Multiaxial Fatigue Criteria Applied to Medium Speed Diesel Engines

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ABSTRACT. *Due to the complexity both in the geometry and in the loading conditions, a multiaxial stress state will arise in most engine parts. The loads on a medium speed diesel engine are dynamic with billions of cycles. However, the loads are usually quite stable and many components are therefore sized for a nominally infinite life. Some years ago it was decided to develop a better understanding of multiaxial fatigue and to develop appropriate design tools. After an examination of different alternatives, the critical plane criteria according to Findley and Dang Van were chosen as a base for this development work, although, especially the Findley criterion requires much computing. Care had therefore to be taken to create efficient algorithms. Some experience has now been gained from the use of these criteria. The Findley criterion has proved to be quite accurate for most practical purposes. The estimates obtained by it agree well with both fatigue test results and with the experience from the observations of failures in the field. However, some difficult problems in its application remain. Among these problems are, e.g., how this criterion should be applied in situations with much yielding as in thread roots. Also the transformation to an equivalent uniaxial stress needed for the Palmgren-Miner type cumulative damage analysis is problematic in some special loading conditions.*

INTRODUCTION

Figure 1 shows a cross section of a big medium speed diesel engine. The complicated geometry of many of the engine parts is clearly visible. This is true, especially, for the cast parts as the cylinder head, engine block and liner. Forged parts, as the crankshaft and connecting rod, have also an intricate shape with sharp notches. Already this complexity in geometric shape will give birth to a multiaxial stress state in many critical points. The many different load types acting in the engine will further strengthen this multiaxial effect. Typical loads on the crank mechanism are the firing pressure and the inertia forces. In addition to the dynamic stresses caused by the firing pressure, temperature stresses arise in the cylinder head and liner every time the engine is started. This produces a highly multiaxial stress state in these components. Moreover, these stresses have variable amplitudes, and a situation with cumulative damage is initiated.

Because the operational life of this kind of engines should be at least 150 000 hours, it means that the number of high cycles can be counted in billions. In marine applications where the engine is used for main propulsion, even the high cycle stresses caused by the firing pressure and inertia forces can vary with the required power output. The number of starts and stops can be tens of thousands. As a contrast, in power plant operation the power output is mostly high and nearly constant and the number of starts and stops is considerably lower.

Usually the acting stresses are proportional. However, in some cases they can be nonproportional. This is true, especially, for vibrating engine parts. The diesel engine generates a multitude of exciting frequencies which can cause resonance problems. Moreover, impulsive or near to impulsive excitations are also present causing transient decaying vibrations in some parts. The resulting load in, e.g., a crankshaft or a camshaft can therefore partly consist of superposed vibrations with many different out-of-phase frequencies.

Less than a decade ago the so-called signed von Mises criterion was usually applied in the evaluation of multiaxial fatigue. In some cases the principal stresses were used, even neglecting some possible rotation of the principal axes. The accuracy of both these pseudo-criteria will vary a lot depending on the situation. Moreover, these criteria have special points where the result is not unique. In the signed von Mises criterion the sign of the mean stress is chosen to be the same as the sign of the trace of the von Mises mean stress matrix. Sometimes the trace of the matrix is close to zero and even a small shift of some normal stress component can cause the mean stress to jump between a big negative or positive value. To apply these criteria to nonproportional load cases seems therefore even still more risky. The need to find accurate, testable and reliable multiaxial fatigue criteria for the sizing of the engine parts is therefore obvious.

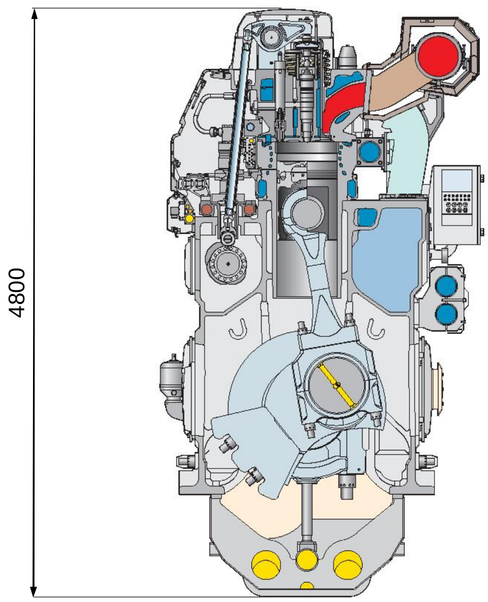


Figure 1. Cross section of a medium speed diesel engine.

In the beginning of the 2000's, a study of different multiaxial fatigue criteria began at the authors' company. Because of the high number of loading cycles in an engine, the focus was put on examining critical plane criteria as the Findley, Dang Van, Papadopoulos and McDiarmid criteria [1, 2 and 3]. It was thought that these criteria were suited for a fatigue analysis aiming at a nominally infinite life. Below a summary of these investigations, a short description of the program development work as well as results and conclusions will be given.

CHOICE OF CRITERIA

There exists a huge number of different suggested multiaxial fatigue criteria. The authors always give good arguments in favour of their favourite, and an objective choice is difficult. This investigation was limited to the four critical plane criteria mentioned above. The critical plane means a plane in the stress space where the combined effect of the shear stress "amplitude" $\Delta\tau/2$ or τ_a and a fraction of the normal stress σ_n has its maximum. This combined effect is called damage and it should be below the allowed fatigue limit in shear. To be able to compare these criteria, a first crude computer program was created which conducted both proportional and nonproportional fatigue analyses. Although a lot of fatigue tests had been done, they were primarily uniaxial tests on plain or notched specimens and very little real multiaxial test data was at hand.

A good multiaxial criterion should also provide accurate results for uniaxial stress cases. To begin with, the four investigated criteria were therefore applied on the results of some uniaxial fatigue tests. In all four criteria, except the McDiarmid, there are two constants that have to be determined. The outcome of uniaxial fatigue tests at two different stress ratios R, e.g., $\sigma_{aR=-1}$ in fully reversed tension-compression and $\sigma_{aR=0}$ for a stress fluctuating from zero to max, can be used to determine these constants. The investigated criteria and the corresponding equations to determine the constants are summarized in Table 1.

Table 1. Examined criteria.

Criterion	Damage D	Constant for normal σ_n or hydrostatic stress σ_h	Fatigue limit in shear ($f, \tau_{af}, \gamma_\infty$ and t)
Findley	$\left(\frac{\Delta\tau}{2}+k\sigma_n\right)_{\max} \le f$	$\frac{k+\sqrt{1+k^2}}{2k+\sqrt{1+4k^2}}=\frac{\sigma_{aR=0}}{\sigma_{aR=-1}}$	$f=\frac{\sigma_{aR=-1}}{2}\left(k+\sqrt{1+k^2}\right)$
Dang Van	$\tau_a(t)+a\sigma_h(t)\le\tau_{af}$	$a=\frac{3(\sigma_{aR=-1}-\sigma_{aR=0})}{2(2\sigma_{aR=0}-\sigma_{aR=-1})}$	$\tau_{af}=\frac{\sigma_{aR=-1}\cdot\sigma_{aR=0}}{2(2\sigma_{aR=0}-\sigma_{aR=-1})}$
Papadopoulos	$\tau_{a,\max}+\alpha_\infty\sigma_{h,\max}\le\gamma_\infty$	$\alpha_\infty=\frac{3(\sigma_{aR=-1}-\sigma_{aR=0})}{2(2\sigma_{aR=0}-\sigma_{aR=-1})}$	$\gamma_\infty=\frac{\sigma_{aR=-1}\cdot\sigma_{aR=0}}{2(2\sigma_{aR=0}-\sigma_{aR=-1})}$
McDiarmid	$\frac{\Delta\tau_{\max}}{2}+\frac{t}{2R_m}\sigma_{n,\max}\le t$	a) $t=\frac{\sigma_{aR=-1}}{2[1-\sigma_{aR=-1}/(4R_m)]}$ or b) $t=\frac{\sigma_{aR=0}}{2[1-\sigma_{aR=0}/(2R_m)]}$	

A tested Haigh diagram for quenched and tempered steel is shown in Fig. 2. If the tested uniaxial median fatigue limit 509.6 MPa at the stress ratio R = -1 is used as load on the reference specimen, then the calculated safety factor should be close to one for a good multiaxial criterion. In fact, due to the small notch, a peripheral stress of 15.9 MPa is also generated at the critical point. The calculated "radial" safety factors defined as the ratio of the shear fatigue limit and the damage are shown in Table 2 for the four criteria in Table 1. The differences in these safety factors are of course small in this case. However, it will be shown later with uniaxial tests on sharply notched test specimens that these differences can become significant.

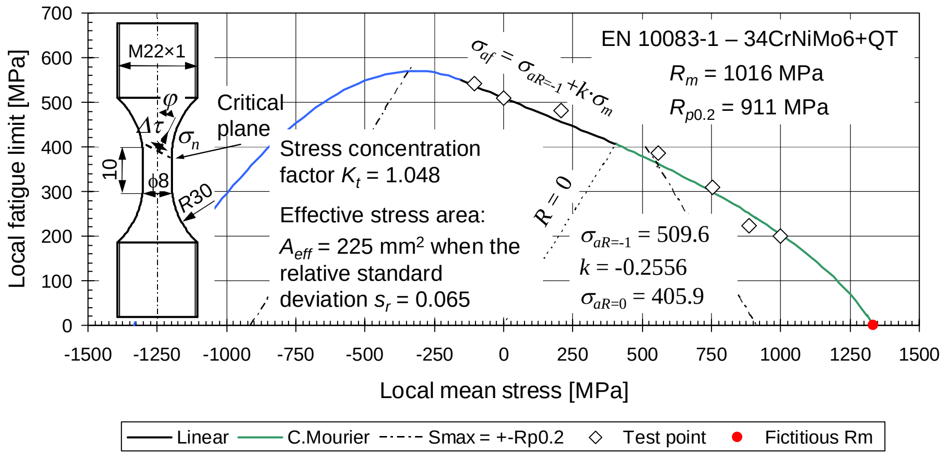


Figure 2. Haigh diagram for the reference specimen based on staircase test results considering the statistical size factor, Rabb [5], and tensile strength.

Table 2. "Radial" safety factor at the tested uniaxial fatigue limit $\sigma_{aR=-1}$ = 509.6 MPa

Stress components at the critical point	Criterion	Safety factor S_F	Direction φ of the critical plane [°]
- Axial stress	Findley	1.000	38.2
σ_a = 509.6 MPa	Dang Van	0.992	45
- Peripheral stress	Papadopoulos	0.992	45
σ_f = 15.9 MPa	McDiarmid	a) 1.000 b) 0.885	45

The result of these investigations is that the Findley criterion gives the best agreement with test results in all tested cases. In fact, Dang Van [4] stresses that the hydrostatic stress used in his criterion is only a simplifying approximation of the normal stress on the critical plane. As illustrated in Fig. 3, only the Findley criterion is able to logically explain the anisotropy found in uniaxial tests between specimens with axial and transversal grain flow.

The Findley criterion works reasonably well also when applied to nodular cast iron components. A Haigh diagram for EN 1563 – GJS-500-7 tested on plain specimens is shown in Fig. 4. The fatigue limit of notched specimens in the same material was also tested at fully reversed tension compression giving a biaxial stress state in the notch. A summary of the results of applying the four examined criteria to the tested fatigue limit

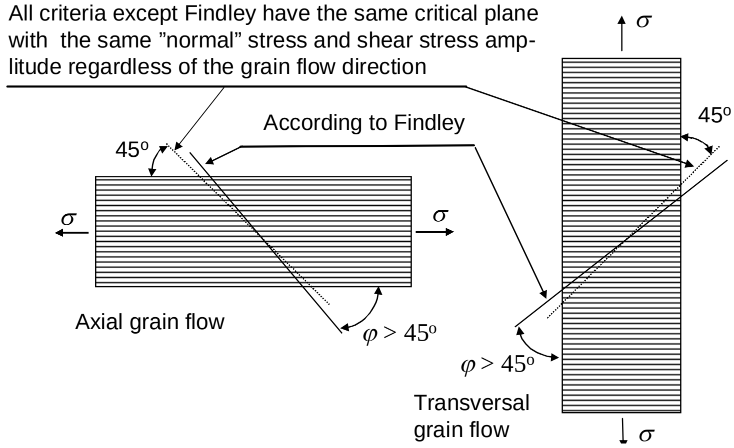


Figure 3. Findley criterion is consistent with the anisotropy found in the fatigue limit.

for the notched specimen is given in Table 3. Again, the Findley criterion gives excellent agreement whereas the other three criteria are quite inexact. The main drawback with the McDiarmid criterion is the seeming ambiguity or that it provides different safety factors depending on at which mean stress the uniaxial fatigue limit has been determined. The previously used signed von Mises criterion underestimates clearly the actual safety factor, although it gives a somewhat better estimate than the one calculated with the Dang Van and Papadopoulos criteria.

Also when these criteria were applied to the results of a fatigue test on tubular specimens of GJS-500-7 in torsion, only the Findley criterion could provide reasonably accurate estimates, see Fig. 5 and Table 4. The values in Table 4 are only indicative because only 9 specimens were used at stress ratio -1 and even 6 specimens at stress ratio 0 in the test of the shear fatigue limit.

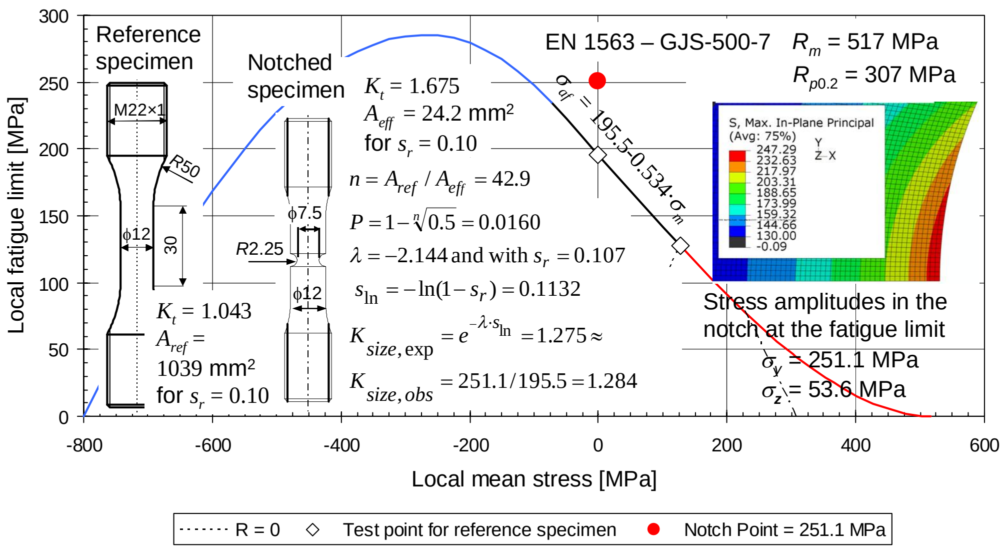


Figure 4. Examined criteria applied to the tested fatigue limit of a notched specimen.

Table 3. Safety factor of the notched GJS-500-7 specimens at the tested uniaxial fatigue limit $\sigma_{aR=-1}$ = 251.1 MPa

Stress components at the critical point	Criterion	Safety factor S_F	Direction φ of the critical plane [°]
- time 1: σ_y = -254.9 and σ_z = -53.2 MPa - time 2: σ_y = 247.3 and σ_z = 54.0 MPa	Findley	1.001	25.8
	Dang Van	0.872	45
	Papadopoulos	0.872	45
	McDiarmid	a) 0.993 b) 0.636	45
	Signed von Mises	0.893	-

The tested shear fatigue limits are therefore rather inaccurate, and the statistical size factor given in Fig. 5 has not been applied in the evaluation of the values in Table 4.

On the basis of these test results, it was decided to base the fatigue analyses of both steel and cast components in multiaxial stress states mainly on the use of the Findley criterion both for proportional and nonproportional load cases. Because the Dang Van criterion is widely used in Europe, it was decided to create an efficient algorithm also for this criterion to be able to compare the results if needed.

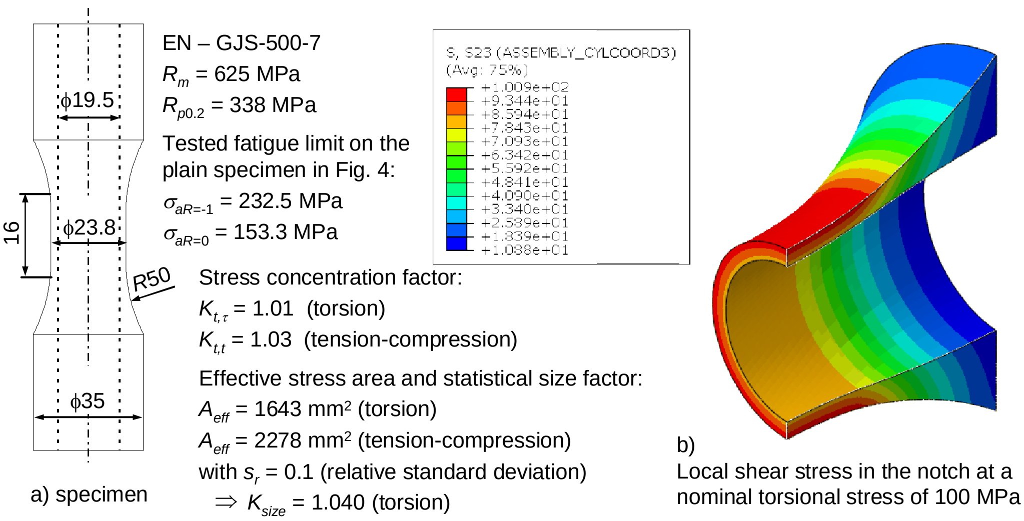


Figure 5. Fatigue test in torsion of tubular specimens of nodular cast iron.

Table 4. Examined criteria applied to the test outcome on tubular specimens in nodular cast iron EN - GJS 500-7

Criterion	Safety factor with tested fatigue limit as load	
	Stress ratio R = -1 with tested fatigue limit in torsion $\tau_{af} \approx$ 183 MPa	Stress ratio R = 0 with tested fatigue limit in torsion $\tau_{af} \approx$ 117 MPa
Findley	0.934	1.133
Dang Van	1.314	2.056
Papadopoulos	1.314	2.068
McDiarmid	a) 0.700 b) 0.477	a) 1.096 b) 0.745
Signed von Mises	0.733	0.631

PROGRAM MULTIV2

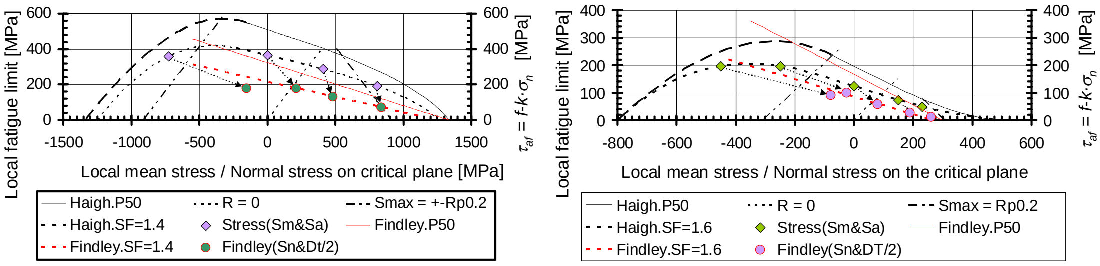
An efficient computer program called MultiV2 was created for an automatic execution of a multiaxial fatigue analysis according to the Findley, Dang Van and the traditional signed von Mises criteria on the stress output files of the ABAQUS finite element program. This program has also been presented at recent ABAQUS Users' conferences [6]. The use of the Findley criterion in nonproportional load cases on large stress output files presents high demands on computer capacity. By discretizing the stress space with varying angle increments in a way that keeps the subsurface areas constant and by using the down-hill simplex optimization algorithm, it was possible to create a fast and user friendly program. The shear stress range on the critical plane can either be determined as the distance between two time steps giving the maximum chord or as the diameter of the enclosed circle.

Although the Dang Van criterion is very easy and straightforward to use in proportional load cases, its use in nonproportional load cases requires the determination of the minimum radius of the hypersphere in 6- or 9-dimensional space that enclose the whole stress history. This is of course a demanding task, and to facilitate the programming work a commercial subroutine *miniball* that can be downloaded from the internet was used [7]. Usually the use of a 6-dimensional space gives the maximum damage and should be preferred.

A still unclear problem is how to calculate exactly the multiaxial damage in the plastic domain. The Haigh diagrams in Figs. 2 and 4 show that the slope of these diagrams change in this domain. However, the Findley criterion is given as a straight line, as shown in Fig. 6, with its slope determined with two values from the linear part of the Haigh diagram. The introduction of the concept of equivalent uniaxial stress σ_{eq} has provided a tool to handle also the plastic domains in a logical way. The Findley criterion, like all other criteria, considers only the total damage caused by the shear amplitude and the normal stress. It is therefore clear that it is almost always possible to find an equivalent uniaxial stress state giving the same damage as the actual multiaxial case. By transforming the calculated damage to this uniaxial equivalent, it is possible to calculate a meaningful safety factor also in situations with yielding. It should also be noted that the safety factor is iteratively calculated as the needed reduction of the fatigue diagram to cross the actual stress point, see Fig. 6.

It is also needed to change the multiaxial damage into an equivalent uniaxial stress when a cumulative damage analysis should be conducted. The transformation of the Findley damage is made using the following equations where $\Delta\tau$ is the shear stress range on the critical plane, σ_n its normal stress, and k is the constant for the normal stress sensitivity. This transformation seems also to work when $\sigma_{eq,max}$ exceeds the yield stress. The transformation is not of course defined if $4k\sigma_n+\Delta\tau<0$, but this happens rather seldom.

$$\alpha = \tan^{-1}\left(\pm\sqrt{\frac{\Delta\tau}{4k\sigma_n+\Delta\tau}}\right) \Rightarrow \begin{array}{c} \sigma_{eq,\max} = \sigma_n / \cos^2\alpha \\ \sigma_{eq,\min} = \sigma_{eq,\max} - 2\Delta\tau / \sin 2\alpha \end{array}$$



a) 34CrNiMo6+QT b) EN – GJS-500-7

Figure 6. Comparison of the safety factor according to Findley to the safety factor obtained using the equivalent uniaxial stress in different areas of the fatigue diagrams.

It is seen in Fig. 6 that the safety factor calculated directly with the “linear” Findley criterion will deviate from the straight line in the plastic domain. Already for maximum stresses approaching the yield stress, the corresponding stress in the Findley diagram for nodular cast iron will go beyond the valid range. However, the use of the equivalent uniaxial stress seems to provide logical safety factors for all situations.

DISCUSSION

For an accurate sizing of the engine components, it is essential to account for the multiaxial stress state generated by the complex geometry and loads. Through a thorough assessment of some available critical plane criteria, it was found that the Findley criterion is one of the most suitable for sizing engine components. However, there is still some uncertainty about the correct use of this criterion, e.g. in situations with plastic deformation.

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