Brittle crack initiation at a V-notch under mixed mode loading

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Abstract *The failure criterion at reentrant corners in brittle elastic materials presented in [1,2] validated in [3] for mode I loading is being extended to mixed mode loading and is being validated by experimental observations. We present all quantities involved in the computation of the failure criterion and validate it by comparison of the predicted critical load and crack initiation angle to experiments on PMMA (polymer) and Macor (ceramic) V-notched specimens under mixed mode loading on.*

INTRODUCTION

Failure laws for brittle materials containing V-notches of variable opening angles have become of major interest because of failure initiation in electronic devices. A reliable law for predicting the failure initiation instance (crack formation) in these cases in the vicinity of singular points, especially when a complex state of stress is present in the vicinity of the V-notch tip, is still a topic of active research and interest. At such points the stress tensor is infinity under the assumption of linear elasticity. A typical example of a singular point is the V-notch tip, for which a crack tip is a particular case when the V-notch solid angle is $\omega = 2\pi$.

For the simplified mode I state of stresses in the vicinity of a V-notch tip, i.e tension perpendicular to the V-notch bi-sector alone, several failure criteria have been proposed and verified by experimental observations, as in [1,4,5,6,7]. A comparison of several of the presented failure criteria (and a newly proposed one) against experimental observations is presented in [3].

For a mixed mode stress state in the vicinity of a V-notch tip, the number of failure initiation criteria suggested and validated via experimental observations is much smaller.

Among these are [8,9,10]. The failure criterion in [8] is restricted to low values of mode mixity when mode I dominates and the failure criteria investigated by Seweryn et al., although predicting well the failure initiation, have been shown to be inferior to Leguillon's criterion for mode I loading. Therefore, we herein extend the failure criterion presented by Leguillon in [1], based on finite fracture mechanics concept, to mixed mode loading – see for details [11]. This criterion, shown to predict very well failure initiation under mode I loading for various V-notch angles (see e.g. [1,12]), satisfies both the classical Griffith criterion and the strength criterion.

We consider a domain containing a V-notch with a solid angle ω (the opening angle is $2\pi - \omega$) with a small crack of length l_0 that initiates at an angle θ_0, see Figure 1.

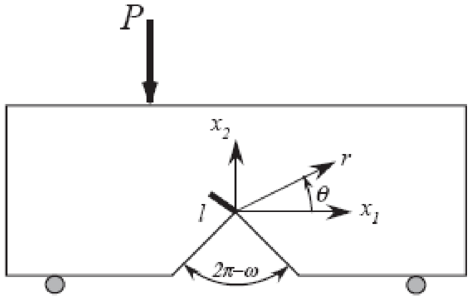

Figure 1: The V-notched specimen with a crack at its tip.

We extend Leguillon's failure initiation criterion for mode I loading [1] to a mixed mode loading, and validate it by experimental observations. The failure criterion depends on two material's parameters, the 1-D stress at brittle fracture, σ_c (strength), and the fracture energy release rate per a unit surface f, G_c (toughness), the first two generalized stress intensity factors A_1 and A_2 associated with the V-notch tip and three geometrical functions H_{11}, H_{12}, H_{22} that depend on the V-notch geometry alone.

THE FAILURE INITIATION CRITERION FOR MIXED MODE LOADING

One of the corner-stone of Leguillon's failure criterion is the postulate that a finite crack length l_0 has to be instantaneously created so to satisfy both the strength and toughness requirements. For a given small load, a large crack is required to satisfy the toughness criterion, whereas only at small distance from the notch tip the tangential stress reaches σ_c (the strength criterion is satisfied). To satisfy both criteria simultaneously, the load has to be increased so that the upper bound of the crack length decreases to satisfy the toughness criterion, and the distance from the notch tip where tangential stress reaches σ_c increases. Only when the load is increased to a level where the lower bound of a crack to satisfy the toughness criterion equals the upper bound of the distance to satisfy the stress criterion, a crack of this length is created satisfying simultaneously the two criteria.
To this end let us first consider the Griffith energy criterion as it is manifested for finite fracture mechanics when a small crack of length l_0 is spontaneously formed at a V-notch tip. In this case, for a crack to be formed the following inequality has to hold:

$$-\frac{\delta\Pi}{l} \geq G_c \tag{1}$$

where $\delta\Pi$ denotes the difference in the potential energy between a V-notched cracked domain and the V-notched un-cracked domain:

$$-\delta\Pi \equiv -\left(\Pi(l) - \Pi(l=0)\right) \quad (2)$$

and $G_c = \dfrac{K_{IC}^2}{E/\left(1-\nu^2\right)}$ for plane strain.

In order to calculate this change in potential energy between the two states one has to specify the behavior of the displacement field for the V-notched cracked domain.

This can be done by an asymptotic analysis of the displacement field when a small finite crack is introduced at the V-notch tip.

Using a coordinate transformation $y_i = x_i / l$, $\rho = r / l$, we observe the very close neighborhood of the V-notch tip and match the terms of the solution of this "inner expansion" with the terms of the "outer expansion" (as $r \to 0$ and $\rho \to \infty$).

Matching the solutions it can be shown that:

$$u_l\left(l_{y1}, l_{y2}\right) = u(0,0) + A_1 l^{\alpha_1}\left(\rho^{\alpha_1} u^{(1)}(\theta) + \hat{v}_1\right) + A_2 l^{\alpha_2}\left(\rho^{\alpha_2} u^{(2)}(\theta) + \hat{v}_2\right) + \ldots \quad (3)$$

Where $\hat{v}_1, \hat{v}_2 = 0 \; as \; \rho \to \infty$

We now need to solve for $\hat{v}_1$ and $\hat{v}_2$ elasticity problems with prescribed vanishing behavior at infinity:

$$\sigma\left(\hat{v}_i\right) \underset{\rho\to\infty}{\approx} 0 \quad or \quad \hat{v}_i \underset{\rho\to\infty}{\approx} 0 \quad (4)$$

Using Betti's theorem one may show that:

$$\delta\Pi = \Pi(l) - \Pi(l=0) = \psi\left(u_l, u_0\right) \; where \; \psi\left(u_l, u_0\right) \equiv \frac{1}{2}\int_\Gamma \left[T(f)g - T(g)f\right] dS \quad (5)$$

Inserting (3) in to (5) one obtains:

$$\begin{aligned} &\delta\Pi = A_1^2 l^{2\alpha_1} H_{11} + A_1 A_2 l^{\alpha_1+\alpha_2}(H_{12} + H_{21}) + A_2^2 l^{\alpha_2} H_{22} \\ &H_{ii} \equiv -\psi\left(\hat{v}_i, \rho^{\alpha_i} u^{(i)}\right) \quad H_{ij} \equiv -\psi\left(\hat{v}_i, \rho^{\alpha_j} u^{(j)}\right) \quad H_{ji} \equiv -\psi\left(\hat{v}_j, \rho^{\alpha_i} u^{(i)}\right) \end{aligned} \quad (6)$$

It is important to note that the H_{ij}'s are only a function of the notch geometry and material elastic parameters. See [11] for details on the different functions H_{ij} obtained by FE analyses.

Following the initial analysis proposed in [13], inserting (6) in to (1) we can obtain a lower limit for l :

$$A_1^2 l^{2\alpha_1 - 1} H_{11}(\omega,\theta) + A_1 A_2 l^{\alpha_1+\alpha_2-1}\left(H_{12}(\omega,\theta) + H_{21}(\omega,\theta)\right) + A_2^2 l^{\alpha_2 - 1} H_{22}(\omega,\theta) \geq G_c$$

$$\Downarrow \tag{7}$$

$$l \geq \left(\frac{G_c}{A_1^2 \left[H_{11}(\omega,\theta) + m\left(H_{12}(\omega,\theta) + H_{21}(\omega,\theta)\right) + m^2 H_{22}(\omega,\theta) \right]} \right)^{\frac{1}{2\alpha_1 - 1}}$$

where

$$m = \frac{A_2}{A_1} l^{\alpha_2 - \alpha_1} \tag{8}$$

The upper limit can be obtained from:

$$\sigma_{\theta\theta}(l,\theta) = A_1 l^{\alpha_1 - 1} \sigma_{\theta\theta}^{(1)}(\theta) + A_2 l^{\alpha_2 - 1} \sigma_{\theta\theta}^{(2)}(\theta) \geq \sigma_c$$

$$\Downarrow \tag{9}$$

$$l \leq \left(\frac{A_1 \left(\sigma_{\theta\theta}^{(1)}(\theta) + m \sigma_{\theta\theta}^{(2)}(\theta) \right)}{\sigma_c} \right)^{\frac{2}{2 - 2\alpha_1}}$$

If both the toughness and strength criteria have to hold at the instance of failure then the upper and lower bounds have to coincide, yielding:

$$A_{1C} = \left(\frac{G_c}{H_{11}(\omega,\theta) + m\left(H_{12}(\omega,\theta) + H_{21}(\omega,\theta)\right) + m^2 H_{22}(\omega,\theta)} \right)^{\alpha_1 - 1} \left(\frac{\sigma_c}{\sigma_{\theta\theta}^{(1)}(\theta) + m \sigma_{\theta\theta}^{(2)}(\theta)} \right)^{2\alpha_1 - 1} \tag{10}$$

and

$$l_0 = \frac{G_c}{H_{11}(\omega,\theta) + m\left(H_{12}(\omega,\theta) + H_{21}(\omega,\theta)\right) + m^2 H_{22}(\omega,\theta)} \left(\frac{\sigma_{\theta\theta}^{(1)}(\theta) + m \sigma_{\theta\theta}^{(2)}(\theta)}{\sigma_c} \right)^2 \tag{11}$$

The crack initiation direction θ_c is the direction which for a given A_2/A_1 ratio gives the smallest A_{1C}. Failure will initiate once for a given load $A_1 \geq A_{1C}^{\min}$ and will take the form of the creation of a finite crack with the length of l_0.

VALIDATION OF THE FAILURE CRITERION BY EXPERIMENTAL OBSERVATION

Experiments on three point bending specimens made of PMMA

Three point bending (3PB) experiments were preformed on PMMA notched specimens at room temperature $(E = 3100[MPa], \sigma_c = 111.8[MPa], K_{IC} = 1.03 - 1.25[MPa\sqrt{m}])$ loaded so to produce a mixed mode state at the notch tip. Different asymmetries in the loading allows generating different mode mixity ratios (Figure 2).

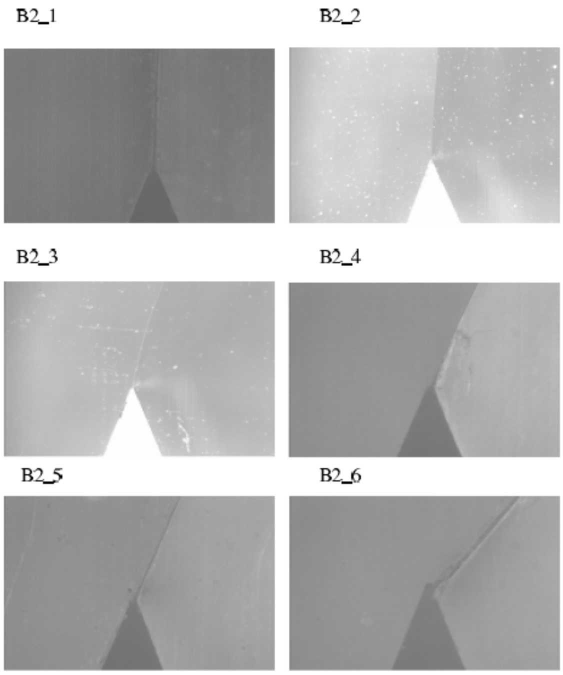

Figure 2: Pictures of the cracks at the 315^0 V-notch tip 3PB PMMA specimens

Figures 3 and 4 show the experimental and prediction crack initiation angle and critical load.

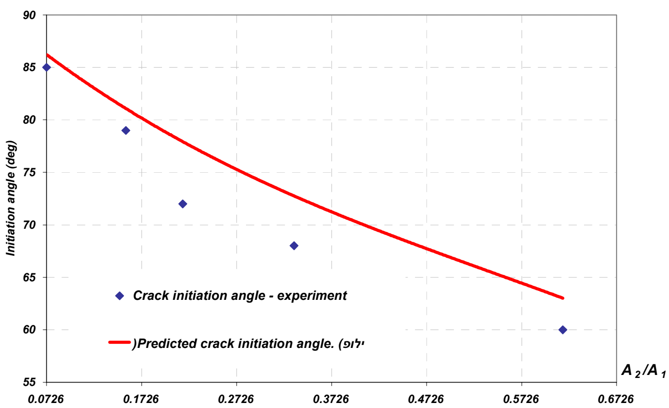

Figure 3: Crack initiation angle for the PMMA specimens experimental and prediction

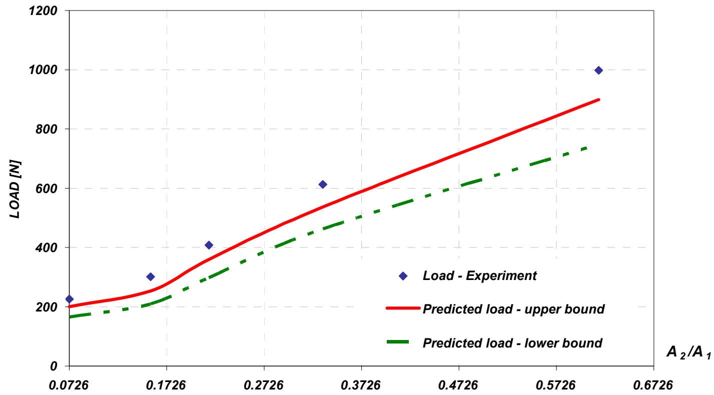

Figure 4: Critical load experimental and prediction for the PMMA specimens.

Experiments on four-point bending specimens made of MACOR

Four point bending (4PB) experiments were conducted on Machinable Ceramic (MACOR- manufactured by Aremco) notched specimens at room temperature ($E = 66900[MPa], \sigma_c = 103[MPa], K_{IC} \approx 1.2[MPa\sqrt{m}]$) were loaded so to produce a mixed mode loading state at the notch tip. In Figure 5 we show pictures of the crack created at the V-notch tip in the various specimens.

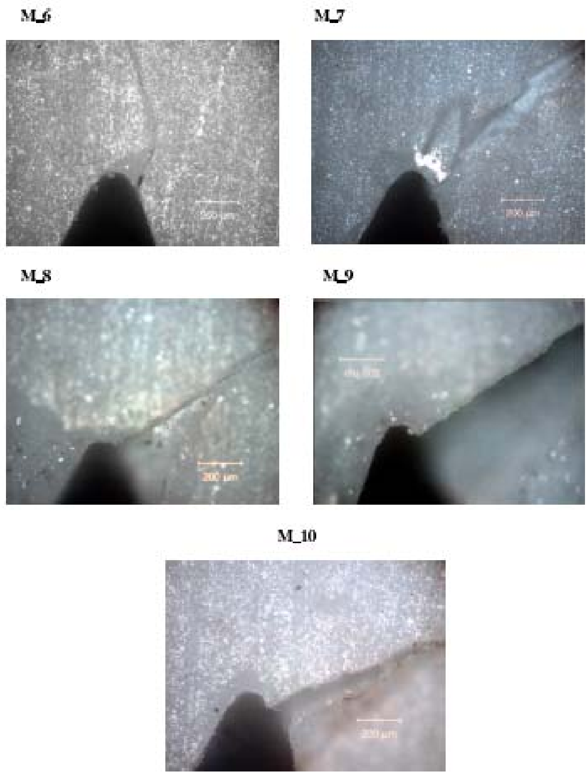

Figure 5: Pictures of the crack at the 315^0 V-notch tip in 4PB MACOR specimens

Figure 6 and 7 show the experimental and prediction crack initiation angle and critical load.

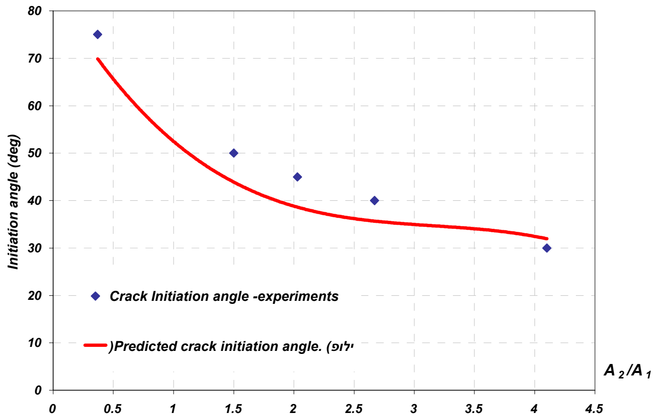

Figure 6: Crack initiation angle for the MACOR specimens experimental and prediction

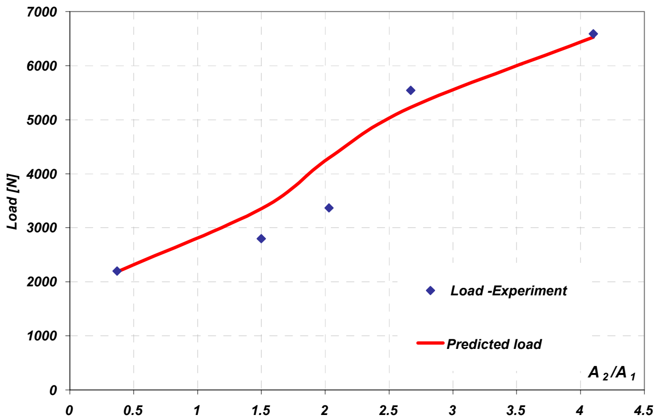

Figure 7: Critical load experimental and prediction for the MACOR specimens

SUMMARY AND CONCLUSIONS

The failure criterion of Leguillon at reentrant corners in brittle elastic materials was extended herein to mixed mode loading. The validity of the proposed failure criterion is examined by comparing predicted failure loads and crack initiation angles for a range of mode mixity ratios to these measured in experiments. Two sources of experimental results were considered for validating the failure criterion: a) Experiments performed on PMMA specimens with different V-notch opening angles and 4PB V-notched Macor specimens.

A very good predictability is demonstrated for the experiments conducted by us on 3PB 315^o V-notched specimens made of PMMA. Because the failure criterion assumes a mathematical sharp V-notched tip the predicted failure loads are expected to be the lower bound to the experimental failure loads, as indeed is the case in Figure 7. A very good predictability is also demonstrated for the 4PB 315^o V-notched specimens made of Macor.

Future investigations will assess the validity of the failure criterion for a wider range of brittle materials and V-notch angles, as well as the influence of the V-notch tip radius (as done for mode I loading in [7]).

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