ON THE USE OF THE WEIGHT FUNCTION APPROACH TO STUDY FATIGUE CRACK GROWTH IN CAST IRON COMPONENTS

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ABSTRACT: Large mechanical components subjected to cyclic loading quite often spend a considerable part of their life in the propagation of one or more fatigue cracks. Therefore the knowledge of materials crack propagation behaviour under cyclic loading appears of considerable interest. During tests conducted for the determination of the classical S-N fatigue curves, useful information on crack growth can usually be collected and a reliable evaluation of the stress intensity factor can be realised. In this paper the results of fatigue tests performed on large cast irons specimens are discussed. During each test the crack initiation was detected and several crack propagation measurements were collected. On the basis of the weight function approach, using a solution proposed for the evaluation of the stress intensity factor (K) in case of corner cracks with elliptical front, ΔK values was calculated and correlated with the corresponding da/dN values, with the intent of determining the Paris laws for the studied material.

KEY WORDS: weight function, corner crack, cast iron, bending fatigue test

INTRODUCTION

The information obtained from the study of fatigue crack propagation can have useful application for the design and the damage evolution analysis of mechanical components characterised by large dimensions, which usually spend a considerable part of their life in the propagation of a crack. In these conditions the possibility of determining reliable fatigue crack growth relationships results very interesting for the assessment of the component functionality.

Quite often however, taking in account the difficulties in the use of standard specimens (i.e. CT), mainly because of the poor representation of the effective propagation conditions during service, it is still preferable to perform the tests in the typical configurations designed for the determinations of the classical S-N curves.

In fact, due to the usually large specimen dimensions, is possible to collect useful information on crack initiation and propagation, that enable the identification of the crack shape evolution in dependence of the applied cyclic loads during the test up to the

specimen failure. In this case, the principal problem for a suitable analysis of data consists in the reliable evaluation of the stress intensity factor (S.I.F.) at the crack tip.
When the crack shape is known, the weight functions approach can represent an efficient calculation tool. In fact it enables the analysis of the crack length measurements for the evaluation of the applied stress intensity factor range (ΔK). This can then be correlated with the corresponding crack propagation rate (da/dN), thus enabling the determination of reliable fatigue crack growth relationships.
As follows from the original formulation (Bueckner, 1970) and from the subsequent generalisation (Rice,1972), the S.I.F. can be calculated by an integration of the nominal loading conditions multiplied by an opportune function depending on the geometry only, which is defined weight function (W.F.).
For 3-D problems, that represents the most general but however also the less treated situation, the solution can be obtained by the following integral, whose parameters are described in figure 1 a) and 1 b):

$$K(\gamma) = \int_{\Omega} W(x,y,\gamma,a,c,t,h,b.....) \cdot q(x,y) \cdot d\Omega \qquad (1)$$

K is a function of the position along the crack front (trough the curvilinear coordinate γ). W(x,y,...) is the 3-D weight function defined in a two dimensional domain (Ω), which depends on the main geometrical parameters of the body. Finally q(x,y) represents the applied nominal loading conditions.

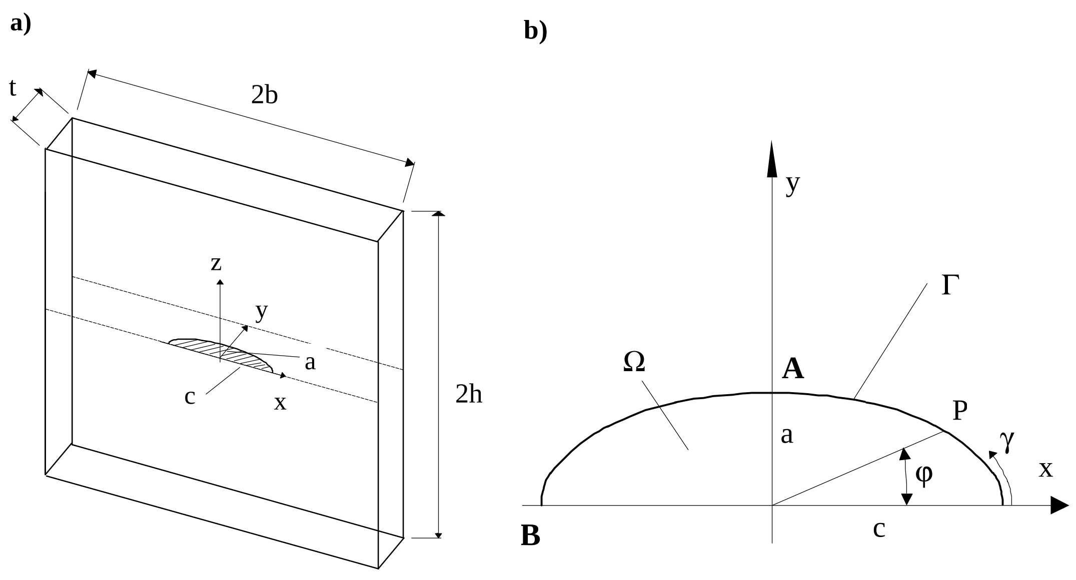

Figure 1: geometrical parameters for the description of a 3-D surface crack

The configuration of semi-elliptical surface crack represents a particular condition, for which, due to the practical importance, some solution were proposed in the literature with the aim of providing the evaluation of the S.I.F. in specific points of the crack front

(Zheng et alii 1996, Zhao et alii 1990) or directly of calculating the values of K at each point of the crack front (Beghini et alii, 1997, Orynyak et alii, 1994).

These approaches are assumed as a basis for the analysis of the experimental results reported in this paper, which were obtained through bending fatigue tests realised on cast irons specimens machined from particular mechanical components.

The experimental program was initially planned with the primary purpose of realising classical bending fatigue tests in order to determine the S-N curves for different cast irons grades. However, during the test, by using opportune instruments, the crack initiation was clearly detected and subsequently crack length measurements during propagation up to rupture on each specimen for one of the studied materials were realised.

Considering that the crack front remained essentially elliptic during a considerable part of the crack propagation, it was possible to analyse the measured data on the basis of the W.F. approach, adopting in particular a simplified solution (Zheng et alii, 1996), that was verified to be reliable for the evaluation of the S.I.F. in case of surface and corner cracks.

It was then possible to correlate the values of da/dN and ΔK and therefore to suggest plausible Paris relationships for the studied material. The results were finally compared with typical data found in literature.

MATERIAL AND EXPERIMENTAL PROCEDURE

The fatigue crack growth behaviour was studied for a cast iron having the chemical composition reported in table I:

Table I: chemical composition of the studied material (Kaufmann et alii,1996)

C	**Si**	**Mn**	**P**	**S**	**Cr**	**Ni**	**Mo**	**Mg**
2,92	1,91	0,86	0,02	0,02	0,44	0,67	0,05	0,003

The material was characterised by a perlitic microstructure and a morphology of graphite typically compact. Interdendritic segregations of very hard steadite were also present. This kind of material is quite often adopted for the production of large dimensions cylinders used in several mechanical processes.

Because of the loading conditions these components undergo frequently to fatigue failure. Therefore an extensive fatigue characterisation was planned and taking in account the in service prevailing bending loading conditions, the testing programs was conducted on bending specimens characterised by notches having shape factor respectively α_{kb} = 1,10 and α_{kb} = 1,75, as shown in the figures 2 and 3. The fatigue tests were conducted at a frequency of nearly 26 Hz on a Schenk testing machine, working in resonance. Stress ratio R was set 0 and -1.

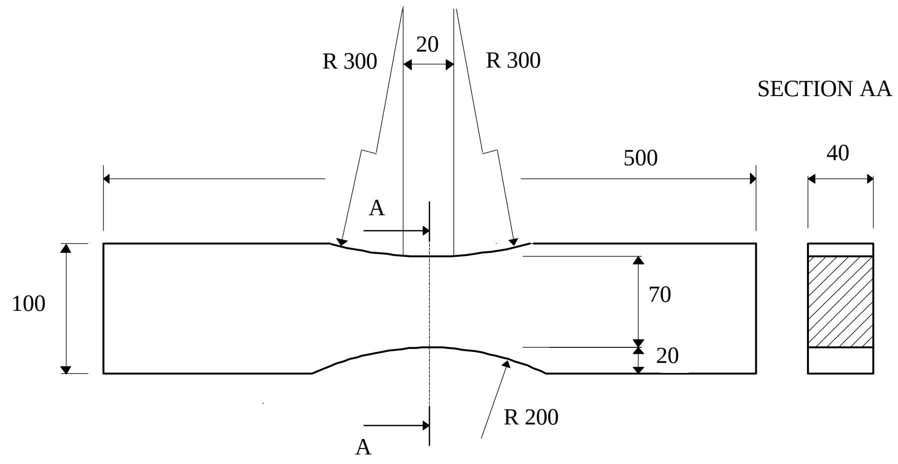

Fig. 2 : bending specimen characterised by shape factor α_{kb} = 1,10 (geometrical dimensions expressed in mm)

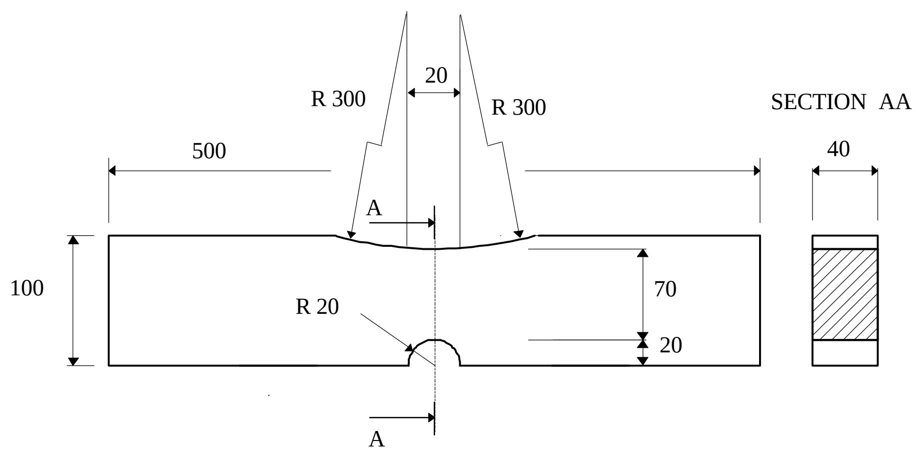

Fig. 3 : bending specimen characterised by shape factor α_{kb} = 1,75 (geometrical dimensions expressed in mm)

The schematic representation of the testing configuration and the experimental apparatus are reported in figure 4. In the figure the instruments used during the first part of the test for the identification of the crack initiation are shown. The apparatus consists of an accelerometer (Delta Analyser) for the acquisitions of frequencies spectra, that are then analysed by a computer software. At the beginning of the test a reference spectrum is acquired; this, more or less, presents a sharp peak at a frequency near to the frequency of the testing machine working in resonance. The initiation of a crack introduces a progressive displacements from the initial spectrum and the difference is assumed as a control parameter for the detection of the initiated specimen damage. When this difference exceeds an a priori defined threshold, the test is temporarily interrupted for the identification and the measurement of the initiated crack. The detection of the crack trace is made clear and easier by the use of a glycerine and zinc-oxide paste spread on the specimen surfaces.

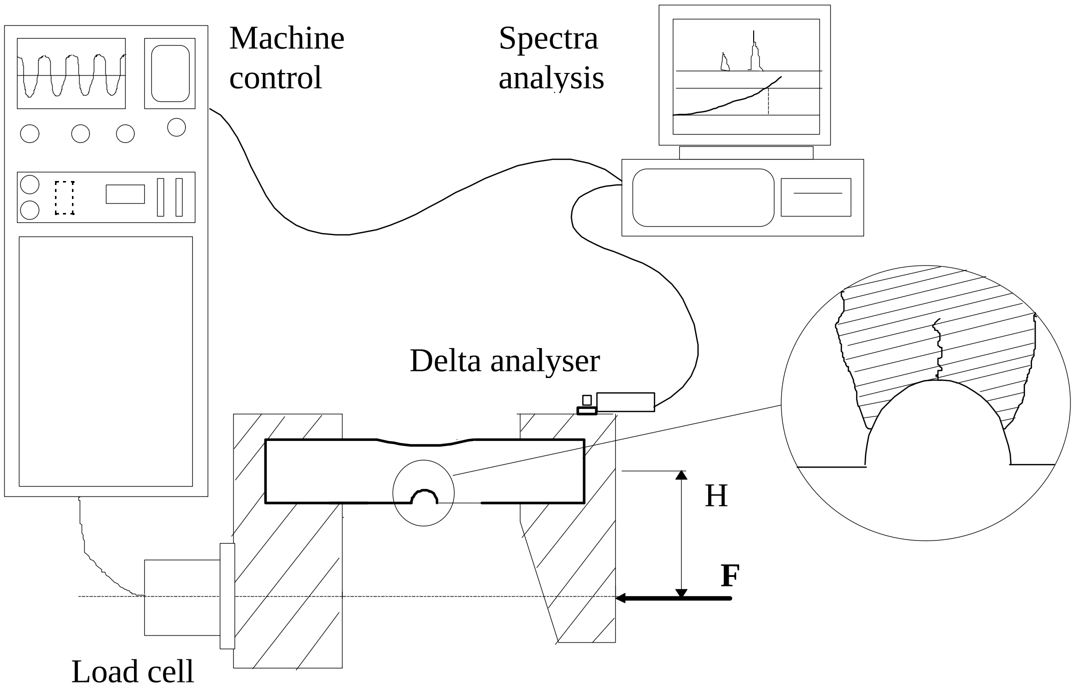

Figure 4: scheme of the fatigue test configuration and of the instruments for the detection of crack initiation

After detection, the crack can be measured during their growth until the final specimen rupture, that happens usually after a propagation phase during which a reduction of nearly 40% of the bearing section is reached. The instrument, adequately set at the beginning, can identify cracks having a surface extension lower than the 0,5-1 % of the total area.

RESULTS AND DISCUSSION

Fracture phenomenology

The tests, conducted at different loading levels and in presence of the two different notches, showed generally the initiation of a corner crack after a fraction of specimen life, ranging between 50 % and 75 % of the total fatigue life.
In the final part of the propagation, secondary cracks were detected. Because of the interaction between these cracks and the principal one, several of the last cracks measurements were not used in the analysis.
The graphite distribution in the perlitic matrix and the presence of the interdendritic brittle steadite showed to have an influence on the crack propagation introducing irregularities in the crack paths. In spite of that, these remained sufficiently regular and from the observation of the fractured surface it was possible to qualitatively confirm the elliptical shape of the crack front during a considerable fraction of the life expended in the propagation. In figure 5 a typical fractured surface is reported, in which the traces for a sequence of crack front positions are highlighted.

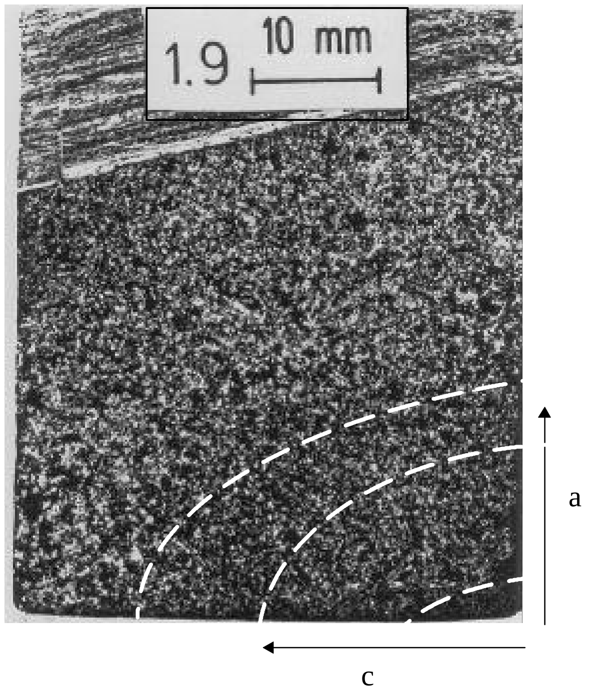

Figure 5: fracture surface for one of the specimens. In evidence the traces of a sequence of crack front positions.

During each test the length *(a)* of the crack emerging on the lateral surface and the length *(c)* emerging on the lower surface of the specimen were measured. As an example in figure 6 the evolution of lengths *a* and *c* in function of the number of cycles are reported for the same specimen as that in figure 5.

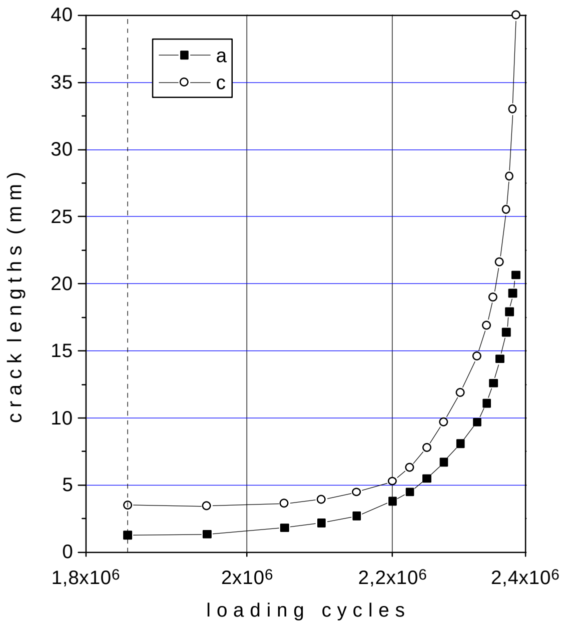

Figure 6: evolution of cracks lengths a and b as a function of loading cycles for one of the tested specimens

<u>*Analysis of results*</u>

Considering the shape of the initiated cracks, for the analysis of the experimental data, the use of the simplified solution proposed by Zheng (Zheng et alii 1996) appeared preferable. Such a solution consider to be valid a general weight function (Glinka et alii,1991) proposed for semi-elliptical crack and consents the calculation of two values of the S.I.F. in correspondence of the two surface points A and B indicated in Fig. 1.
In particular for the point A the W.F. can be expressed by the following equation:

$$h_A(x,a) = \frac{2}{\sqrt{2\pi(a-x)}} \cdot \sum_{i=0}^{3} M_{iA} \cdot \left(1-\frac{x}{a}\right)^{i/2} \tag{2}$$

whereas for the point B:

$$h_B(x,a) = \frac{2}{\sqrt{2\pi \cdot x}} \cdot \sum_{i=0}^{3} M_{iB} \cdot \left(\frac{x}{a}\right)^{i/2} \tag{3}$$

By developing the model in case of nominal stress distributions represented by power expansion up to the third degree, the following relationships can be written:

$$\frac{K_A}{\sigma_0 \cdot \sqrt{\pi a / Q}} = \frac{\sqrt{2Q}}{\pi} \cdot \sum_{i=0}^{3} c_i \cdot M_{iA} \tag{4}$$

$$\frac{K_B}{\sigma_0 \cdot \sqrt{\pi a / Q}} = \frac{\sqrt{Q}}{\pi} \cdot \sum_{i=0}^{3} d_i \cdot M_{iB} \tag{5}$$

In these equations the parameters M_{iA} ed M_{iB} can be determined as functions of a/t and a/c (in particular $M_{0A}=1$ and $M_{0B}=1$), whereas the coefficients c_i e d_i are expressed as functions of the ratio a/t. Finally the term Q can be written through the following equation:

$$Q = 1+1.464\cdot(a/c)^{1.65} \tag{6}$$

The limits for a/c and a/t in order to assess the validity of the proposed expressions are respectively to be set to $0,2\leq a/c\leq 1$ e per $0,1\leq a/t\leq 0,8$. It was verified that inside those ranges the maximum error is restricted to 1,5%.
In order to apply these solutions to the analysis of the experimental data it was finally necessary to develop a Finite Element linear elastic model, which gave the stress distribution on the specimen section. For the specimens characterised by a shape factor of α_{kb} = 1,10 the approximation with a linear stress distribution was considered acceptable (error less than 2%), whereas in the case of notches having α_{kb} = 1,75 it was necessary to adopt a cubic polynomial expression.

By introducing the stress distribution, it was possible to determine for each specimen the couples of values da/dN vs. ΔK_A and dc/dN vs. ΔK_C associated to every crack measurement. In figure 7 the calculated data are presented for two of the tested specimens characterised respectively by α_{kb} = 1,10 and α_{kb} = 1,75.
In the figures it can be observed that the fatigue crack growth rates (F.C.G.R) at point A and point B show approximately superposed trends. Also the differences between the values determined for α_{kb} = 1,10 and for α_{kb} = 1,75 result quite small.

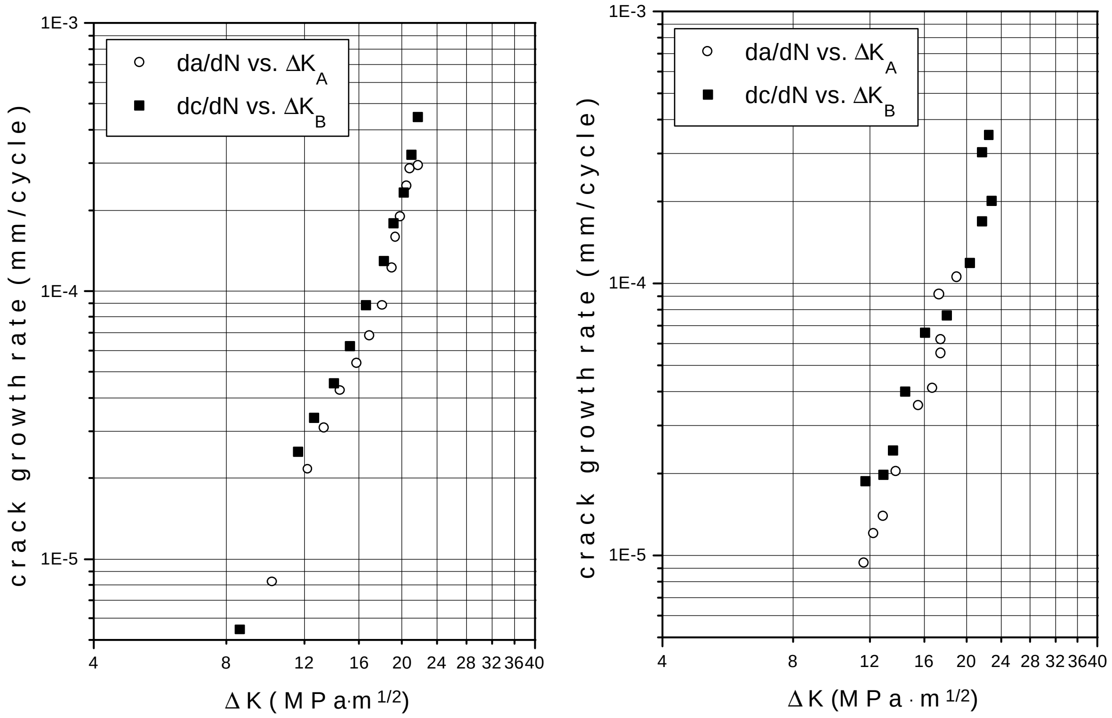

Figure 7: Fatigue crack growth rate vs. DK for two specimens having respectively shape factor a) a_{kb} = 1,1 and b) a_{kb} = 1,75.

The results obtained from the different tests realised at different loading levels were characterised by comparable F.C.G.Rs and S.I.Fs. It was then possible to plot the whole data together as reported in the diagrams of figure 8a and 8b and to perform an apparent linear fitting in order to determine the coefficients C and n for the Paris relationship. The obtained values for the two point A and B are reported in the following table:

Table II: coefficients of the Paris relationship

	C (*)	n	R (correlation coefficient)
Point A	$10^{-9,457}$	4,38	0,9842
Point B	$10^{-9,275}$	4,30	0,97574

(*) da/dN → mm/cycle; ΔK→ MPa·√m

Taking in account the uncertainties in the measurements and the simplification introduced in the calculations, the correlation factors for the regression lines can be considered substantially acceptable.
As expected the parameters show values approximately similar for the F.C.G.R. calculated at the two point of the crack front. The small difference could be done by the

irregularities of the crack paths that can have a stronger effect on the propagation at point A.
The values of the coefficients *n* appears quite high but comparable with data found in the literature (Motz et alii,1980, Peter et alii 1994, Tokaij et alii 1994, Clement et alii 1984) for materials having similar microstructural characteristics.

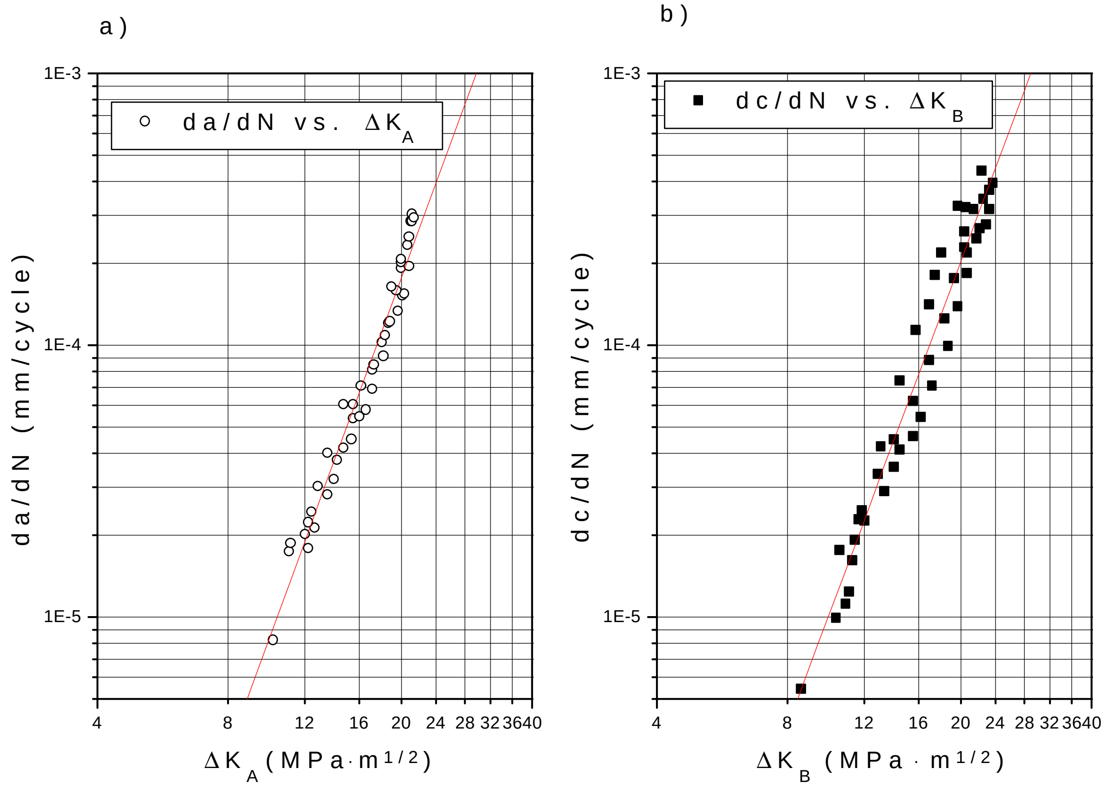

Figure 8: calculated values of fatigue crack growth rate vs. DK

CONCLUSION

By using opportune instruments, the initiation and the propagation of fatigue cracks were determined during fatigue tests on large dimensions specimen performed for the determination of the classical S-N curves.
The W.F. approach were adopted for the analysis of the experimental data, with the purpose of calculating the S.I.Fs in correspondence of particular points on the crack front.
By correlating the S.I.Fs with the corresponding fatigue crack growth rate, Paris relationships were determined for the studied material. The data can be considered comparable with data found in the literature on cast irons with similar characteristics.
The obtained results confirm the usefulness of the W.F. approach for the study of practical situations such as the determination of materials crack growth behaviour from non-standard specimens and tests.

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