilistic character of the approach, the volume of the element has to be sufficiently small when compared to the volume of the meshed structure or to the zone size where stress gradients can develop (i.e. the fracture process zone). This can lead to very small ratios V_s / V_g which fall out of the domain of validity supported by the experimental campaign [2]. More recently, [4] has shown that the evolutions of the mean values and the standard deviations given by the empirical formulas with respect to the compressive strength become meaningless for ratios $V_s / V_g < 1$. An inverse analysis has then been proposed to determine the extrapolation of the empirical formulas to the small ratios V_s / V_g domain. In this paper, the extrapolations issued from the inverse analysis are taken into account for $V_s / V_g < 1$. The original size-effect law is therefore updated and will be used in the finite element analysis. Some questions arise also in the applicability of the discrete-explicit model. For more global approaches, at the scale of a whole structure for example, such model leads to prohibitive computational costs as the use of contact elements doubles the number of nodes. This is even more sensitive in the case of 3D modeling. These considerations justify an enhancement towards a continuum based approach. Such a model seems more adequate in many situations and in particular when dealing with real structures. If compared to a discrete model, a continuum model does not require contact elements, i.e. no pre-oriented cracks (any crack direction is favored). Moreover, in a continuum model, the cracking of a finite element corresponds to the cracking of a volume of material, i.e. the failure of a material volume can be associated to the idea of the fracture process zone (FPZ).

Continuum approach

The continuum based approach is defined at a macroscopic scale where stress and strain states are defined. At this scale, it is theoretically possible to establish a constitutive relation between stress and strain defining the macroscopic behavior of the material. Cracking processes can be then taken into account by considering a dissipative mechanism at the material scale. Two important facts have to be pointed out:

- Usually, the identification of the material behavior is performed on laboratory samples which size has to be larger than the Representative Elementary Volume (REV) in order to properly take into account the material heterogeneity. However, when dealing with concrete this size is not often in accordance with the size of the finite elements used in the modeling. It is thus necessary to perform an extrapolation of the identified experimental behavior to the scale of the finite element. This requires taking into account scale changes, i.e. volume effects must be considered at this stage.
- The localization of cracks, generally occurring at the peak, has to be carefully taken into account. Before localization, material integrity is quite preserved even if the material is severely damaged. After localization, material integrity fails such that it is impossible to consider the post-peak softening behavior as representative of the behavior of the material. In other words, after the peak we shift from a material behavior to a structural behavior [5]. Numerical translations of these problems are mostly leading to strong mesh sensitivities and non objective responses [6].

The model takes into account at the finite element level these aspects as follows:

- It is assumed that it is possible to define macroscopic quantities whatever the size of the finite element, whether it is material representative or not. It is then supposed that the mechanical behavior of the finite element depends on its size and position, i.e. the behavior of each finite element is prone to random variations, thus taking account the material heterogeneity.
- The mechanical behavior of the finite element (pre- and post-localization) is replaced by an equivalent material behavior. Since it is considered as a material behavior, this equivalent behavior does not have a softening branch after the peak. A dissipative mechanism is chosen to represent the whole cracking process, pre- and post-localization. The equivalent behavior is defined via equivalence in deformation energy. It can be argued that the local dissipative mechanism is not representative of the local energy amount really dissipated by the material during cracking. However, one should not forget that the key point is to replace the material behavior with a structural behavior by means of an equivalent material. In other terms, the local mechanism is approximated in favor of a proper global response. At the end of the cracking process, when the total amount of available energy is dissipated, failure of the finite element is assumed to be brittle.

The dissipative mechanism is represented via perfect plasticity. This choice is justified by the simplicity of the approach together with the well established theoretical framework and the robust numerical implementations. The principle of the energy equivalence is depicted in fig.2. Details are given in [3].

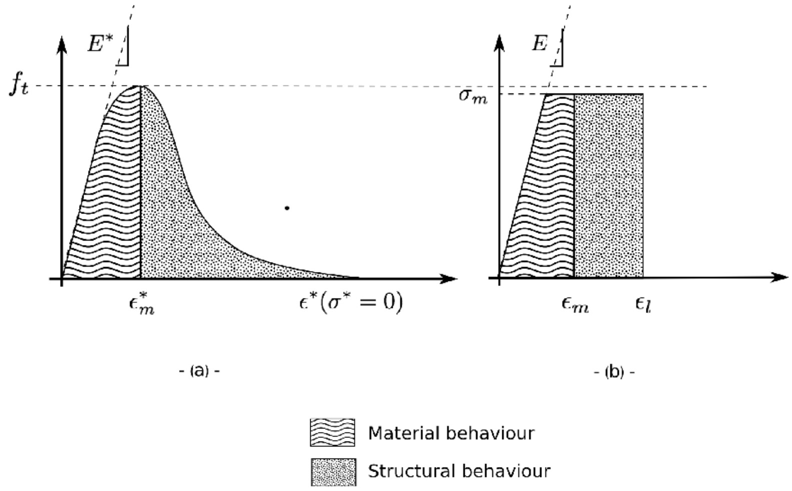


Figure 2: Principle of the equivalence for a uniaxial tensile behavior

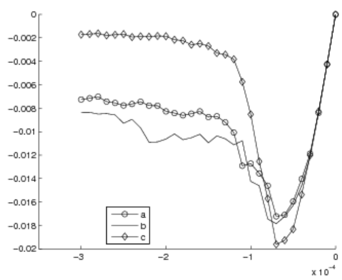
As far as the uniaxial behavior depends on the stressed volume of material and presents some randomness, the area under the curves is also a random quantity influenced by volume effects. Consecutively, σ_m and w_d can be considered as random parameters of the elastic-plastic equivalent model (also influenced by volume effect). The general laws defining the characteristics of the probabilistic distributions for σ_m and w_d have to be then identified via an experimental campaign or via a numerical campaign using the discrete approach. In such a case, the identification is performed following these steps:

- Choice of one type of concrete (i.e. V_g and f_c are fixed)
- Choice of one mesh size (this fixes the ratio V_s / V_g)
- Execution of *n* different computations
- Identification of the pre-peak behavior (material behavior) on each of the *n* computations followed by the computation of $m(\sigma_m)$ and $s(\sigma_m)$
- Identification of the mean value of w_d on the mean curve of the *n* computations, according to the principle depicted fig.2

The post-peak energy dispersion (and the standard deviation) identification, can be alternatively achieved via an inverse analysis on the equivalent model.

NUMERICAL SUPPORT

The choice of a numerical support is important as it should combine the relative simplicity of implicit models (which are particularly suitable for being used in the description of large structures) together with the capacity of giving some extra information necessary for a proper crack description. Three finite element approaches are therefore considered for the study: a Rashid-like [7] model (in which element stiffness is reduced to zero as soon as an energy threshold is reached), a fixed crack model [8] and an embedded formulation [9]. The three models have been tested on different configurations in order to evaluate the eventual stress locking (see figs.3 left) and mesh dependence (see fig.3 right)



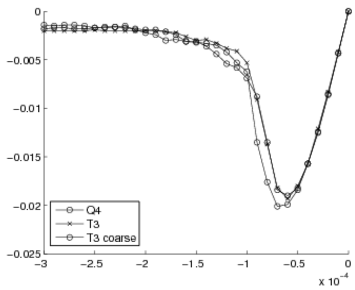


Figure 3
Left: notched beam bending test: fixed crack (b), EFEM (a), Rashid approach (c)
Right: Traction test with Rashid approach and different mesh (T3 regular, T3 coarse, Q4)

According to our test results, the Rashid-like model did not exhibit stress locking and together with the proposed probabilistic approach proved to be mesh independent. For these reasons, this model has been retained for the further probabilistic analysis.

VALIDATION

The modeling strategy presented in the previous sections is here compared to an original experimental test performed at LCPC. The experiment consists of a four point displacement-controlled bending test on a plain concrete beam. The beam geometry is given in fig.5 the concrete used is an ordinary concrete ($E=35GPa$, $f_c=50MPa$, $f_t=3MPa$, values experimentally determined). Displacements are measured on the front face via 6 LVDTs. The numerical probabilistic approach follows these steps:

- 30 computations are executed via the discrete approach for simulating the uniaxial tensile behavior of the concrete. A mean behavior is deduced.
- An inverse analysis is performed on the mean behavior to determine the parameters of the continuum approach
- These parameters are used to the modeling of the bending behavior. Again 30 computations are performed.

The beam has been modeled via T3 regular elements; the Rashid-like model has been used. Results are given in fig.5.

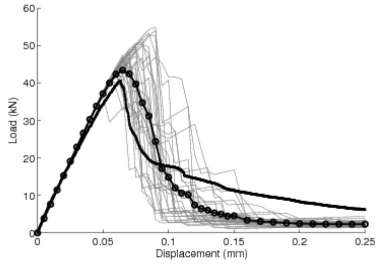


Figure 5: Global behavior: experimental (bold), numerical answer (grey) and mean (circles)

The correlation between the experimental result and the mean curve is quite good as the experimental result is contained in the set of the numerical answers and is very close to the mean answer. The macroscopic model provides not only a global answer but also some local information on cracks opening and distribution. In fig.6 a typical crack pattern and crack opening curve are presented. It is interesting to observe that not only a main macro crack is represented but also the multi-cracking character of the global failure is represented. Finally, a numerical traction test on a cube proves the 3D capabilities of the approach. In fig.7, a typical crack pattern of a traction test performed on a cube is presented.

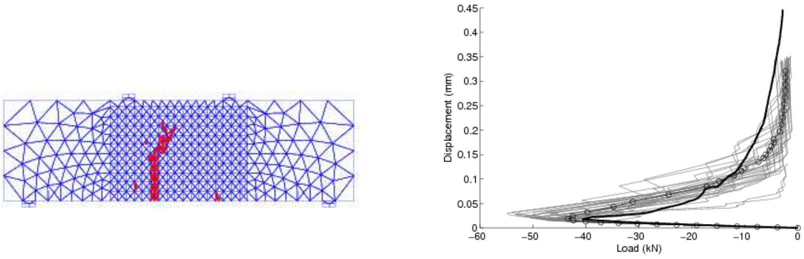


Figure 6: Typical crack patterns and crack opening (numerical -grey-, mean answer -circles- and experimental -bold-)

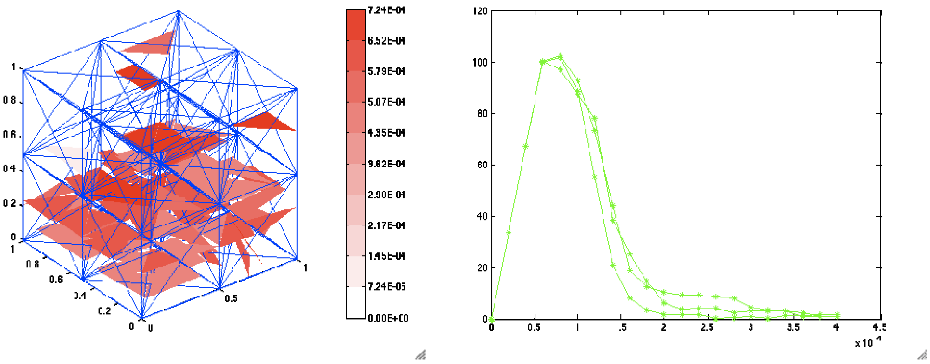


Figure 7: Traction test on a cube: cracks opening and load/displacement global answer

CONCLUSIONS

In this paper, a model representing cracking processes in concrete through a robust continuum approach coupled with a simple numerical modeling of discontinuities is presented. One should not forget that the simplicity of the numerical modeling is meaningful only if the approach is coupled with the statistical distribution of properties and the given scale laws. This solution strategy allows to properly take into account scale effects and the heterogeneous nature of concrete, providing a reliable global answer as well as local information such as crack patterns and openings. The first results confirmed that also 3D modeling is feasible: this enhancement seems a necessary step to take into account the complex nature and geometry of cracks and giving a satisfactory description of the local-global behavior of a real structure.

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