Role of the T-stress on the kink of a crack out of an interface

D. Leguillon[1] and S. Murer[1]

[1] Institut Jean le Rond d'Alembert - CNRS UMR 7190
Université Pierre et Marie Curie - Paris 6
4 place Jussieu, case 162, 75252 Paris Cedex 05 – France
Tel. : +33 144 275 322
Fax : +33 144 275 259
dominique.leguillon@upmc.fr
murer@lmm.jussieu.fr

ABSTRACT. *The presence of complex terms in the Williams' asymptotic expansion of the near tip field of an interface crack makes the analysis of the crack propagation more intricate. Griffith's criterion remains valid for the delamination growth because in this particular case the energy release rate does not involve oscillating terms. This is no longer true if the crack kinks out of the interface. Due to the oscillating terms, the energy release rate is ill-defined and it becomes impossible to extend Grifitth's criterion. Taking the T-stress, i.e. the next term of the asymptotic expansion, into account allows getting rid of this difficulty thanks to a characteristic length derived from a two-fold criterion using both energy and stress conditions.*

INTRODUCTION

It has been shown by one of the authors [1] that the initiation of a crack at a V-notch in a homogeneous material can be accurately predicted by a two-fold criterion based both on energy and stress conditions. Furthermore, the proposed criterion coincides with Griffith's one for a pure crack (i.e. when the V-notch opening vanishes). As a consequence of exponents greater than $1/2$ in Williams' expansion in the general case, this nucleation process is shown to be unstable, the crack jumps a short length. The presence of the T-stress term leads to a similar reasoning which is carried out herein.

Energy condition

The first condition results from an energy balance between two states of the structure prior and after the onset of a short crack increment. It states that, at initiation, the *incremental* energy release rate $G = -\delta W_p / \ell$ has to exceed the toughness G_c of the material, δW_p being the potential energy change and ℓ the newly created crack length (within the plane elasticity framework, the 3D generalization is possible but presents some technical difficulties [2]). Note that the *differential* form used by Griffith is the limit of the incremental energy release rate as $\ell \to 0$. This incremental form derives straightforwardly from the energy balance and is almost unquestionable whereas the differential form is conditional since it requires the existence of the limit. For interface cracks, we see later that the limit exists only in case of delamination growth (i.e. the crack keeps propagating along the interface).

Stress condition
The second condition is based on the maximum tension that a material can sustain before failure. It states that fracture can occur only if the opening stress along the expected crack path exceeds the material tensile strength σ_c. It reads $\sigma_{\theta\theta} \geq \sigma_c$ where $\sigma_{\theta\theta}$ is the hoop stress. It must be pointed out that, if there are oscillations, as it is the case for an interface crack, this inequality requires additional attention.

Mixed criterion
In the V-notch case, the compatibility between the two conditions gives an equation for the crack initiation length ℓ_c provided the (real part of the) exponent λ of the singularity involved in the associated Williams' expansion is strictly greater than 1/2 [1]. Inserted in one or other of the above inequalities and using Williams' expansion again, it leads to the final Irwin-like criterion

$$k \geq k_c = \left(\frac{G_c}{A(\alpha)}\right)^{1-\lambda} \left(\frac{\sigma_c}{s(\alpha)}\right)^{2\lambda-1} \tag{1}$$

Where k is the generalized stress intensity factor (GSIF) of the singular term of Williams' expansion (i.e. the weight of the singular term), k_c its critical value defined as a function of the material toughness G_c, of the tensile strength σ_c and of the singularity exponent λ. In Eq. 1, A and s are geometric coefficients depending on the direction α chosen by the crack increment.
In the particular case of a pre-existing crack, $\lambda = 1/2$, the second term of Eq. 1 disappears and the above criterion coincides with Griffith's condition.

The presence of complex terms in the asymptotic expansion of the near tip field of an interface crack makes the analysis of the crack propagation more difficult as shown in the next section.

THE INTERFACE CRACK - THE LEADING TERMS OF THE EXPANSION

The near tip field expands with two conjugate terms associated with the exponents $\lambda = 1/2 \pm \mathrm{i}\varepsilon$ [3,4] (in Eq. 2 and in the sequel the upper bar denotes the complex conjugate)

$$\underline{U}(x_1, x_2) = \underline{U}(O) + Kr^{1/2+\mathrm{i}\varepsilon}\underline{u}(\theta) + \overline{K}r^{1/2-\mathrm{i}\varepsilon}\overline{\underline{u}}(\theta) + \ldots = \underline{U}(O) + 2Re\left(Kr^{1/2+\mathrm{i}\varepsilon}\underline{u}(\theta)\right) + \ldots \tag{2}$$

Where r and θ are the polar coordinates emanating from O (figure 1) and where K holds for the complex stress intensity factor (SIF). It can be rewritten, introducing the mixed mode parameter $m(r)$ [4,5]

$$\underline{U}(x_1, x_2) = \underline{U}(O) + Kr^{1/2+\mathrm{i}\varepsilon}\left(\underline{u}(\theta) + m(r)\overline{\underline{u}}(\theta)\right) + \ldots \text{ with } m(r) = \frac{\overline{K}}{K} r^{-2\mathrm{i}\varepsilon} \tag{3}$$

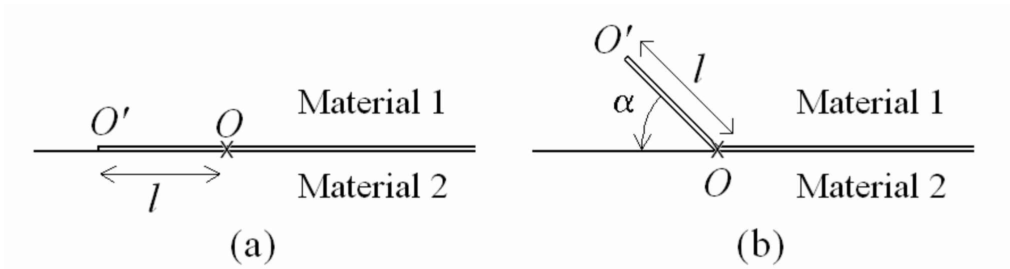

Figure 1. The interface crack growth, (a) delamination, (b) kink out of the interface.

The incremental energy release rate for a crack increment ℓ in the direction α now writes [5,6]

$$G = 2K\overline{K}\ Re\left(A(\alpha) + m(\ell)A'(\alpha)\right) + \ldots \tag{4}$$

Here the complex terms A and A' play the role of the single real scaling coefficient A in Eq. 1. For a delamination crack $\alpha = 0$ and $A'(0) = 0$, then it is clear that the energy condition $G \geq G_c$ (using either the incremental or the differential form) is meaningful and can be used to predict the delamination growth. Note that G_c denotes here the interface toughness, a “material” parameter which is difficult both to define (because of the mixity of modes) and to determine experimentally [7,8].

On the other hand, for a crack kinking out of the interface $A'(\alpha) \neq 0$. Then, due to the term $m(\ell)$ (Eq. 3), the limit as $\ell \to 0$ does not exists, the differential energy release rate is ill-defined. Different authors have tried to overcome this difficulty by assuming either that the imaginary part ε of the exponent, which is usually small, can be neglected or by prescribing a given characteristic distance [9,10,11]. In the next section we show that the next term of Williams’ expansion (Eqs. 2 and 3) can be used to avoid this problem.

THE ROLE OF THE T-STRESS

The pioneering work in this domain is due to Cotterell and Rice [12] who investigated the influence of the T-stress on the kinking of a crack submitted to a biaxial loading

(mode I plus T-stress) in a homogeneous material. This topic has been recently revisited [13,14] leading to define a positive threshold of the T-stress below which no branching can occur as observed in some experiments [15].
In the case of an interfacial crack, Williams' expansion takes the form

$$\underline{U}(x_1,x_2)=\underline{U}(O)+Kr^{1/2+\mathrm{i}\varepsilon}\underline{u}(\theta)+\overline{K}r^{1/2-\mathrm{i}\varepsilon}\overline{\underline{u}}(\theta)+Tr\underline{t}(\theta)+\ldots \tag{5}$$

Where $r\underline{t}(\theta)$ is the first non singular term of the expansion, the associated stress field does not grow to infinity but remains constant as $r\to 0$. The coefficient T is the corresponding intensity factor. Introducing a new mixed mode parameter $M(r)$ allows writing

$$\begin{cases}\underline{U}(r,\theta)=\underline{U}(O)+K\ r^{1/2+\mathrm{i}\varepsilon}\ (\underline{u}(\theta)+m(r)\ \overline{\underline{u}}(\theta)+M(r)\ \underline{t}(\theta)+\ldots)\\ \underline{\underline{\sigma}}(r,\theta)=Kr^{-1/2+\mathrm{i}\varepsilon}(\underline{\underline{s}}(\theta)+m(r)\ \overline{\underline{\underline{s}}}(\theta)+M(r)\ \underline{\underline{\tau}}(\theta)+\ldots)\end{cases}\quad \text{with}\quad M(r)=\frac{T}{K}r^{1/2-\mathrm{i}\varepsilon} \tag{6}$$

The energy condition takes the following form

$$G=K\overline{K}\ X(\alpha,m(\ell),M(\ell))+\ldots\geq G_c \tag{7}$$

With

$$X(\alpha,m(\ell),M(\ell))=2Re\left[A(\alpha)+m(\ell)A'(\alpha)+M(\ell)B(\alpha)\right]+M(\ell)\overline{M(\ell)}\ C(\alpha)+\ldots$$

The new complex term B and the real one C play roles similar to A and A' (Eq. 4). The stress condition derived from Eq. 6_2 leads to

$$K\overline{K}\ Y(\theta,m(\ell),M(\ell))\geq \ell\ \sigma_c^2 \tag{8}$$

With

$$Y(\theta,m(\ell),M(\ell))=\left|s_{\theta\theta}(\theta)+m(\ell)\ s_{\theta\theta}(\theta)+M(\ell)\ \tau_{\theta\theta}(\theta)+\ldots\right|^2$$

The compatibility between the two inequalities 7 and 8 gives an equation for the crack initiation length ℓ_c as a function of α

$$\ell_c\,\frac{X(\alpha,m(\ell_c),M(\ell_c))}{Y(\alpha,m(\ell_c),M(\ell_c))}=\frac{G_c}{\sigma_c^2} \tag{9}$$

It was not possible to extract such a length from Eq. 4 where ℓ appears only through the oscillating term in $m(\ell)$. It is now feasible thanks to the additional T-stress term which involves an exponent larger than 1/2.
Finally Eq. 7 with $\ell=\ell_c$ gives a condition on $|K|$ for crack initiation in the direction α :

$$|K| \geq K_\alpha = \sqrt{\frac{G_c}{X(\alpha, m(\ell_c), M(\ell_c))}} \tag{10}$$

The critical value K_α depends on α and the actual kink angle α_c maximizes the denominator, i.e. minimizes K_α giving K_f (i.e. $|K|$ at failure).

$$X(\alpha_c, m(\ell_c), M(\ell_c)) \geq X(\alpha, m(\ell_c), M(\ell_c)) \quad \forall \alpha \tag{11}$$

And

$$K_f = \sqrt{\frac{G_c}{X(\alpha_c, m(\ell_c), M(\ell_c))}} \tag{12}$$

Note that Eq. 11 is not trivial since ℓ_c is a function of α (Eq. 9). This reasoning follows step by step that of the real case [1,5] (homogeneous material for instance) but the physical background is less rigorous because of the oscillating terms met in G and $\sigma_{\theta\theta}$.

APPLICATION

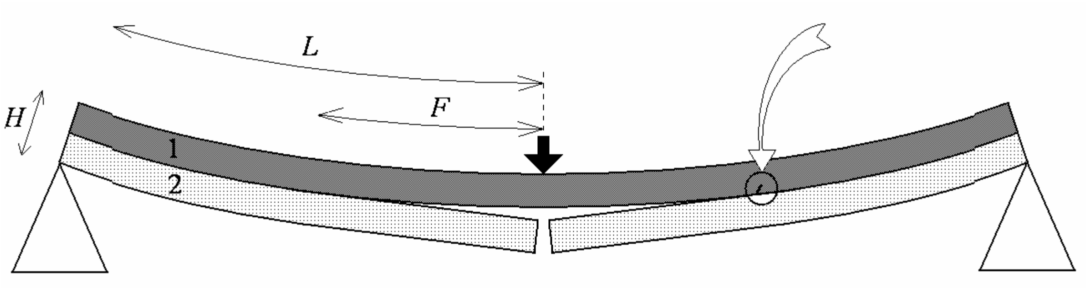

Figure 2. 3-point bending test, $F/L = 0.25$, $H/L = 0.2$.

Elastic simulations are carried out on a notched bimaterial specimen in flexion (figure 2) with a long debonding of the interface. The stiffer material is alternatively in the upper ($R > 1$) and lower ($R < 1$) position, R is the Young's modulus ratio, $\nu = 0.3$ in both materials. Two different materials are considered: Alumina, $E_1 = 300$ GPa, $\sigma_{1c} = 400$ MPa and $G_{1c} = 0.05$ MPa.mm ($K_{Ic} = 4.06$ MPa.m$^{1/2}$), and PMMA $E_2 = 3$ GPa, $\sigma_{1c} = 75$ MPa and $G_{1c} = 0.35$ MPa.mm ($K_{Ic} = 1.07$ MPa.m$^{1/2}$). When Alumina is in the upper position ($R > 1$) various contrasts are analysed for R varying from 100 to 2. Similarly, when PMMA is in the upper position ($R < 1$) various contrasts are analysed for R varying from 0.5 to 0.01. The particular case $R = 1$ (no contrast) is given for comparison taking $K=\frac{1}{2}\left(K_I+\mathrm{i}\,K_{II}\right)$ and $\underline{u}(\theta)=\underline{u}_I(\theta)-\mathrm{i}\,\underline{u}_{II}(\theta)$ where the indices I and II hold for the classical modes I and II associated with a crack tip located in a homogeneous material. A constant force is applied in all cases.

Results are presented in table 1. It is worth noting that the sign of ε is a convention since both + and – occur in the expansion (Eq. 2). The two terms in the last two rows correspond on the left to the present analysis and on the right to a simplified analysis where T is neglected (i.e. with $T=0$ in the equations). Results are almost similar for a kink of the interface crack in the more compliant material (i.e. R varying from 0.01 to 0.5). The ratio $K_f\,/\,K_{Ic}$ remains the same and the kink angle does not exceed 50 deg. On the contrary the difference becomes significant when the kink occurs in the stiffer material. The ratio $K_f\,/\,K_{Ic}$ is twice as high (for large contrasts) if T is neglected. This means that the T-stress has a significant role in the crack kinking and omitting this term leads to overestimating the applied load at failure. Moreover, the kink angle quickly reaches 90 deg. as the contrast increases.

Table 1. Numerical results. The right hand side term in the last two rows are obtained by neglecting T, i.e. with $T=0$ in the equations.

R	ε	$\mathrm{Im}(K)/\mathrm{Re}(K)$	$T/\mathrm{Re}(K)$ ($\mathrm{m}^{-1/2}$)	α_c (°)	$K_f\,/\,K_{Ic}$
100	0.091	0.41	37.4	90 / 30	0.07 / 0.17
75	0.090	0.38	31.6	90 / 30	0.09 / 0.21
50	0.089	0.41	37.8	90 / 40	0.12 / 0.22
10	0.075	0.49	62.8	70 / 40	0.23 / 0.28
2	0.030	0.75	65.2	60 / 50	0.33 / 0.35
1	0.000	0.93	81.8	60 / 50	0.38 / 0.39
0.5	-0.030	-1.14	47.7	50 / 50	0.47 / 048
0.1	-0.075	-1.49	15.8	40 / 40	0.63 / 0.63
0.02	-0.089	-1.58	11.9	30 / 30	0.73 / 0.73
0.13	-0.090	-1.59	-35.4	30 / 30	0.75 / 0.75
0.01	-0.091	-1.59	-38.1	30 / 30	0.76 / 0.75

CONCLUSION

When considering a kink in the stiffer material, neglecting the T-stress can lead, in case of strong contrast, to a significant discrepancy in terms of critical load and kink angle prediction. In particular, the load at failure is overestimated and leads to a non-conservative prediction. Nevertheless there are only few experiments to corroborate these results.

An interesting feature to point out is the influence of the T-stress on the kink angle. It is almost 90 deg. if the crack grows in the stiff layer whereas it does not exceed 50 deg. if the crack grows in the compliant one. It can be compared to the fracture pattern of a laminated ceramic (figure 3) made of compliant (light) and stiff (dark) layers [16]. However, the comparison has its limits since there are no large deflections of the crack along the interface as assumed in the present analysis (figure 2).

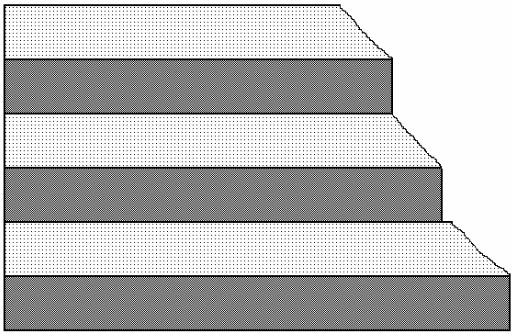

Figure 3. Schematic fracture pattern of a laminated ceramic

For a complete analysis the competition between the deflection mechanism and the delamination growth should be analysed. This requires the knowledge of the interface toughness and strength, which determination is difficult since it is commonly admitted that they depend on the mixity of modes [7,8] involved at the interface crack tip.

REFERENCES

1. 1. Leguillon D. (2002). Strength or toughness? A criterion for crack onset at a notch. Eur. J. Mech. A/Solids, 21, 61–72.
2. Leguillon D. (2003). Computation of 3D singular elastic fields for the prediction of failure at corners, in "*Advances in Fracture and damage Mechanics*", Key Engineering Materials, vol. 251-252, 147-152.
3. Leguillon D., Sanchez-Palencia E. (1987). *Computation of singular solutions in elliptic problems and elasticity*. J. Wiley, New-York Masson, Paris.
4. Rice J R (1988). Fracture mechanics concepts for interfacial cracks. J. Appl. Mech., 55, 98–103.
5. Leguillon D., Siruguet K. (2001). Finite fracture mechanics – Application to the onset of a crack at a bimaterial corner. In *IUTAM Symposium on Analytical and Computational Fracture Mechanics of Non-Homogeneous Materials*, B.L. Karihaloo ed., Solid Mechanics and its Applications vol.97, Kluwer Academic Publishers, Dordrecht, 2002, 11-18.
6. Leguillon D. (1989). Calcul du taux de restitution de l'énergie au voisinage d'une singularité, C.R. Acad. Sci. Paris, série II, 945-950.

7. Liechti K.M., Chai Y.S. (1991). Biaxial loading experiments for determining interfacial fracture toughness, J. Appl. Mech., 58, p.681-687.
8. Wang J.S., Suo Z. (1990) Experimental determination of interfacial toughness curves using brazil-nut-sandwiches. Acta Metall. Mater. 38(7), 1279-1290.
9. He M.Y., Hutchinson J.W. (1989). Kinking of a crack out of an interface. J. Appl. Mech., 111, 270-278.
10. He M.Y., Bartlett A., Evans A.G., Hutchinson J.W. (1991). Kinking of a crack out of an interface: role of in-plane stress. J. Am. Ceram. Soc., 74(4), 767-771.
11. Lee W., Shin H., Yoo Y.H. (2004). Revisited criterion to prevent kinking of a crack out of a bi-material interface under the presence of in-plane residual stresses – a global approach, Int. J. Engng. Sci., 42, 257-270.
12. Cotterell B. and Rice J.R. (1980). Slightly curved or kinked cracks. Int. J. Fract., 16(2), 155–169.
13. Leguillon D., Murer S. (2008). Crack deflection in a biaxial stress state. Int. J. Fract., 150, 75-90.
14. Smith D.J., Ayatollahi M.R., Pavier M.J. (2006). On the consequences of T-stress in elastic brittle fracture, Proc. R. Soc. A, 462, 2415-2437.
15. Selvarathinam A.S., Goree J.G. (1998). T-stress based fracture model for cracks in isotropic materials. Eng. Frac. Mech., 60(5-6), 543–561.
16. Tariolle S. (2004). Carbure de Bore monolithique poreux et composites lamellaires. Elaboration, propriétés, renforcement. PhD thesis, Univ. of St. Etienne, France.