IDD (Integration of Damage Differentials): General Representation

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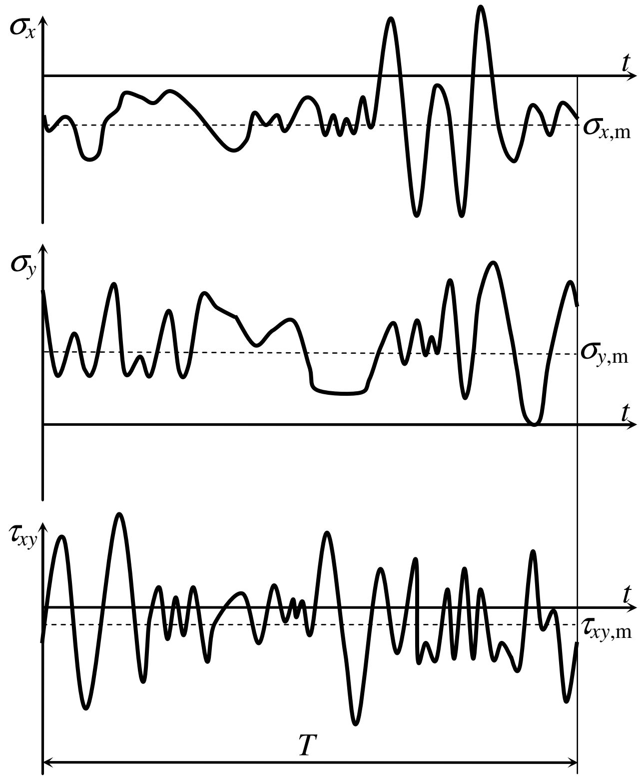
ABSTRACT. *IDD is a completely different concept than any other knowledge for fatigue life assessment under any loading including the general case of plane stressing: any stress-time functions $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$, not restricted to be cyclic, neither with specified variations, nor two of them zero (uniaxial loading), nor the three of them proportional, nor deterministic (but random), and so on. Then, no reversals and cycles could be distinguished along the loading path in the σ_x-σ_y-τ_{xy} coordinate space. Hence, any trial to apply again the well-known notions of loading cycles (amplitudes) and damages per cycles would conceptually fail. That is why, damage differentials* d*D are introduced to be summed (integrated) instead of finite damages per cycles. Each damage differential* d*D is per loading (stressing) differential* d*s that replaces the notion of loading cycle. There is an implementation of the IDD concept as a concrete IDD method and software. The present definitions of* d*s and* d*D are rerepresented. Two empirical factors of multiaxial loading non-proportionality, f_c and f_τ, are involved in the definition of dD. Methodical clarifications to latest results of IDD verifications under multiaxial non-proportional loadings are presented. The very results are reported in three other concomitant ICMFF9 papers with participation of coauthors.*

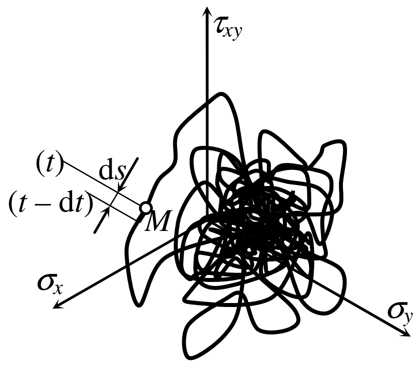
INTRODUCTION

This paper is aimed at ICMFF9 as a forum for general representation of IDD and advancing it as a completely different concept for fatigue life evaluation than the cycle counting approach.

All the other investigations relating to fatigue life are grounded on the notion of loading cycle and follow the cycle counting approach. After the treatment of constant-amplitude (cyclic) loading in the 19th century, the next step of the researches in the 20th century was to solve the fatigue life problem under variable-amplitude loading. It was all-accepted to decompose a non-cyclic stress-time function/history (shortly named "oscillogram") into a multitude of cycles of different amplitudes (and to build an amplitude spectrum). Each cycle is considered to cause fatigue damage 1/*N* (relative, as a part of 1) where *N* is taken from a corresponding *S*-*N* line. While cycles run, damages per cycles 1/*N* are summed. The life ends when the growing sum of the addends 1/*N*, i.e. the accumulated fatigue damage, reaches 1. This is the Palmgren-Miner rule of using the linear damage accumulation hypothesis. The rain-flow procedure of the cycle counting approach was all-accepted. Thus, the fatigue life assessment problem seems conceptually solved under variable-amplitude uniaxial, or multiaxial but proportional, loading: a single oscillogram can be cycle-counted and a single *S*-*N* line is used.

But what to do in the general case of varying plane-stress state (Fig. 1)?





(b) Loading path (“trajectory”) showing the mutual run (the mutuality) of the three oscillograms; a variant loading differential d*s*

(a) Three arbitrary oscillograms $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$

Figure 1. Multiaxial non-proportional and non-cyclic loading (the general case)

This general case of fatigue loading means arbitrary oscillograms $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$: not restricted to be cyclic (with constant amplitude), neither with specified variations, nor two of them zero (uniaxial loading), nor the three of them proportional, nor deterministic (but random), and so on. This is actually to be expected while obtaining unknown oscillograms by means of strain gages attached to a real object (real structural component). And, for this case, a concept and a general method for fatigue life evaluation should be available.

Then, no reversals and cycles could be distinguished along the loading path in the σ_x-σ_y-τ_{xy} coordinate space, Fig. 1(b). Any trial to apply again the notions of loading cycles (amplitudes) and damages per cycles would be logically inconsistent. The oscillograms, Fig. 1(a), cannot be separately cycle-counted since their mutuality, Fig. 1(b), is of first importance. For example, if two simplest sinusoidal bending and torsion oscillograms run, it is important whether they are in-phase or out-of-phase: they separately remain the same but their mutual run differs in the two cases, and hence the life differs, as well.

After all, no uniform, all-acknowledged and universal approach to the fatigue life assessment was established in the general loading case. Instead, a great number of inconsistent methods were proposed and a great scatter of research efforts occurred.

The reason for this failure is considered here to lay in the very inductive way of thinking: proceeding from the notion of cycle in the particular case of the simplest cyclic loading and trying to generalize under loading which becomes more and more complicated. This inductive approach is upside-down from calculus's point of view. To think from the particular to the general, based on finite quantities, was a manner prior to the time when Newton, Leibniz et al. introduced the infinitely little quantities, the differentials. As a result, all the sciences stormily developed grounded on relations between differentials (differential equations) integrated under arbitrary conditions. The inductive way was totally replaced by the deductive way starting from differential level which is free of integration (boundary) conditions and then integrating under any kind of those conditions. This was, in fact, the revolutionary concept of the calculus.

Upon transferring this concept to the fatigue life assessment, relations on differential level are to be revealed and integrated under arbitrary loading conditions. The notion of loading cycle is substituted by loading differential ds and the notion of damage per cycle $1/N$ is substituted by damage differential dD. The computed fatigue life results from integration of the damage differentials dD up to reaching 1. The life is again denoted by N, but now N is a number of repetitions of the representative oscillograms, i.e. of the time interval T, Fig. 1(a), until the cumulative damage reaches 1.

Thus, the loading does not need any preliminary (rain-flow etc.) cycle-counting but the very ordinates of the oscillograms automatically and directly run as integration (boundary) conditions while integrating. Hence, the oscillograms can be any: the concept is universally applicable under any loading. The preliminary factor "kind of loading" is not decisive now. The basis of notions is rearranged: cycle, S-N line and damage per cycle $1/N$ are not basic notions now but particular cases of cyclic loading integration conditions.

Hence, IDD does not need to preliminarily know any next wave-form of any of the oscillograms in Fig. 1(a). Neither is it necessary to know whether the oscillograms are proportional or non-proportional, etc. What is necessary is to have IDD software to just sum (integrate) the cumulative damage by adding every next damage differential dD regardless of how the next $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$ ordinates appear.

The present IDD method needs, though, some preliminary loading information. The mean (static) levels $\sigma_{x,\mathrm{m}}$, $\sigma_{y,\mathrm{m}}$ and $\tau_{xy,\mathrm{m}}$, Fig. 1(a), should be known in advance to take into account the mean-stress effect (under some discussable conditions). Or, if the loading is pulsating, this should also be known preliminarily.

Mentioning software, the IDD concept needs the contemporary computers and could not have been implemented earlier than 30 years ago. But for the last 30 years, as a concrete version of implementation of the IDD concept, the present IDD method and software have been developed. An IDD site, http://www.freewebs.com/fatigue-life-integral, has also been created. Any additional detail can be found there.

SHORT GENERAL REPRESENTATION OF THE PRESENT IDD METHOD

The Definition of the Invariant Loading Differential* d*s

Figure 1(b) shows a running point M describing the trajectory in the σ_x-σ_y-τ_{xy} coordinate space. Between the current (running) time t and the preceding time $t_p = t - dt$, a geometrical differential ds is described. It has components dσ_x, dσ_y and dτ_{xy} that form a loading (stressing) differential. However, the problem arises that the stresses σ_x, σ_y and dτ_{xy}, as well as ds and the whole trajectory in Fig. 1(b), are variant of (depend on) the orientation of the stress axes x and y. This orientation is not to prescribe: it is a choice of one who obtains the oscillograms $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$. If another one makes a different choice, the oscillograms, Fig. 1(a), and the trajectory, Fig. 1(b), will look different: a change of the x-y orientation makes σ_x, σ_y and τ_x change according to the well known equations of the stress theory. Obviously, the principal cubic volume (infinitesimal cuboid) with its invariant principal stresses in the principal directions (principal axes) should be used in some way without skipping their rotation. The principal axes are denoted as ' and " and, respectively, the principal stresses are σ' and σ'' (Fig. 2).

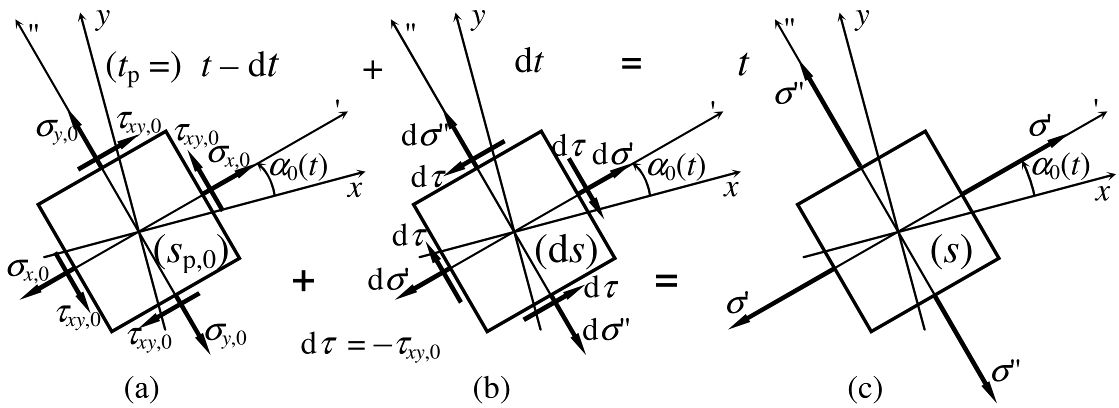


Figure 2. The invariant (ds) differential (b) as a difference between the principal (s) state of stress (c) at t and the preceding ($s_{p,0}$) state (a) at t – dt on the same cuboid

Thus, another idea for an invariant ds differential is applied, as follows. Figure 2(a) shows, at the preceding time $t_p = t - dt$, the cuboid ($s_{p,0}$) which will be principal at the current t time (cuboids with stresses are labeled in parentheses). That (immovable) cuboid stays at the angle α_0 valid for the t time. Since the cuboid is non-principal during dt and its state of stress is very close to the principal one, an infinitesimal shear stress $\tau_{xy,0} = -d\tau$ acts on it at t_p together with normal stresses $\sigma_{x,0}$ and $\sigma_{y,0}$. Figure 2(b) shows the appearance of dσ', dσ'' and $d\tau = -\tau_{xy,0}$ on the same cuboid during dt. They compose the invariant loading differential (ds) in physical meaning. By adding (ds) to ($s_{p,0}$), the invariant principal state of stress (s) results at t, Fig. 2(c) (no shear stress on the cuboid). Respectively, (ds) = (s) – ($s_{p,0}$) i.e. $d\sigma' = \sigma' - \sigma_{x,0}$, $d\sigma'' = \sigma'' - \sigma_{y,0}$ and $d\tau = -\tau_{xy,0}$.

The next physical loading differential from t to t + dt will look similar to Fig. 2(b) but valid for another immovable cuboid with an infinitesimally different orientation. Thus, consecutive invariant loading differentials are considered as lots of immovable cuboids at consecutive values of α_0 during consecutive time differentials dt. Actually, the continuous smooth function $\alpha_0(t)$ of rotation of the principal axes is replaced with a stepped one. Correspondingly, the rotation of the principal axes is followed in a stepped way. Hence, together with dσ' and dσ", Fig. 2(b), the third component $\mathrm{d}\tau = -\tau_{xy,0}$ of (ds) also appears. This dτ is owing to, and namely takes into account, the rotation of the principal axes during dt. Indeed, if there is no rotation, then $\mathrm{d}\tau = -\tau_{xy,0}$ will be zero.

The physical invariant loading differential (ds) introduced above, Fig. 2(b), is subject to geometrical interpretation as represented next. It is not obligatory (the entire IDD theory can be built only based on cuboids with stresses) but is very suggestive.

It proves (see the IDD site) that the well-known calculation of the principal stresses in the stress theory means geometrically the following: the current point $M(\sigma_x,\sigma_y,\tau_{xy})$ of the σ_x-σ_y-τ_{xy} trajectory, Fig. 1(b), goes along an ellipse of transformation (Fig. 3) and reaches the σ_x-σ_y plane which becomes σ'-σ" plane. Such an ellipse is always parallel to the ξ-τ_{xy} plane where ξ is the bisector of the quadrants II and IV. The major axis of the ellipse is parallel to ξ and the minor axis is parallel to τ_{xy}. The ratio between the axes is

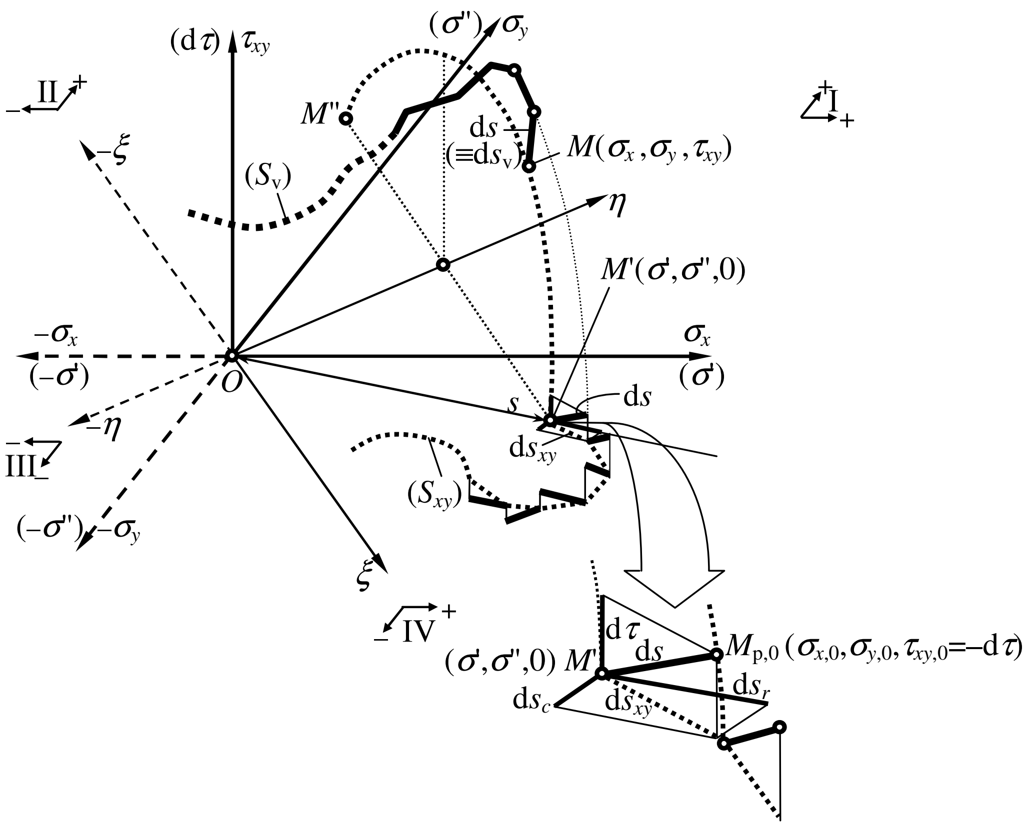


Figure 3. Transformation of ds element from σ_x-σ_y-τ_{xy} coordinate space into σ'-σ"-dτ coordinate space

always $\sqrt{2}$. The center of the ellipse is on the bisector η of the quadrants I and III.

Thus, the determination of σ', σ'' and ds at t time means the following. The current (new) end of the current variant element $\mathrm{d}s \equiv \mathrm{d}s_\mathrm{v}$ in the σ_x-σ_y-τ_{xy} space is brought to a contact with the plane σ'-σ'' and becomes the point $M'(\sigma',\sigma'',0)$ (or M'' if exchanging σ' and σ''). And the determination of $\sigma_{x,0}$, $\sigma_{y,0}$ and $\tau_{xy,0}$ at the time t – dt means that the preceding end of the transformed element ds, the point $M_{\mathrm{p},0}$ $(\sigma_{x,0},\sigma_{y,0},\mathrm{d}\tau)$, is obtained. It remains at the distance dτ from the σ'-σ'' plane. The so-transformed current variant element $\mathrm{d}s \equiv \mathrm{d}s_\mathrm{v}$ becomes ds: already invariant of the choice of the stress axes x and y.

Figure 3 also shows the appearance of the preceding loading differentials. The so-built sequence of the invariant ds elements is denoted as (S) and is called invariant "trajectory" (although it is torn to infinitesimal ds fragments in case $\mathrm{d}\tau \neq 0$). The length of (S), i.e. the sum (the integral) of the ds lengths, is S.

The component (projection) of any invariant element ds onto the σ'-σ'' plane is labeled as ds_{xy} (Fig. 3). All the ds_{xy} elements form a trajectory (S_{xy}). Its length S_{xy} is the sum of the ds_{xy} lengths. It turns out that the differentials ds_{xy} can be considered connected with the allowance of second-order infinitesimal deviations. Respectively, the (S_{xy}) trajectory, in infinitesimals, can be considered as a smooth curved line (it is a straight radial line under proportional loading, in particular).

Together with the trajectories (S) and (S_{xy}), it is also to introduce a "trajectory" (S_τ) built by the elements dτ. Its length S_τ is the sum of their lengths. So, if the principal axes rotate, the elements ds disconnect from each other since their components dτ appear. In case the principal axes are immovable, the elements dτ disappear, and the elements $\mathrm{d}s \equiv \mathrm{d}s_{xy}$ are exactly connected. Then they compose a continuous smooth trajectory $(S) \equiv (S_{xy})$ which entirely lies in the σ'-σ'' plane.

Meanwhile it is to remark that d differentials are represented (substituted) by little Δ finite differences (finite elements): d ~ Δ. The integration is numerical.

As a matter of fact, a special three-dimensional coordinate space σ'-σ''-dτ ($\mathrm{d}\tau \sim \Delta\tau$) is introduced in which the third dimension dτ is infinitesimal. And, with the transition from the original three dimensions σ_x-σ_y-τ_{xy} to the new again three dimensions σ'-σ''-dτ, no loading information is lost. The transformation provides a reversible one-to-one correspondence: the original variant continuous trajectory can be restored from the invariant torn one. On the other hand, as the third dimension is infinitesimal, the further analysis is practically two-dimensional. This is a significant convenience, also for computer visualization: the (S_{xy}) trajectory will only be displayed on the computer screen that represents the σ'-σ'' plane, and, on a separate place on the screen, the corresponding $\Delta\tau$ element will be displayed simultaneously with every Δs_{xy} element.

In particular, if the original variant trajectory coincides with a transforming ellipse, then the elements $\Delta\tau$ only exist ($\Delta s \equiv \Delta\tau$). They gather all onto one point $M'(\sigma',\sigma'',0)$ (or M''), Fig. 3, and then $(S) \equiv (S_\tau)$. This is the case of constant principal stresses in rotating principal directions.

IDD also suggests statistical treatment of random loading (without any amplitudes). Two-dimensional density of distribution of the Δs elements onto the σ'-σ'' plane can be done. This subject, relating to random general loading, cannot be included in this paper (see the IDD site instead).

Components of the Loading Differential, Basic IDD Types of Loading and Two Practical Categories of Non-Proportional Loading

The invariant geometrical ds differential is "zoomed in" in Fig. 3. It is resolved into a radial component ds_r, a circumferential component ds_c, and a "perpendicular" dτ component. Correspondingly, two other "trajectories" are introduced: (S_r) of length S_r as a sum of the Δs_r elements, and (S_c) of length S_c as a sum of the Δs_c elements. Trajectory ratios $t_r = S_r/S$, $t_c = S_c/S$ and $t_\tau = S_\tau/S$ are also introduced. Each of them is ≤ 1.

Hence, the present IDD method suggests three basic types of loading, as follow.

First type: proportional loading (including uniaxial stressing and pure shear) with a trajectory (S) $\equiv$ (S_r). It is also called r-loading. Radial elements Δs_r exclusively appear and the ratio of the principal stresses $k = \sigma''(t)/\sigma'(t)$ remains constant. The elements $\Delta s \equiv \Delta s_r$ lie on a radial line through the coordinate origin in the σ'-σ'' plane. Each proportional loading has its own k radial line on which the trajectory (S) $\equiv$ (S_r) oscillates. All the elements $\Delta s \equiv \Delta s_r$ are connected, all the elements Δs_c and $\Delta\tau$ are zeros, $t_r = S_r/S$ is exact 1, and the principal axes are permanently immovable. The trajectory ratio t_r may also not be exact 1 but close to 1: the loading is nearly proportional.

Second type: non-proportional loading with immovable principal directions and a trajectory (S) $\equiv$ (S_{xy}) $\equiv$ (S_c) that is an (arc of a central) circumference in the σ'-σ'' plane. It is also called c-loading. In laboratories, cruciform or thin-wall tubular specimens can be exposed to such loading. The pure c-loading has the trajectory ratio $t_c = S_c/S = 1$. Otherwise, every non-proportional loading with immovable principal axes relates to the second type and may have $t_c = S_c/S$ close to 1.

Third type: non-proportional loading having constant principal stresses in rotating principal directions mentioned above. It is also called dτ-loading or $\Delta\tau$-loading.

On the other hand, in fact, there are very rare cases in the engineering practice when really all the three oscillograms could be non-proportional. Much more often, two "practical" non-proportional loading categories are most important: combined bending or axial loading and torsion (with rotating principal axes), and biaxial tension-compression (with immovable principal axes). The first practical category relates to a beam (or a shaft) as the most popular model of a structural component. The second category concerns plates and shells. This paper is finally directed to the first practical loading category.

The Present Damage Differential with Damage Intensities in It

Publications on the present IDD method started in 1978. On international level, in English, the first publications were [1, 2, 3, 4]. At the latest time, coauthors were also involved: [5, 6], and [7, 8, 9] presented together with this paper at ICMFF9.

The basic equation postulated is d$D(s)$ = $R(s)$ds at any $k = \sigma''/\sigma'$. The s argument is the distance from the current principal point M' of the trajectory to the coordinate origin

(Fig. 3). The function $R(s)$, as a derivative $R(s) = \mathrm{d}D(s)/\mathrm{d}s$, is damage intensity. $D(s)$ is damage function and is the primitive function of the damage intensity.

Apart from the main argument s at any k, R is influenced by additional arguments:

$$R = R[s;\ k;\ \sigma_{x,\mathrm{m}}(t), \sigma_{y,\mathrm{m}}(t), \tau_{xy,\mathrm{m}}(t);\ \mathrm{d}s_r/\mathrm{d}s, \mathrm{d}s_c/\mathrm{d}s, \mathrm{d}\tau/\mathrm{d}s;\ t_r(t), t_c(t), t_\tau(t);\ \ldots] \tag{1}$$

More details about the additional arguments can be found on the IDD site. Some of them are current: they vary with t (by the way, the current cumulative damage is also an additional argument which is responsible for non-linear damage accumulation).

Depending on how many additional arguments will be taken into account and insofar thoroughly, different IDD methods can be created, more or less complicated. The present IDD method envisages steady (no varying) static (mean) stresses (levels) $\sigma_{x,\mathrm{m}}$, $\sigma_{y,\mathrm{m}}$, $\tau_{xy,\mathrm{m}}$ in Eq. 1, as shown in Fig. 1(a). Details about how such static levels are taken into account can be found on the IDD site and/or in [9]. How the dependence of R on $\mathrm{d}s_r/\mathrm{d}s$, $\mathrm{d}s_c/\mathrm{d}s$ and $\mathrm{d}\tau/\mathrm{d}s$ is considered is cleared below. The next possible additional arguments in Eq. 1 are not taken into account so far.

If the loading is proportional (r-loading), then R is denoted as R_r and $\mathrm{d}D$ becomes $\mathrm{d}D_r = R_r\mathrm{d}s_r$. Under pure c-loading and pure $\mathrm{d}\tau$-loading, the respective damage differentials are $\mathrm{d}D_c = R_c\mathrm{d}s_c$ and $\mathrm{d}D_\tau = R_\tau\mathrm{d}\tau$. If the loading is of mixed type, the key question arises of how to formulate the damage differential $\mathrm{d}D$ per $\mathrm{d}s$? Or: how are $\mathrm{d}D_r$, $\mathrm{d}D_c$ and $\mathrm{d}D_\tau$ combined into a general differential $\mathrm{d}D$? The present postulation is:

$$\mathrm{d}D = \sqrt{\mathrm{d}D_r^{\,2} + \mathrm{d}D_c^{\,2} + \mathrm{d}D_\tau^{\,2}} \ \text{ i.e. } \ \mathrm{d}D = \sqrt{(R_r\mathrm{d}s_r)^2 + (R_c\mathrm{d}s_c)^2 + (R_t\mathrm{d}\tau)^2}\,. \tag{2}$$

If introducing damage intensity ratios $f_c = R_c/R_r$ and $f_\tau = R_\tau/R_r$, then

$$\mathrm{d}D = R_r\sqrt{\mathrm{d}s_r^{\,2} + f_c^{\,2}\mathrm{d}s_c^{\,2} + f_\tau^{\,2}\mathrm{d}\tau^2}\,. \tag{3}$$

Upon integrating all the differentials $\mathrm{d}D$ and (according to the linear damage accumulation hypothesis) taking the reciprocal of the accumulated damage within T, the life is computed:

$$N = \left[\int_{(S)} R_r\sqrt{\mathrm{d}s_r^{\,2} + f_c^{\,2}\mathrm{d}s_c^{\,2} + f_\tau^{\,2}\mathrm{d}\tau^2}\right]^{-1}. \tag{4}$$

In finite Δ elements,

$$N = \left[\sum_{(S)} R_r\sqrt{\Delta s_r^{\,2} + f_c^{\,2}\Delta s_c^{\,2} + f_\tau^{\,2}\Delta\tau^2}\right]^{-1}. \tag{5}$$

In the above equations, zero R_r, R_c and R_τ areas can be introduced in the σ'-σ'' plane round the coordinate origin. This is next IDD notion instead of notion of fatigue limits.

Equation 5 is what the present IDD software performs in the general loading case. The ratios $f_c = R_c/R_r$ and $f_\tau = R_\tau/R_r$ are given (selected) by the IDD user (discussed below). The intensity R_r is computed for each trajectory's element Δs as cleared below.

IDD Procedures and IDD Application

For setting R_r at any k radial line in the σ'-σ'' plane, empirical S-R_r lines should have been investigated valid for proportional loadings (both cyclic and non-cyclic). However, the fatigue knowledge was not initiated in the IDD way in the 19th century. Instead, the empirical *S-N* lines, easier for realizing, were introduced under cyclic loadings and all the fatigue knowledge was grounded on them. As they are integral results under cyclic loading integration (boundary) conditions, then vice versa, by differentiation, S-R_r lines can be determined from *S-N* lines.

Thus, for IDD, input *S-N* lines are entered under different cyclic proportional (in-phase) loadings (*r*-loadings) with different values of the constant $k = \sigma''/\sigma'$. The number of the input *S-N* lines is ≤ 9. Each of them is a double-logarithmic straight *S-N* line described by the well-known equation $s^m N = A$ = constant (where $s \equiv s_{\max}$, and $s_{\max} \equiv s_a$ for reversed stress cycle). The mostly used input *S-N* lines are for uniaxial stressing ($k = 0$) and pure shear ($k = -1$); and, as well, they can be for any k between +1 and –1.

To determine R_r from such input *S-N* lines means to make Eq. 4 reproduce them (then f_c and f_τ do not interfere since $ds_c = d\tau = 0$). In other words, Eq. 4 should turn into $N = A/s^m$ for each of the input *S-N* lines. Based on this, by applying the basic Newton-Leibniz theorem of the calculus (see the IDD site), the following equation is deduced for the R_r damage intensity along a straight radial k line in the σ'-σ'' plane:

$$R_r = R_r(s) = \frac{ms^{m-1}}{i^* A} \tag{6}$$

where m and A are from the *S-N* line equation $s^m N = A$; s, as already known, is the distance to the coordinate origin; i^* is a corrective determined by an equation (see the IDD site); it proves 4 if the mean (static) level s_m is 0; otherwise, i^* is between 4 and ≈2 if s_m is close to 0, and $i^* \approx 2$ if s_m is far from 0; or, if the loading is pulsating, $i^* = 2$.

As to the determination of $R_r = R_r(\sigma', \sigma'')$ in the whole σ'-σ'' plane, a heavy mathematical expression of R_r is deduced. Its solution is only possible by successive approximations done by the IDD software.

In terms of a zero R_r area, the following considerations apply.

If the IDD user intends to have each input *S-N* line reproduced as breaking in two at a fatigue limit s_l, then the so-called breaking mode of the IDD software is requested and a number of cycles N_l is entered which corresponds to s_l (i.e. $s_l^m N_l = A$).

If the IDD user intends to have each input *S-N* line reproduced as asymptotically (smoothly) bending (curving, turning) to a lower damage intensity limit $s_r < s_l$, the so-called smooth mode is requested and a number of cycles $N_r > N_l$ is entered. Then, Eq. 4 turns into $N = A/(s^m - s_r^m)$ i.e. the reproduced *S-N* line bends to s_r. In such a case, the input line with the equation $s^m N = A$ is called R_r-prototype. In fact, the *S-N* lines entered in the IDD software are, generally speaking, always R_r-prototypes. In particular, an R_r-prototype coincides with the reproduced *S-N* line in the breaking mode only.

If the IDD user does not intend to use either a fatigue limit s_l or a distinct damage intensity limit s_r, then the smooth mode is used but setting N_r as great, e.g. $N_l = 10^9$, as not to have any significant difference between $s^m N = A$ and $(s^m - s_r^m)N = A$. Thus the input R_r-prototypes and the reproduced input *S-N* lines will practically coincide and continue downwards as still straight ones.

Once R_r is determined so that the input *S-N* lines are reproduced in the form the IDD user prefers, then the same R_r damage intensity is ready for application of the Eq. 5 or Eq. 4 to any variable amplitude uniaxial or multiaxial but proportional loading (f_c and f_τ do not interfere). And then no rain-flow etc. cycle counting is necessary but the original loading directly runs under the computation.

Even this feature already makes the IDD approach topical: there had been no idea before that fatigue life assessment under variable-amplitude uniaxial or proportional loading is possible without cycle counting. And now it is possible and it was successfully verified in [5]. With that, instead of Eq. 5, much supplier equation and software are used [5] based on the Newton-Leibniz theorem only.

As to the ratios f_c and f_τ in Eq. 5, they were still not introduced in [1, 2, 3] as constants. Instead, R_c and R_τ were independently searched in the σ'-σ'' plane as to satisfy a part of the experimental life data. Then they served very well to assess the rest of data. However, for mass application of the present IDD method into practice, it is better to introduce f_c and f_τ as constants and recommend verified values of them.

On this occasion, f_c and f_τ are called factors of loading non-proportionality. To set them in Eq. 5 is the main theoretical and practical problem while applying IDD to non-proportional loading. It is expected that these factors will be comparatively easy to be envisaged. Indeed, a relation like Eq. 4 on the differential level, even if based on parameters being problematic for the present, will be more reliable than any other relation on integral level influenced by the integration conditions.

The loading non-proportionality factors f_c and f_τ are considered as new empirical material and component characteristics together with the traditional fatigue characteristics like (conditional) fatigue limits, parameters of *S-N* lines, etc. This paper, together with [7, 8, 9] reveals that, for the most popular first loading category (relating to beams), f_c and f_τ can be set as 2 and 3 so far. In the future, a thorough data-bank for recommendable values of f_c and f_τ and their application conditions should be created. In any case $f_c \geq 1$ and $f_\tau \geq 1$ are expected. For the second loading category (relating to shells and plates), f_τ does not interfere (no rotation of the principal axes), but f_c may, instead, reach a higher value, e.g. 10 [3].

SHORT GENERAL REPRESTNTATION OF THE PRESENT IDD SOFTWARE

Equation 5 is only performed by means of computers with original IDD software called *Ellipse*. It is freeware and includes FORTRAN programs which can be downloaded from the IDD site. Demos are available there as a manual for self-teaching. There are many options at the input and at the output: besides the main output result of the computed life, several more additional results can be requested and interpreted.

One, two or three arbitrary stress oscillograms are entered as a file containing their ordinates. It is called Current Data File or C-file. It may contain either several ordinates only or millions of ordinates.

As Δ elements should be short enough in order to substitute d differentials, intermediate ordinates are generated, if necessary, between every two successive original ordinates. For this purpose, trigonometric or cubic-spline interpolation is done. The program enabling the trigonometric interpolation is called *EllipseT*. It can also create C-files of sinusoidal oscillograms (in-phase or out-of-phase) at the request by the user. The program enabling the spline interpolation is called *EllipseS*. It is mainly envisaged for long C-files containing comparatively dense ordinates.

Each *Ellipse* program is able to work in text mode only or in graph mode. The latter is enabled by a subprogram called *Graf*. In the graph mode, the σ'-σ'' plane is displayed on the computer screen and the run of the loading trajectory (S_{xy}) (Fig. 3) can be seen. Thus, the consecutive Δs_{xy} elements and the accumulation of the ΔD differentials can be followed until the sum in Eq. 5 is finally computed. Simultaneously with the Δs_{xy} elements, the $\Delta\tau$ elements also appear on a separate place of the computer screen [7, 8].

The processing of the C-file is led by another input file called Leading Data File or L-file. It contains the input R_r-prototypes and the number N_r which defines the borderline between the zero and non-zero R_r areas of the σ'-σ'' plane in the smooth mode (or N_l is entered for N_r in the breaking mode). Together with the R_r-prototypes, analogous so-called R_c-prototypes and R_τ-prototypes are also included in the L-file in a way as to form the factors f_c and f_τ. Other more input data are numbers N_c and N_τ, analogous to N_r, defining the borderlines between the zero and non-zero R_c and R_τ areas.

CONCLUSIVE NOTES

A conviction is expressed that IDD as a concept is the most adequate for fatigue life assessment in the general loading case. Newton and Leibniz would have recommended it. Any skepticism on the part of some researchers would put them in the position of mathematicians prior to the discovery of the calculus who did not admire the idea of the infinitesimals. However, introducing the infinitely little quantities, the differentials, was the infinitely great progress of the human being's thinking. Only then did the stormy scientific and technological progress starts.

As to the present IDD method as a version of implementation of the IDD concept, it does not claim for perfection. Anyway, it is the high time now for the fatigue life researchers to readjust their thinking for IDD as a new way to re-interpret and use the fatigue life knowledge accumulated from Wöhler's time to nowadays. Out of the question any idea is that IDD would depreciate the cycle counting approach and its practical application to particular loading cases. The question is that new followers should implement IDD, also in other versions of dD, and develop it parallel with the cycle counting approach: fruitful struggle of ideas will lead to better solutions.

The above definition of the invariant loading differential ds is considered to be the single possible one. Any other method created by followers of the IDD concept may differ from the present method, but the ds definition, Figs 2 and 3, would hardly change.

If following everything presented on the IDD site about the R_r, R_c and R_τ intensities in the σ'-σ'' plane, it will become apparent that the IDD way of thinking suggests surprising re-interpretations of well-known old fatigue life problems and puts new ones. For example, the three loading types introduced are conceptually new. They do not refer to specific oscillogram's variations as previous names of loading kinds refer (the new loading types are released from the specific integration conditions of the wave-forms). And again for example, so formulated, the pure third type of loading (dτ- or $\Delta\tau$-loading) had not been considered before. However, it proves to be of first-rate importance now. It is a maximized case in the meaning that all the planes parallel to z are equally critical. It is a "touch stone", an "acid test" that may invalidate many of the fatigue life theories proposed: see [4] or/and the IDD site.

Still in the vein of surprising re-interpretations, it is conceptually not correct for the cycle counting approach to take into account the mean stress effect at every cycle [5]: this makes discontinuities of the new function $R_r(s)$; whereas, $R_r(s)$ and the whole process of damage accumulation is physically continuous [5].

Concretized representation of the present IDD method and its software continues in the papers [7, 8, 9]. They report the latest verifications for f_c and f_τ, as well as for the numbers N_r, N_c and N_τ.

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