On the Generation of a Loading History with Zero Mean for Structures Subjected to Multiaxial Variable Stresses

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ABSTRACT. *When a component is subjected to an excitation that has a nonzero mean, this mean value could have a noteworthy effect on the stresses and thus on the fatigue life of the component. In fatigue calculations, mean stress effect can be taken into consideration rather easily but experimental verification is troublesome since with traditional testing equipment like vibration shakers, any mean value of an acceleration excitation other than 1g in the direction of gravity is difficult to simulate. In this study, a method is proposed to create a modified input loading history with a zero mean which causes fatigue damage approximately equivalent to that created by input loading with a nonzero mean. For this purpose, a mathematical procedure is developed to apply three dimensional mean stress correction to the output stress power spectral density data. A modified input acceleration power spectral density is generated by means of transfer functions calculated via frequency response analysis. In the implementation of the developed method, a plate subjected to base excitation is considered. The base excitation leads to a fully three dimensional multiaxial state of stress in the plate. It is shown that the developed method is useful in creating a modified input acceleration data with a zero mean that can simulate damage for a selected point in the structure under consideration. Equivalent input loading obtained by means of the aforementioned method is convenient for experimental applications. Besides being a time saving method, it is suitable to be implemented for any type of loading.*

INTRODUCTION

Lifetime prediction assessment for components under random loading is an important concern in engineering. In fatigue calculation algorithms, a cycle counting method, commonly used ones are rainflow, peak, level crossing and range counting procedures according to [1], is implemented to identify loading cycle occurrences, amplitudes and mean values while a damage accumulation theory, mainly Palmgren–Miner rule, is employed to determine accumulated damage.

Mean value of the loading is an essential issue since it results in mean stress on the structure which causes a decrease in fatigue life. Several mean stress and strain correction procedures are introduced in the literature. The effectiveness of damage parameters to determine the mean stress effect on fatigue life of test specimens are evaluated considering the results of strain-controlled fatigue tests with and without mean strain and mean stress [2]. Gerber and Goodman formulations are among the most widely used mean stress correction procedures. Time domain fatigue life calculation techniques directly utilize mean stress correction methods to amplify the stress ranges to take the mean stress effect into account. In frequency domain fatigue life calculation approaches, when the power spectral density of the stress is determined, only the variable stress part is considered, however, the constant mean component is ignored [3].

In this study, the mean stress correction is accomplished on stress power spectral density to be able to include the loading's bias effect. Goodman correction technique is preferred to Gerber since it is reported in [4] that Gerber relationship generally gives higher fatigue lives compared to the experimental results. Dirlik method is chosen for application of vibration fatigue approach. According to [5], Dirlik approach leads to better results in comparison to the corresponding time domain and frequency response methods. The main goal of this study is to create an input power spectral density of the loading with a zero mean which causes fatigue damage approximately equivalent to that created by input loading with a nonzero mean. This will enable to perform fatigue tests conservatively. Therefore, a mathematical approach is developed to modify the output von Mises stress power spectral density to take the multiaxial stresses into account at the point of consideration and to provide the needed input loading power spectral density. The corrected input with zero mean value enables experimental verification while the mean value of the input cannot be applied in standard vibration tests. As a case study, an aluminum plate fixed and excited at one end is used for the implementation which is shown schematically in Figure 1.

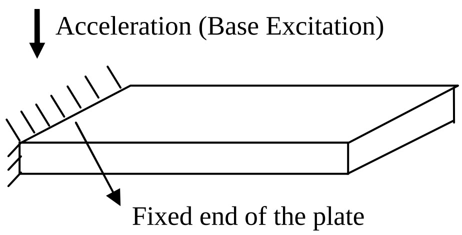

Figure 1. Plate Subjected to Base Excitation

THEORY

In this section, the mathematical approach proposed to calculate equivalent zero mean loading for a structure subjected to random loading with nonzero mean is described. The developed method is based on the frequency domain fatigue calculation techniques. Vibration-Fatigue calculations and its mathematical background is not explained in detail since it is out of scope of this work, but the interested readers are referred to the recent publications by Sherratt et al. [6] for preliminary information on the current methods of this concept and Sherratt [7] for a standard vibration-fatigue approach.

In vibration fatigue calculations, the frequency content of the stresses is accounted for by a probability density function of rainflow stress ranges, *p(S)*. The fatigue calculations with this approach are performed by utilizing (1) given below in which *T* is the fatigue life in seconds and *E[D]* is expected value of damage that is set to unity.

$$E[D] = \sum_i \frac{n_i(S)}{N(S_i)} = E[P]\frac{T}{C}\int_0^{\infty} S^b p(S) dS \tag{1}$$

In (1), *n(S)* is the number of cycles at the particular stress level *S* , *N(S)* is the total number of cycles at the stress level *S* that cause failure according to Woehler curve, *E[P]* is the expected number of peaks per second and *C* and *b* stand for material constant and Basquin exponent respectively.

To calculate the probability density function of rainflow stress ranges, several different empirical solutions are proposed by researchers like Wirsching et al (1990), Chaudhury and Dover (1885), Tunna (1986), Hancock Kam and Dover (1988), and Kam and Dover (1988) [8]. A survey by Bishop et al. [9] can be referred to for detailed investigation of some of the methods developed. In the present study, the broadband formulation (2) developed by Dirlik [10], which is accepted widely, is employed to assess the stress range probability density function.

$$p(S_i) = \frac{\frac{D_1}{Q} e^{-\frac{Z_i}{Q}} + \frac{D_2 Z_i}{R^2} e^{-\frac{Z_i^2}{2R^2}} + D_3 Z_i e^{-\frac{Z_i^2}{2}}}{2\sqrt{m_0}} \tag{2}$$

In (2), m_0 is the 0th order spectral moment of the output PSD; whereas the other parameters, *Zi*, *D1*, *D2*, *D3*, *R* and *Q* are formulated as stated in detail in [11].

As it is clear from the formulation carried out to calculate (2), *p(S)* is calculated through utilization of the stress amplitude, *S*, and spectral moments up to forth degree, m_0, m_1, m_2 and m_4. These spectral moments are calculated from the response stress PSD, $G_r(f)$, which is given in (3) below, where *H(f)* and *H*(f)* stand for the transfer function between $G_r(f)$ and $G_i(f)$, and its complex conjugate respectively.

$$G_r(f) = H(f) \times H^*(f) \times G_i(f) \tag{3}$$

Transfer function can be obtained via frequency response analysis of the structure where for each frequency a different transfer function matrix is calculated.

To compute a zero mean input acceleration PSD, which is mentioned as the modified acceleration input PSD ($G_{im}(f)$) here, first of all, mean stress correction is performed on the von Mises stress output PSD, $G_r(f)$. Goodman's mean stress correction shown in Equation (4) below is used to obtain the modified output stress PSD, $G_{rm}(f)$.

$$S_{ar} = \frac{S_a}{1 - \frac{S_m}{S_u}} \tag{4}$$

Note that Goodman's formula is widely used for mean stress correction for stress amplitude modification [12], but in this study it is utilized to correct PSD stress output data. If the PSD is split into equal strips, the area of each strip can be utilized to obtain an equivalent sine wave. The amplitude of each equivalent sine wave is equal to the square root of the area times $\sqrt{2}$. This is because of the fact that the root mean square of a sine wave is equal to its amplitude divided by $\sqrt{2}$, and the root mean square of each strip in the PSD is equal to the square root of its area [13]. In this study, the stress PSD value, $G_r(f)$, is multiplied by the corresponding frequency resolution and the square root of it is taken and multiplied with $\sqrt{2}$ to obtain the alternating stress amplitude, S_a, for each frequency. Then, mean stress correction is performed with (4) to calculate the modified alternating stress amplitude, S_{ar}. Afterwards, the calculations are carried out in the reverse direction to reach corrected output stress PSD, $G_{rm}(f)$.

Theoretically, there should be an input acceleration PSD that will result in the modified output stress PSD, $G_{rm}(f)$, when applied to the structure since the transfer function of the structure remains the same regardless of input type. This physical relation can be represented by the following equation which is rewritten from (3) for a different input-output couple.

$$G_{rm}(f) = H(f) \times H^*(f) \times G_{im}(f) \tag{5}$$

To obtain $G_{im}(f)$, (5) is re-ordered which results in (6):

$$G_{im}(f) = \left[G_{rm}(f)^{-1} x G_r(f) x G_i(f)^{-1} \right]^{-1} \tag{6}$$

The variables in equation (6) are vectors, which in turn makes regular matrix inversion impossible. Therefore, the inverse calculations in equation (6) are carried out by using pseudo-inverse. A corrected input loading with zero mean but modified alternating stress, which creates fatigue damage equivalent to the damage caused by the processed loading with nonzero mean, is extracted by means of this technique. It should be noted that, this approach is focusing to find a modified loading that simulates damage for a selected point on the structure, which in turn will be acceptable in the vicinity of that point.

CASE STUDY

In this section, the proposed correction method is demonstrated on an application analysed according to the method presented in the Theory section.

The plate shown in Figure 1 is excited from its cantilevered base with stationary, Gaussian random 10 second acceleration time history with a mean value other than zero as shown in Figure 2. The input loading consists of harmonics that possess different amplitudes with broadband frequency range, up to 2000 Hz, involving the first 20 of the natural frequencies of the specimen. The deformation patterns include bending and torsional mode shapes that induce a multiaxial stress state in the structure.

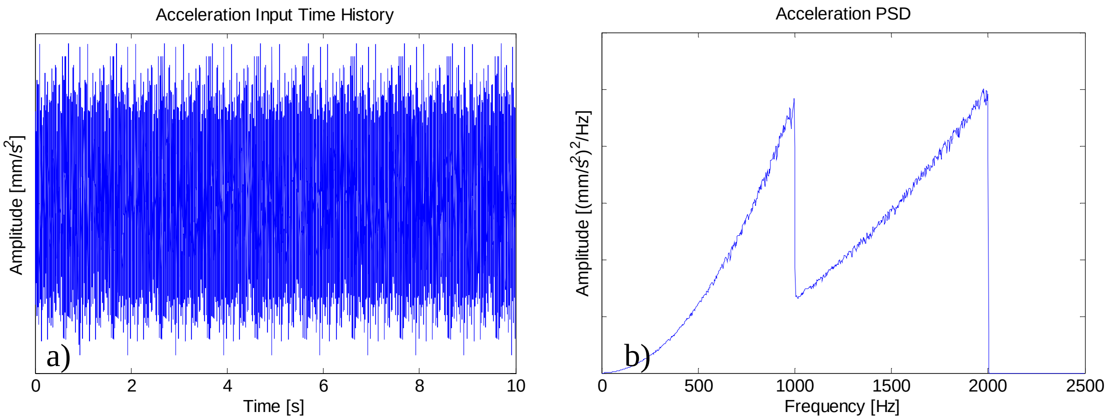

Figure 2. Input Acceleration Loading: a) Time History; b) PSD

MSC Patran/Nastran and MSC Fatigue software tools are used for finite element modeling and analysis. Frequency response analysis is carried out to obtain transfer functions between input acceleration and output von Mises stresses needed for fatigue calculations. The first analysis is conducted using Dirlik's method for determination of probability density function of rainflow stress ranges stated in [7] without mean stress correction. Then, the second analysis is conducted with Goodman mean stress correction proposed by MSC Fatigue. Afterwards, mean stress correction technique mentioned in Section 2 is performed using output von Mises stress PSD taken from the first analysis. Von Mises stress caused by mean acceleration of the input loading at the specific point whose life is the least is used for the mean stress correction. Specified point is actually close to the cantilever support for the given case study where fatigue failure is expected to occur. Mean von Mises stress at the desired vicinity is found via linear static analysis with an inertial loading of magnitude equal to the mean value of the input acceleration loading shown in Figure 2. After calculating modified output stress PSD as described in Section 2, corrected input loading PSD with amplified amplitudes (Figure 3) is determined via (6). Finally, fatigue analysis is performed using modified input loading PSD for comparison.

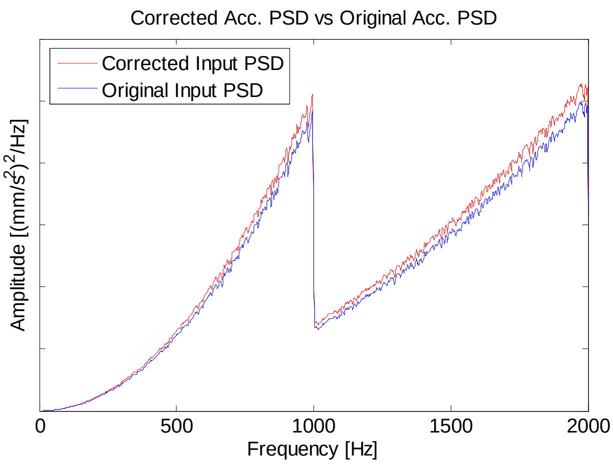

Figure 3. Modified Input Loading and Original Input Loading

Finite element analysis results regarding fatigue life are presented in Figure 4 and compared for the points in the vicinity of the point of failure (nodes shown in Figure 5) in Table 1.

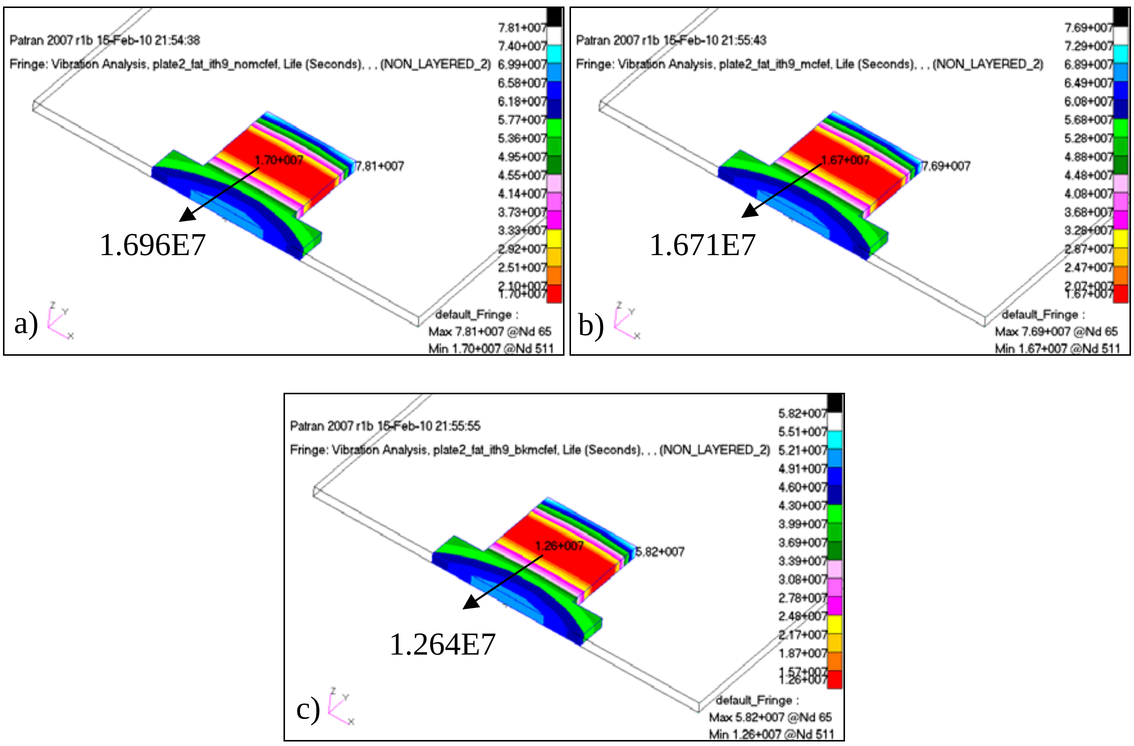

Figure 4. Fatigue Lives for the Plate (in seconds): a) without Mean Stress Correction; b) with Goodman Mean Stress Correction; c) with Proposed Mean Stress Correction

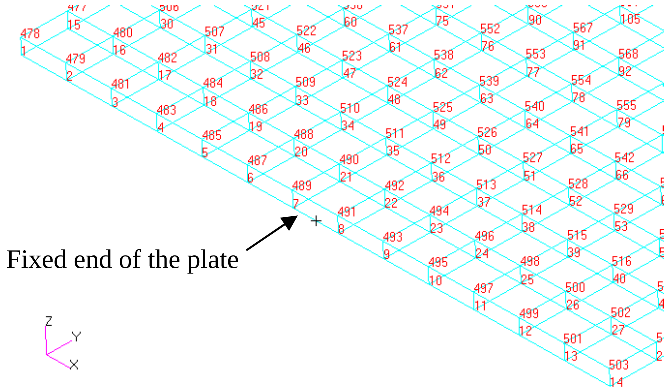

Figure 5. Finite Element Model of Displaying the Plate-Node Numbers

Table 1. Comparisons of Fatigue Lives

Life (s)	Analysis 1 (w/o mean correction)	Analysis 2 (w/ Goodman mean correction)	Analysis 3 (w proposed mean correction)
Node 511	1.696E7	1.671E7	1.264E7
Node 35	1.698E7	1.672E7	1.265E7
Node 512	1.7E7	1.675E7	1.267E7
Node 36	1.7E7	1.674E7	1.266E7
Node 510	1.716E7	1.69E7	1.278E7
Node 34	1.721E7	1.695E7	1.282E7

CONCLUSIONS

This paper presents a new mathematical approach that applies Goodman mean stress correction to power spectral density of von Mises stress to obtain an equivalent input power spectral density of the loading with a zero mean which leads to damage approximately equivalent to that caused by an input with a nonzero mean.

A case study is presented to show the implementation of the proposed method. A plate exposed to base excitation is examined and fatigue life is calculated for the excitation shown in Figure 2 without mean stress correction, by considering Goodman mean stress correction (in MSC Fatigue) and also by considering the loading generated by means of the proposed technique. Developed method gives a more conservative fatigue life compared to that obtained by the Goodman mean stress correction. Therefore, the input loading obtained by means of the aforementioned method is concluded to be safe, practical and convenient for experimental applications.

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