DELAYED CRACK INSTABILITY OF VISCOELASTIC BODIES

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INTRODUCTION

Neither elastic nor elastic-plastic fracture mechanics can treat such time-dependent problem as delayed unstable crack extension. The behaviour of many engineering materials (e.g., polymer, rock, concrete, etc.) exhibit time-dependence even at room-temparature. Hence, only when we properly select rheological models as the basis for our study of crack problems, can we obtain results close to objective reality. Recently, a theory of the rheological fracture of idealized linear viscoelastic cracked bodies has been developed[1-3]. This theory is in fact a direct generalization for the Griffith-Irwin approach of idealized linear elastic cracked bodies. It plays the same important role as the linear elastic theory does for elastic bodies. For many practical engineering problems, it is of great interest to know what level of load could be applied in order to avoid the instantaneous or delayed catastrophic failure of the structure members rather than to know the velocity of the stable crack growth. In dealing with the viscoelastic crack problems, it is of great importance to distinguish two approaches of treatment, based on different assumptions to the logical model of the medium. The first one is based on the assumption that the medium is linearly viscoelastic at all of its points and no plastic deformation occurs at the crack edge. The second one assumes that a crack is propagating in a linearly viscoelastic medium in the presence of a thin plastic zone ahead of the crack tip. As we know, in the former case, there is no slow stable crack growth under constant loads even for those cracks with $dK/da>0$. The critical state (the initiation of unstable crack propagation) occurs at a certain time t_c after the application of the load. It is well worth notice that during this period ($0 \leq t \leq t_c$) the initial size of the crack length remains unchanged, but the displacement on the crack surface increases gradually on account of the time-dependent behaviour of the material. As a result, the energy release rate $G_I(t)$ for a viscoelastic cracked body increases also with time[1]. As $G_I(t)$ reaches G_{Ic}, the equilibrium of the crack becomes unstable and the crack will begin to propagate in the idealized viscoelastic medium with an unbounded velocity in the absence of inertial constraints. If the creep compliance of a material possesses an upper limit, then there exists a lower limit load/crack size below which a crack will not propagate. We refer to this load/crack size as the critical load/crack size of delayed crack instability. In the latter case, the approximate crack growth velocity was derived by Schapery[4], McCartney[5] by using the generalized Barenblatt-Dugdale crack-tip model. They showed that there is also a delay or incubation period which must elapse before crack growth takes place. The crack length is also constant during this period. Nuismer[6] pointed out that for (idealized) linearly viscoelastic materials, the thermodynamic power balance as a fracture criterion can only be used to predict crack initiation but not crack growth.

Of course, it should be noted that this conclusion is valid only for those cracks with $dK/da>0$. Nuismer arrived at a criterion in agreement with that reached by Graham[7]. However, in his derivation he used an expression for the crack surface displacement [6,Eq.(2)], which is based on the quasi-static theory and implies that the crack moves immediately as the stress is applied. As we have mentioned above, for idealized viscoelastic cracked bodies, the crack-tip will not move until $t = t_c$, and once t reaches t_c [6, Eq.(2)] is not applicable for those cracks with $dK/da>0$. Thus, his result is questionable.

Since when $t \leq t_c$ the crack problem does not involve a time-dependent boundary region, the viscoelastic correspondence principle is then applicable only for $t \leq t_c$. By using this principle and the equation of the thermodynamic power balance, the main results of LEFM have been generalized to the idealized viscoelastic cracked bodies.

CRITERION FOR DALAYED CRACK INSTABILITY

Consider a fixed region R occupied by a linear viscoelastic material. Let us restrict our consideration to infinitesimal displacement and velocity. Thus the partial derivative with respect to time is equal to the material derivative within the first order and the difference between the Eulerian and Lagrangian strain tensor can be neglected. Let $u_i(x,t)$, $\varepsilon_{ij}(x,t)$ and $\sigma_{ij}(x,t)$ denote the Cartesian components of the displacement, infinitesimal strain and stress respectively, which are defined for all (x,t) on $R\times[0,\infty)$. The constitutive relations describing the deformation of isotropic linear viscoelastic solids may be written in the form of the creep integral law

$$e_{ij}(x,t) = s_{ij}*dJ_1 = s_{ij}(x,t)J_1(0)+\int_0^t s_{ij}(x,t)\partial J_1(t-\tau)/\partial(t-\tau)d\tau$$
$$\varepsilon_{kk}(x,t) = \sigma_{kk}*dJ_2 = \sigma_{kk}(x,t)J_2(0)+\int_0^t \sigma_{kk}(x,t)\partial J_2(t-\tau)/\partial(t-\tau)d\tau \quad (1)$$

where e_{ij} and s_{ij} denote the derivatoric components of strain and stress respectively, and $*$ denotes the Stieltjes convolution. And

$$e_{ij}(x,t) = \varepsilon_{ij}(x,t)-\frac{1}{3}\delta_{ij}\varepsilon_{kk}(x,t), \quad s_{ij}(x,t) = \sigma_{ij}(x,t)-\frac{1}{3}\delta_{ij}\sigma_{kk}(x,t) \quad (2)$$

under uniaxial load the constitutive equation reduces to

$$\varepsilon(x,t) = \sigma*dD = \sigma(x,t)D(0)+\int_0^t \sigma(x,t)\partial D(t-\tau)/\partial(t-\tau)d\tau \quad (3)$$

J_1, J_2, D are the creep functions in shear, in isotropic compression and in uniaxial tension respectively. $\phi_s(t) = \partial J_1(t)/\partial t$, $\phi_d(t) = \partial J_2(t)/\partial t$, $\phi_1(t) = \partial D(t)/\partial t$ are the corresponding creep kernels. ν, μ, k, E stand for Poisson's ratio, the instantaneous shear, bulk and Young's moduli respectively while $1/(2\mu) = J_1(0)$, $1/(3k) = J_2(0)$ and $1/E = D(0)$. Let the Laplace transform of u be written as u^*. According to the correspondence principle, the necessary substitution of material constants are[1-3]

$$2\mu^* = 1/[pJ_1^*(p)] = 2\mu/(1+2\mu\phi_s^*), \quad E^* = 1/[pD^*(p)] = E/(1+E\phi_1^*) \quad (4)$$

Since the crack fields of elastic bodies are known, the corresponding viscoelastic solution can be obtained by an inversion[3]. Using these crack-tip fields and the equation of the thermodynamic power balance and following Irwin's method for caculating the energy release rates G(t) at any fixed time t, we arrived at a result in the form

$$G_i(t) = G_i(0)f_{ig}(t), \quad f_{ig}(t) = f_{i\sigma}(t)f_{iu}(t) \qquad (i = I,II,III) \quad (5)$$

which is different from that of Nuismer[1]. Here $f_{ig}(t)$, $f_{i\sigma}(t)$, $f_{iu}(t)$ are the time factors for the energy release rates, stress intensity factors and

displacement components respectively. It can be shown that if the applied stress is assigned and if the stess intensity factors can be written in the form

$$K_i(t) = K_i f_{i\sigma}(t) \qquad (i = I,II,III) \quad (6)$$

then

$$f_{iu}(t) = f_{i\sigma}(t)*L^{-1}(E'/E^{*\prime}) = [f_{i\sigma}(t)*dC'(t)]/C'(0) \quad (i = I,II)$$
$$f_{IIIu}(t) = f_{III\sigma}(t)*L^{-1}(\mu/\mu^*) = [f_{III\sigma}(t)*dJ_1(t)]/J_1(0) \quad (7)$$

where C'(t) denotes the creep function for plane stress D(t) or that for plane strain C(t), which are defined through

$$pC'^*(p) = \begin{cases} 1/E^* = pD^*(p), & \text{for plane stress} \\ \dfrac{1-\nu^{*2}}{E^*} = \dfrac{1}{2\mu^*}\left(2-\dfrac{E^*}{2\mu^*}\right) = pJ_1^*(p)\left[2-\dfrac{J_1^*(p)}{D^*(p)}\right] & \text{for plane strain} \end{cases} \quad (8)$$

so that

$$C'(t) = \begin{cases} L^{-1}D^*(p) = D(t) & \text{for plane stress} \\ L^{-1}[J_1^*(p)(2-J_1^*(p)/D^*(p)] = C(t) & \text{for plane strain} \end{cases} \quad (9)$$

where

$$L^{-1}[F(p)] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}F(p)e^{pt}dp$$

is the inverse Laplace transformation of F(p), the constant c being any number greater than the abscissa of convergence.

Let us write down the fracture criterion for mode I Griffith crack of length 2a under in-plane loading $\sigma f_\sigma(t)$ applied at infinity:

$$G_I(t) = G_I(0)f_{Ig}(t) = G_{Ic} \quad (10)$$
$$f_{Ig}(t) = f_{I\sigma}(t)[f_{I\sigma}(t)*L^{-1}(E'/E^{*\prime})] = f_{I\sigma}(t)[f_{I\sigma}(t)*dC'(t)]/C'(0) \quad (11)$$

where $G_I(0) = (\sigma^2\pi a)/E'$ is the instantaneous energy release rate. For a given material G_{Ic} is assumed to be a constant. For those cracks with $dK/da>0$, if $G_I(0) = G_{Ic}$, the limiting equilibrium of the crack will become unstable and the crack will propagate immediately as the load is applied. If $G_I(\infty) = G_I(0)f_{Ig}(\infty)<G_{Ic}$, the crack will not propagate forever. If $G_I(t_c) = G_I(0)f_{Ig}(t_c) = G_{Ic}$, then at a certain time t_c, delayed instability will occur. The critical state for delayed instability is determined by the condition

$$G_I(\infty) = f_{Ig}(\infty)(\sigma^2\pi a)/E' = G_{Ic} \tag{12}$$

If the crack size/load is assigned. the critical load/crack size of delayed crack instability may be determined from Eq.(12) provided $f_{Ig}(t)$ is known. If the load/crack size is smaller than this critical value, the state of delayed crack instability can never be reached. For Maxwell bodies, standard linear bodies and Burgers bodies, the time factors are given in the tables of [1,2].

In particular, if $f_{I\sigma}(t) = U(t)$ is the unit Heaviside step function, according to the theory of final value and Eq.(11), (9), (4), we obtain

$$f_{Ig}(\infty) = \begin{cases} \lim\limits_{t\to\infty} D(t)/D(0) = \lim\limits_{p\to 0} pD^*(p)/D(0) = \lim\limits_{p\to 0} (1+E\phi_1^*), & \text{for plane stress} \\ \lim\limits_{t\to\infty} C(t)/C(0) = \lim\limits_{p\to 0} pC^*(p)/C(0) & \\ \quad = \lim\limits_{p\to 0} pJ_1^*(p)[2-J_1^*(p)/D^*(p)] & \\ \quad = \lim\limits_{p\to 0} (1+2\mu\phi_s^*)[1+\frac{1+\nu}{1-\nu}(1-\frac{1+2\mu\phi_s^*}{1+E\phi_1^*})] & \text{for plane strain} \end{cases} \tag{13}$$

Generalized Kelvin Solids

The shear and uniaxial creep functions take the form

$$\begin{aligned} J_1(t) &= J_1(0)[1+\sum_{i=1}^{m} \mu W_i(1-e^{-\beta_i t})] \\ D(t) &= D(0)[1+\sum_{i=1}^{n} EC_i(1-e^{-\lambda_i t})] \end{aligned} \tag{14}$$

From Eq.(13), we have

$$f_{Ig}(\infty) = \begin{cases} 1+E\sum_{i=1}^{n} C_i & \text{for plane stress} \\ (1+\mu\sum_{i=1}^{m} W_i)[1+\frac{1+\nu}{1-\nu}(1-\frac{1+\mu\sum_{i=1}^{m} W_i}{1+E\sum_{i=1}^{n} C_i})] & \text{for plane strain} \end{cases} \tag{15}$$

The final value of time-factor for standard linear solid may be obtained from Eq.(15) by setting $m = n = 1$. If the Poisson's ratio ν remains constant, time-factors for plane strain will reduce to the same results as those for the plane stress.

Solids for Which Creep Kernel Has a Singularity at $t = 0$

Sometimes better results can be obtained if we take the creep kernel in the form

$$\phi_I(t) = Ae^{-\beta t}/t^{1-\alpha}, \quad 0<\alpha<1, \quad A>0, \quad \beta>0 \tag{16}$$

Substituting Eq.(16) into Eq.(13) yields

$$f_{Ig}(\infty) = 1+EA\Gamma(\alpha)/\beta^{\alpha} \qquad \text{for plane stress} \tag{17}$$

where $\Gamma(\alpha)$ is the Gamma function.

CONCLUSIONS

By using the viscoelastic correspondence principle, the main results of LEFM have been generalized to the idealized viscoelastic cracked bodies. It is shown that

1. The stress intensity factors, the displacements and the energy release rates for viscoelastic cracked bodies can be obtained by mutiplying those for elastic cracked bodies by some time-factors. These factors may be found in [1,2].

2. For an idealized linear viscoelastic cracked body, there is no slow stable crack growth under constant load even for those cracks with $dK/da>o$. The equalibrium of a crack may become unstable instantaneously or at a certain time t_c after the application of stress, depending upon the magnitude of the applied stress (crack size) when the crack size (applied stress) is assigned. t_c may be determined from Eqs. (10) and (11), it depends on the entire history of loading and the time-dependent behaviour of the material.

3. If the creep function of a material possesses an upper limit, then there exists a critcal load (crack size) of delayed crack instability. From Eqs. (13), (15), and (17) it is seen that this critical load/crack size does not depend upon the intermediate values of $J_1(t)$, $D(t)$ and is comple-

tely determined by their final values.

4. For Maxwell and Burgers bodies $G_I(t)$ will increases with time unboundedly because of the existance of viscous flow. In this case, conditional critical load/crack size of delayed crack instability may be useful.

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