Numerical Modelling of Fatigue Damage in Gears Tooth Root

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ABSTRACT. *Structural components, such as gears, can be subjected to short–time overloads due to dynamically acting loads on gear teeth during torque transmission and because of the other excitations in gearbox that cause additional loads on teeth and leads to low–cycle fatigue damage in material. Although gears are, like most engineering structures, influenced by complex multiaxial stresses that arise from loading and geometry, laboratory fatigue testing very often uses simple specimens that are subjected to uniaxial states of stress. The data recorded through these tests are used for material behaviour estimation of multiaxial non-proportional stress-strain states within the range of elastic-plastic deformations. In this paper the material behaviour in low–cycle fatigue regime is described by complex material model that combines isotropic and kinematic hardening and/or softening with mechanics of material damage to simulate elastic-plastic material behaviour and damage nucleation and accumulation. Chosen material model is implemented into finite element code to enable numerical modelling of material response in gears tooth root, considering stress and strain variation during load cycles as well as damage occurrence and development. Numerical modelling of fatigue damage can provide better fatigue life estimation of gears, focusing on the crack initiation period.*

INTRODUCTION

Gear teeth damage is classified in two main types [1]. Pitting damage is consequence of particles breakage out of teeth flanks. The pits that appear on flanks can be tolerated to some extent, depending mostly on the field of application. The second type of teeth damage is tooth root breakage that immediately ends the service life of a transmission. Tooth root fatigue damage is considered within this paper.

In order to produce optimal gear design with chosen material, numerous running tests on gear samples must be conducted which is expensive, time-consuming and offers very limited possibility for results extrapolation to different gear designs. It is also possible to perform endurance tests on notched and un-notched test circular samples, with the bending strength calculation with included influence factors.

In the tests only the occurrence of final failure in the gear tooth root is considered, but not the damage nucleation and accumulation that results in crack initiation. Furthermore, these tests are usually executed in high-cycle fatigue regime, during which achieved bending stresses in tooth root do not exceed yield stress of the material.

In operating service life the gear teeth are dynamically loaded when transmitting the torque, because the load is periodically applied on teeth during gear cycles. The loads vary throughout the working process, from starting through operation at or near critical speed to the operation in working conditions [2]. Besides these forces, there are numerous other excitations in gearbox induced by the variations in geometry due to manufacturing tolerances and errors, variations in alignment, lubrication, teeth stiffness nonlinearity, stiffness fluctuation during the mesh, teeth clearance, transmission errors, assembly inaccuracies etc., that cause additional loads on teeth [3]. These load alternations in its magnitude and frequency are manifested as short–time overloads and can cause plastic strains and low-cycle fatigue in material [4]. Generally, due to overloads, the damage is initiated at defects in microstructure or because of the residual microstresses that are developed in material during production, assembly or maintenance. Despite the fact that the load spectrum can not be precisely predicted which leads to uncertainty of the life estimation, advanced knowledge of material characteristics and its behaviour during operation can provide great improvement in gears design.

MATERIAL BEHAVIOUR MODELLING

In order to describe phenomena that appear in structural components material during their service life, it is necessary to understand the physical mechanisms within material and choose or develop material model that can be used for modelling of material behaviour in given conditions. In order to analyze material behaviour in this paper a combined elastic–plastic constitutive model is used that takes into account isotropic and kinematic hardening/softening of the material and mean stress relaxation [5]. The damage nucleation and accumulation is modelled based on damage mechanics theories [6]. The material behaviour described by this model is time- and rate-independent, in isothermal conditions and it considers small strains. Considered material model is used to describe material behaviour in low–cycle fatigue regime.

Multiaxial plasticity criteria is described as von Mises criterion [5] which states that in case of three-dimensional principal state of stress the yielding occurs when equivalent stress in the von Mises sense exceeds yield stress which defines elastic domain in uniaxial stress space:

$$\tfrac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]=\sigma_y\,. \quad (1)$$

Based on continuum damage mechanics approach, when considering damaged material, the strain equivalence principle [6] can be used. It assumes that every strain constitutive equation for a damaged material can be expressed in the same manner as the equation of undamaged one with the usage of effective stress:

$$\tilde{\sigma} = \frac{\sigma}{1 - D}. \qquad (2)$$

Constitutive equations

Since elastic–plastic behaviour of material is presumed, total strain is given by the sum of elastic and plastic part:

$$\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{p}}, \qquad (3)$$

which are described by separated constitutive relations. The Hooke's law describes the elastic stress strain relationship:

$$\varepsilon^{\mathrm{e}} = \frac{\sigma}{(1 - D)E}, \qquad (4)$$

while the plasticity is considered when the plastic yield criterion is reached, by taking into account both kinematic and isotropic hardening, induced by plastic flow [6]:

$$f = \left| \frac{\sigma}{1 - D} - X \right| - R - \sigma_{\mathrm{y}} = 0. \qquad (5)$$

It is appropriate to express von Mises yield surface in deviatoric stress space, considering that hydrostatic stress has no effect on the plastic deformation [7]:

$$f = \sqrt{\frac{3}{2}\left(S_{\mathrm{ij}} - X_{\mathrm{ij}}\right)\left(S_{\mathrm{ij}} - X_{\mathrm{ij}}\right)} - R - \sigma_{\mathrm{y}} = 0, \qquad (6)$$

where S_{ij} is deviatoric stress tensor, X_{ij} is back stress tensor that defines the centre of the yield surface, R is the isotropic hardening variable and σ_{y} is initial yield stress. The derivative of the yield surface, $f = 0$, gives the direction of the plastic strain increment. The plastic flow follows the normality rule:

$$\mathrm{d}\varepsilon^{\mathrm{p}} = \mathrm{d}\lambda \frac{\mathrm{d}f}{\mathrm{d}\sigma}, \qquad (7)$$

where plastic multiplier $\mathrm{d}\lambda$ is derived from the consistency condition $\mathrm{d}f = 0$. After the elastic limit is reached, the yield surface evolution through cycles can be described by the law of isotropic hardening [7]:

$$\mathrm{d}R = b(R_{\infty} - R)\mathrm{d}p, \qquad (8)$$

where

$$\mathrm{d}p = \sqrt{\frac{2}{3}\mathrm{d}\varepsilon_{\mathrm{ij}}^{\mathrm{p}}\mathrm{d}\varepsilon_{\mathrm{ij}}^{\mathrm{p}}} \qquad (9)$$

is accumulated plastic strain, R_∞ is the boundary of isotropic hardening (or softening) and b is parameter used to describe evolution of a yield surface over cycles. The integration of Eq 8 leads to exponential expression to describe the maximum stresses variations during cycles, as a function of accumulated plastic strain:

$$\frac{\sigma_{\max} - \sigma_{\max 0}}{\sigma_{\max s} - \sigma_{\max 0}} = 1 - \exp(-2bp). \tag{10}$$

Besides the change in the yield surface, described by the isotropic hardening rule, in order to model material behaviour in terms of modelling translation of the yield surface that covers the Bauschinger effect, the non-linear kinematic hardening model is used. The material behaviour is described by Armostrong–Frederick model by expression [7,8,9]:

$$\mathrm{d}X_{\mathrm{ij}} = \frac{2}{3} C\mathrm{d}\varepsilon_{\mathrm{ij}}^{\mathrm{p}} - \gamma X_{\mathrm{ij}}\mathrm{d}p, \tag{11}$$

where C and γ are the characteristic coefficients of the material. They can be identified from uniaxial tests. The evolution of back-stress is exponential and it saturates at value of $X_\infty = C/\gamma$. Although non-linear kinematic hardening model was proven advanced, related to Prager's linear model, further enhancement was derived in a manner of Chaboche's proposal of a decomposed nonlinear kinematic hardening rule in the form:

$$\mathrm{d}X_{\mathrm{ij}} = \sum_{\mathrm{i}=1}^{n} \mathrm{d}X_{\mathrm{ij}}^{(\mathrm{n})}. \tag{12}$$

Integration of Eq 11, with applying a three-decomposition rule (Eq 12) leads to exponential expression:

$$\frac{\Delta\sigma}{2} = R_\infty + X_\infty^{(1)} \tanh\left(\gamma^{(1)} \frac{\Delta\varepsilon_{\mathrm{p}}}{2}\right) + X_\infty^{(2)} \tanh\left(\gamma^{(2)} \frac{\Delta\varepsilon_{\mathrm{p}}}{2}\right) + X_\infty^{(3)} \tanh\left(\gamma^{(3)} \frac{\Delta\varepsilon_{\mathrm{p}}}{2}\right), \tag{13}$$

that is suitable for identification of kinematic hardening material parameters. The three-decomposition rule is used to better control stress-strain relationship in stabilized cycle, but it is not robust enough to describe ratcheting response of the material proprietly. Therefore in chosen model for the kinematic hardening with mean stress relaxation description Ohno-Wang rule was applied [7,8,9,10]:

$$\mathrm{d}X_{\mathrm{ij}}^{(\mathrm{n})} = \frac{2}{3}\gamma^{(\mathrm{n})} X_\infty^{(\mathrm{n})} (1 - D)\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}} - \left(\frac{X_{\mathrm{eq}}^{(\mathrm{n})}}{X_\infty^{(\mathrm{n})}}\right)^{m_{\mathrm{n}}} X_{\mathrm{ij}}^{(\mathrm{n})}\gamma^{(\mathrm{n})}\mathrm{d}p, \tag{14}$$

where

$$X_{\mathrm{eq}}^{(\mathrm{n})} = \sqrt{\frac{3}{2} X_{\mathrm{ij}}^{(\mathrm{n})} X_{\mathrm{ij}}^{(\mathrm{n})}}. \tag{15}$$

Ohno-Wang parameters are identified in common practice [7,8,9,11,12,13] by adjusting them when controlling a recorded data in uniaxial strain controlled tests with mean stress unequal to zero.

Fatigue damage modelling

Low-cycle fatigue damage within material is accumulating with the plastic strain accumulation after the damage threshold is reached. Damage threshold represents the limit of accumulated plastic strain that is needed for damage nucleation. The damage measurement within this paper is conducted as Young's modulus variation control through cycles [14]

$$D = 1 - \frac{\widetilde{E}^{+}}{E}, \tag{16}$$

while damage threshold p_D, critical value of damage D_c and energy strength of damage S are calculated from the damage – accumulated strain curves recorded through cycles. The crack in the material can be considered initiated when the critical value of damage D_c is reached. It is characterized by its progressive increase that follows with the further plastic strain accumulation. The energy strength of damage S is material parameter that describes linear damage evolution through cycles:

$$S = \frac{\sigma^2}{2E(1-D)^2 \frac{\mathrm{d}D}{\mathrm{d}p}}. \tag{17}$$

TOOTH ROOT DAMAGE MODELLING

Gears, like most engineering structures, are influenced by complex multiaxial stresses that arise from loading and geometry. The gears tooth root is subjected to combined bending, shear and compressive stresses. The fatigue crack in the tensile loaded side of the tooth is likely to originate on the tooth root surface at the location where maximum principal stress is expected. The estimation of fatigue life, considering crack initiation in tooth root in low-cycle fatigue regime, is based on knowledge of material behaviour of test specimens.

In order to investigate low-cycle fatigue that can occur in gears tooth root due to short-time overloads, the material parameters have been identified to model material behaviour of the steel 42CrMo4 in normalised state with hardness of 296 HV and tempered state with hardness of 420 HV and 546 HV. Detailed response of the material during cycle loading was recorded during own experiments on circular shaped un-notched specimens. The strain–controlled fatigue testing was conducted, following standard procedure [15] and it serves as a base for modelling of material behaviour. The procedure for determining the material parameters of material is based on genetic algorithm [16,17].

The material fatigue testing on standard test specimens is performed to estimate damage initiation and accumulation in gears tooth root material, subjected to similar operating conditions. With the presumption that material response in tooth root is similar to material response in the test specimen and the presumption that both have the same stress history, the defined material model can be applied to damage and stress calculation in the gear tooth root [18] (Fig. 1).

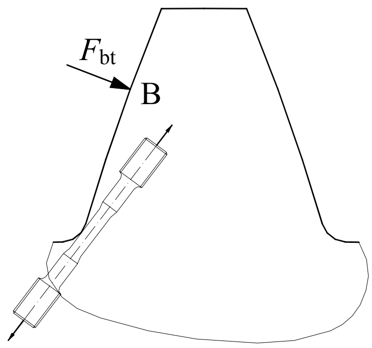

Figure 1. Material model application on gears

The analysis is performed on the gear tooth with geometric characteristics: z = 124; m_n = 6 mm; α_n = 20°; h^*_{a0}=1,25; ρ^*_{a0}=0,25; c^*=0,25; x = 0; b = 1 mm; d_a = 756 mm. After the gear tooth design [19], the numerical damage model is incorporated into a computer programme based on the finite element method with the goal to correlate with experimental data. The gear tooth is discretized by finite elements that have integrated materials' nonlinearities, modelled to follow the material behaviour defined by identified material parameters. Up to the yield stress, the finite element's material exhibits a linear stress – strain relationship, while the stresses beyond it will cause plastic behaviour of material, following the mechanical principle presented earlier. The nonlinear stress – strain relationship of material causes a structure's stiffness to change at different load levels during numerical analysis.

In the calculation procedure the whole nominal transverse load in plane of action F_{bt} is applied on the B point on path of contact, where double tooth contact (two teeth pairs simultaneously in mesh) is changing to single (only one tooth pair is carrying the whole load), according to [1]. The force is incrementally applied dynamically.

SIMULATIONS

Tooth root damage is initiating and accumulating during overload cycles at the tooth root surface at the location of plastic strain occurrence. If the stress values are very high in wider area, the plastic strain can be expected in several finite elements on the tooth root. The damage evolution during cycles for tested material with hardness 420 HV is presented in Fig. 2 for several finite elements on tooth root that deform plastically. The evolution of damage through cycles varies considering its location on the tooth root surface. It does not exhibit linear correlation to number of cycles because of the variation of stresses and strains during cycles. Although the force on the tooth flank is controlled, the plastic deformation of the tooth during cycles causes uneven strains in the tooth root. Besides, plastic deformation that occurs under the toot root fillet results in occurrence of residual compressive stresses on the surface [20].

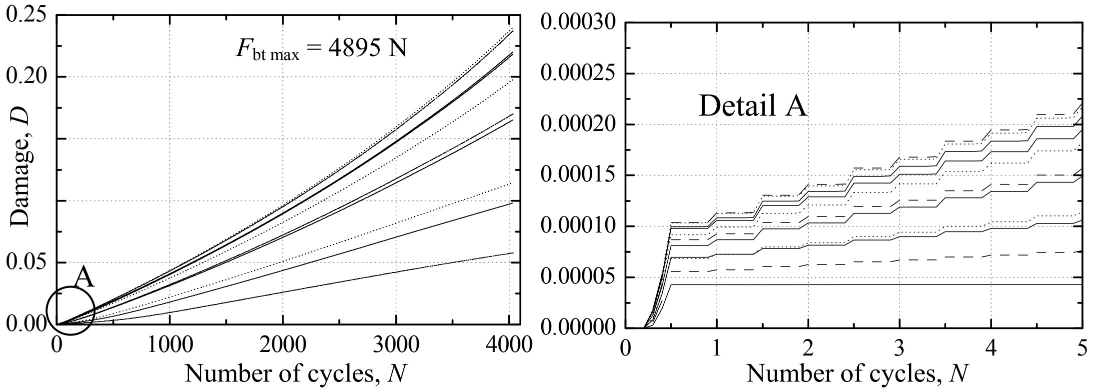

Figure 2. Tooth root damage accumulation, 42 CrMo 4 – 420 HV

Maximum tooth root damage accumulation on the tooth root surface for different hardness levels of material is shown in Fig. 3. Although the value of critical damage decreases with the material hardness increase, it is reached with the higher number of cycles for each of tested materials. The location of critical damage on the tooth root surface also varies with material hardness value.

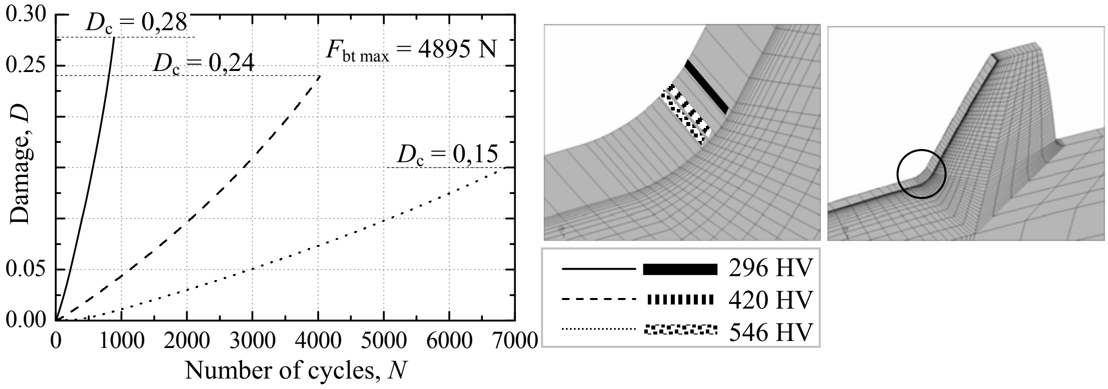

Figure 3. Maximum tooth root damage accumulation, 42 CrMo 4

CONCLUSION

Material behaviour modelling, based on uniaxial tests on circular specimens, can serve as a base to provide effective low–cycle fatigue damage modelling in gears toot root. The model takes into account gears geometrical and material characteristics, considering kinematic and isotropic hardening/softening and also damage initiation and nucleation. Usage of proposed model could easily provide optimization of gear design without necessity of time-consuming and cost-effective tests on numerous gear models.

Presented simulations show damage initiation and accumulation on the tooth root surface and also it's evolution during load cycles. Further simulations with different load values applied on tooth can provide good assessment of fatigue strength at a specified number of loading cycles in low–cycle fatigue regime, considering crack initiation in the tooth root.

This investigation serves as a concept and a motivation for further investigations to be made to utilize presented favourable characteristics of modelling method for fatigue strength estimation of gears. Different material models could be used to model needed fatigue behaviour for numerous gear designs.

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