THERMAL CRACK PROPAGATION IN DIFFERENT SHAPED TWO-PHASE COMPOSITE STRUCTURES: ANALYSIS AND EXPERIMENT

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INTRODUCTION

The failure of composite structures due to thermal loading represents a very important problem in today's fracture mechanics research because in space travel and reactor technology multiphase materials are sometimes subjected to instationary temperature fields causing fracture if the ultimate strength of the material is reached. Further, experimental investigations concerning thermal fracture of compound materials show very often the existence of curved crack paths in the cracked specimens. The reason for this phenomenon should be the non-uniform thermal stress fields existing in the corresponding composite structures. Thereby curved or kinked cracks were investigated already from several authors either as interface cracks along circular inclusions in bimaterial problems [1,2] or in connection with the assessment of existing crack propagation criteria [3,4].

In this paper, curved thermal cracks are considered running along special principal stress trajectories of thermal stress fields originated in different shaped two-phase composite structures with circular, quadratic and hexagonal cross sections, respectively, by a steady cooling process.

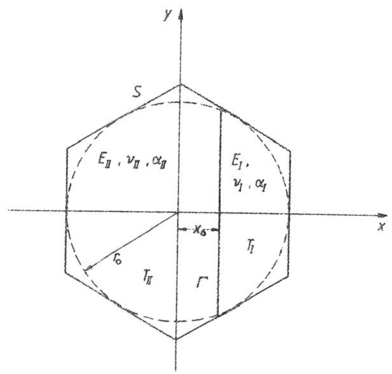

Fig. 1: Hexagonal cross section of an uncracked two-phase solid.

The two-phase compounds consist of homogeneous, isotropic and linearly elastic materials with differing thermoelastic properties varying discontinuously at the straight discontinuity area Γ from the values E_I, ν_I, α_I of the segment I to the values E_{II}, ν_{II}, α_{II} of the segment II (E - Young's modulus, ν - Poisson's ratio, α - linear coefficient of thermal expansion). Moreover, the conditions of perfect contact at the interface Γ are assumed. Fig. 1 shows the hexagonal cross section of an uncracked compound cylinder which has been submitted to a constant temperature distribution $T = T_I = T_{II} \neq T_o$ where T_o represents the temperature of the unstressed initial state. The resulting self-stress problem can be treated as a plane strain state by assuming temperature-independent thermoelastic properties.

FORMULATION OF BOUNDARY-VALUE PROBLEMS

1. Uncracked Two-Phase Composite Structure

Experiments showed that curved thermal cracks began to propagate in those brittle two-phase composite structures if special material combinations (two optical glasses with differing properties) were pasted together by a temperature of nearly 550 deg C and afterwards were cooled down to room temperature. Thereby the experiments give crack paths following in a reasonable agreement a special principal stress trajectory belonging to the self-stress field in the appropriate uncracked bimaterial specimen. This thermal stress field is obtainable from the solution of a boundary-value problem of the stationary plane thermoelasticity. A closed form solution of this boundary-value problem, the Airy stress function $F(x,y)$, for a two-phase compound cylinder with a circular cross section has been given in [5] whereas the solutions for the composite structures with a quadratic and a hexagonal cross section, respectively, were obtained by using the finite element method. Further, the field of the principal stress trajectories can be calculated by a numerical integration procedure of the following ordinary differential equation

$$2\sigma_{xy}dy + \{(\sigma_{xx} - \sigma_{yy}) \mp \sqrt{(\sigma_{yy} - \sigma_{xx})^2 + 4\sigma_{xy}^2}\}dx = 0 \qquad (1)$$

Fig. 2 shows the field of the principal stress trajectories of a plane self-stress state for a compound material consisting of the two optical glasses BK4/BK12. Moreover, a curved thermal crack originated by the above mentioned cooling experiment starts with high initial velocity from one intersection point of the interface Γ with the external boundary S and is running along a special principal stress trajectory located entirely in one of both glass segments (cf. Fig. 2 for illustration).

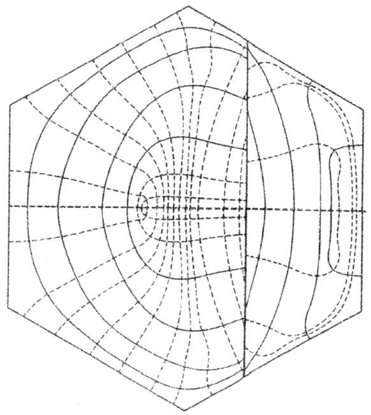

Fig. 2: Principal stress trajectories in the cross section of an uncracked self-stressed two-phase solid.

2. Cracked Two-Phase Composite Structure

The treatment of the crack path prediction as well as the determination of the fracture mechanical data governing the quasistatic growth of the curved thermal crack have been performed by applying the concepts of linear elastic fracture mechanics. Thus, by use of the basic equations

$$\sigma_{ij} = 2\mu\{\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\} \qquad (2)$$

$$\sigma_{ij,i} = 0 \qquad (3)$$

$$\varepsilon_{ijk}\varepsilon_{lmn}\varepsilon_{km,jn} = -\alpha\varepsilon_{ijk}\varepsilon_{lmn}T_{,jn}\delta_{km} \qquad (4)$$

and by assuming the existence of a plane strain state in the two-phase composite structure a mixed boundary-value problem of the plane thermoelasticity has to be solved numerically fulfilling the following boundary conditions:

$$\sigma_{ij}n_j = 0 \qquad (i,j=x,y) \qquad (5)$$

where $\underline{n}$ is the unit normal vector with respect to the external boundary S and to the two crack surfaces $S^+ \cup S^-$, respectively. Besides, the continuity conditions

$$[\sigma_{xx}(x,y)]_{x=x_o} = [\sigma_{xy}(x,y)]_{x=x_o} = 0 \qquad (6)$$

$$[u_x(x,y)]_{x=x_o} = [u_y(x,y)]_{x=x_o} = 0 \qquad (7)$$

have to be satisfied at the material interface Γ, where

$$[\omega(x,y)]_{x=x_o} = \omega^{I}(x_o,y)-\omega^{II}(x_o,y) \qquad (8)$$

means the jump of the corresponding quantity at the discontinuity area Γ.

The theoretical crack path prediction was performed for a composite structure consisting of two segments of the optical glasses BK4 and BK12 and by using different crack propagation criteria [6]. The radius of the circular cross section of the model was chosen to $r_o = 16.5$ mm, while the distance of the discontinuity area Γ to the center of the two-phase solid had the value $x_o = 5.64$ mm and the temperature difference was assumed to be $\Delta T = -560$ deg C. The calculations were carried out on the computer PRIME 550 at the University of Paderborn and by applying the finite element program ASKA. Thereby the numerically calculated crack path shows a reasonable agreement with a special principal stress trajectory of the appropriate uncracked specimen (stable configuration) and with crack paths obtained from cooling experiments, respectively [7].

Finally, the determination of the strain energy release rate G_I at the tip of a quasistatic extending curved thermal crack has been performed applying a method described in [8] which bases on the calculation of a crack closure integral modified by using the finite element method. It should be mentioned that there exists mode I-loading only of the thermal crack during its assumed propagation along a special principal stress trajectory. The finite element mesh-works for the bimaterial specimens consist of numerous triangular linear strain six-node elements with nearly 4000 degrees of freedom focused essentially in the neighborhood of the prospective crack line.

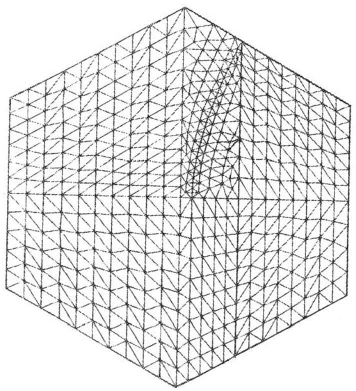

Fig. 3: Finite element mesh-work for a two-phase composite structure with a hexagonal cross section.

Fig. 3 gives a typical finite element mesh for a two-phase solid with a hexagonal cross section.

Further, Fig. 4 shows the strain energy release rate G_I in dependence on crack length a for three two-phase composite structures with circular, hexagonal and quadratic cross sections, respectively. As can be seen from the graph the curves (1) and (2) decrease monotonously whereas the curve (3) decreases after passing a maximum value at $a = 4$ mm. Moreover, there exists a remarkable influence of the shape of the external surface on the values of the strain energy release rate G_I.

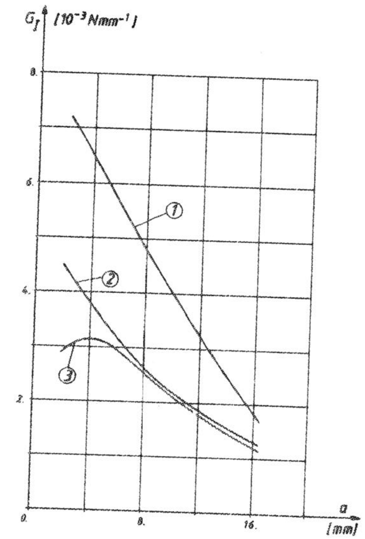

Fig. 4: Strain energy release rate G_I as it depends on crack length a for three two-phase solids with different cross sections
(1): circular, (2): hexagonal, (3): quadratic.

In addition, Fig. 5 gives a comparison of experimentally determined stress intensity factors gained by the optical method of caustics in disk-like two-phase glass specimens and theoretically determined stress intensity factors obtained by a finite element calculation. Thereby a reasonable agreement can be stated between the theoretically and experimentally performed investigations.

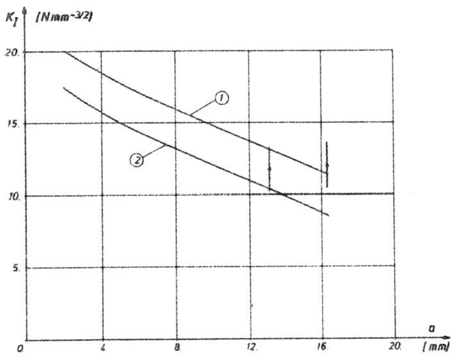

Fig. 5: Theoretically and experimentally gained stress intensity factors for a thermally cracked composite structure (optical glasses F2/ZKN7, $\Delta T = -260$ deg C) with a hexagonal cross section ((1): plane strain, (2): plane stress).

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