ON THE CLEAVAGE FRACTURE CRITERION BASED ON STATISTICAL MODEL

Qu Ding (瞿 定) Cai Qigong (蔡其巩)
Central Iron and Steel Research Institute, China

INTRODUCTION

It is commonly accepted that the cleavage resistance can be defined by a single critical stress parameter — the cleavage fracture stress σ_f measured by fracturing a notched specimen at the temperature range of cleavage initiation. However, the cleavage stress measured by notch specimen is apparently higher than that measured by smooth specimen, and the cleavage stress measured by cracked specimen is even higher, if it is defined as the maximum principal stress in crack tip region at fracture. Further, in order to apply the singe parameter σ_f to predict the cleavage K_{1c} of a cracked specimen, the well-known RKR model postulated that the stress σ_1 in front of the crack tip has to exceed σ_f over a characteristic distance X_0 before fracture could occur. This criterion leads to an expression for cleavage K_{1c} : $K_{1c}\sigma_y^{(N-1)/2} = f^{-(N+1)/2}\sigma_f^{(N+1)/2} X_0^{1/2}$, which predicts the variation of cleavage K_{1c} with temperature[1]. However, there is a large scatter band of measured cleavage K_{1c}, which is an intrinsic property of cleavage fracture of structural steels since it exists even it is measured by using specimens with the same size and geometry. In order to use the RKR model to predict the different values of cleavage K_{1c} measured at a fixed temperature, one should adopt different characteristic distance X_0 (see Fig. 1, with X_0=2d—5d[2] for different

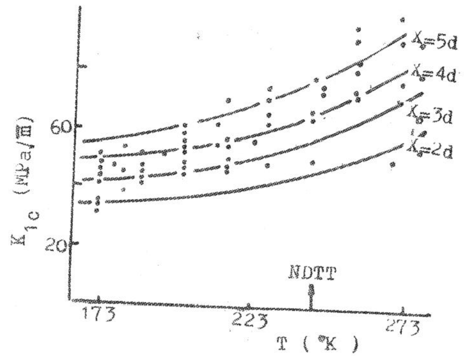

Fig. 1. K_{1c} and the characterisic X_0

specimens, this of course contradicts to the physical interpretation of characteristic distance.

In this paper, a Weibull type cleavage fracture probability function based on an experimentally verified carbide crack size distribution was introduced, the "Weakest Link" theory was then applied to the result to derive the cleavage fracture criterion for notched and cracked specimens. The results of Weibull parameter measured with experimental data were used to analyse the size effect and scatter band of cleavage stress σ_f and cleavage K_{1c} for smooth, notched and cracked specimens in order to show that the criterion is a general and unified one for analysing the cleavage fracture of specimens with different geomtries.

THE WEIBULL STATISTICAL MODEL OF CLEAVAGE FRACTURE BASED ON A GIVEN DEFECT SIZE DISTRIBUTION

By using the same carbide crack size distribution function which was proposed to have the form:

$$f(a) = \frac{c^{n-1}}{(n-2)!} e^{-c/a} a^{-n} \qquad (1)$$

(where a is the semi-crack length, c is a scaling parameter and n is the rate at which the density tends to zero) and the commonly accepted Griffith criterion for cleavage propagation of an inclined carbide crack under the applied principal stress σ_1, a three-dimensional Weibull type expression for the failure probability of the brittle materials has been extended from two-dimensional analysis[3] and the expression for the failure probability $F(\sigma_1)$ of a crack under the applied principal stress σ_1 has been obtained

$$F(\sigma_1) = \frac{1}{(n-1)!(4n-3)} \left[\frac{(1-\nu^2)\pi c\sigma_1^2}{2E\gamma_p}\right]^{n-1} = \left(\frac{\sigma_1}{\sigma_u}\right)^m \qquad (2)$$

with

$$m = 2(n-1) \qquad (3)$$

and

$$\sigma_u = [(n-1)!(4n-3)]^{\frac{1}{m}} \left[\frac{2E\gamma_p}{(1-\nu^2)\pi c}\right]^{\frac{1}{2}} \qquad (4)$$

where γ_p is the effective surface energy of ferrite. Using Eqn. (2), the fracture probability P of a specimen with N crabide cracks under the applied stress σ_1 can be derived based on the weakest link theory:

$$P = 1 - \exp\left[-NF(\sigma_1)\right] = 1 - \exp\left[-\frac{V}{V_u}\left(\frac{\sigma_1}{\sigma_u}\right)^m\right] \tag{5}$$

where $V_u = V/N$ is the average volume shared by a carbide crack and V is the volume of the specimen.

For notched and cracked specimen, the stress field is non-uniform. The fracture probability in notched and cracked specimen is written by

$$P = 1 - \exp\left[-\frac{\int \sigma_1^m dV}{\sigma_u^m V_u}\right]$$

and the statistical criterion of cleavage fracture is therefore

$$\int_{V_p} \sigma_1^m dV = \sigma_u^m V_u \ln\left(\frac{1}{1-P}\right) \tag{6}$$

where V_p is the volume of plastic zone at the notch or crack-tip region, since the carbide cracks only within the plastic zone.

DETERMINATION OF THE WEIBULL PARAMETERS

The above derivation leads to a general cleavage criterion, Eqn. (6) in which the parameters m, σ_u, and V_u are defined by the defect distribution (1) and the material properties by Eqn.(4). They are independent of the specimen geometry. Thus, the Weibull parameters for a steel can be determined by comparing the experimental data of cleavage fracture (σ_f, K_{1c}, etc.) for different specimen geometries (i.e. notched and cracked specimens) through the use of the criterion Eqn.(6).

For cracked specimens, the criterion (6) can be rewritten as:

$$B_{cr}\sigma_y^m\left(\frac{K_{1c}}{\sigma_y}\right)^4 \iint \left[\frac{\sigma_1(u,0)}{\sigma_y}\right]^m \left[\frac{\tilde{\sigma}_1(\theta)}{\tilde{\sigma}_1(0)}\right]^m u\,du\,d\theta = \sigma_u^m V_u \ln\frac{1}{1-P} \tag{7}$$

where B_{cr} is the thickness of cracked specimen, $\sigma_1(u,0)/\sigma_y$ is given by FEM of McMeeking[4], the elastic plastic interface is located at a distances $u = r/\left(\frac{K_{1c}}{\sigma_y}\right)^2 = 0.03$ ahead of the crack tip, the angular factor $\tilde{\sigma}_1(\theta)/\tilde{\sigma}_1(0)$ of maximum principal stress for cracked specimen is calculated by

$$\tilde{\sigma}_\theta(\theta) = \frac{1}{2}\left[\tilde{\sigma}_\theta(\theta) + \tilde{\sigma}_r(\theta)\right] + \left\{\frac{1}{4}\left[\tilde{\sigma}_\theta(\theta) - \tilde{\sigma}_r(\theta)\right]^2 + \tilde{\sigma}_{r\theta}(\theta)^2\right\}^{\frac{1}{2}}$$

with $\tilde{\sigma}_\theta(\theta)$, $\tilde{\sigma}_r(\theta)$ $\tilde{\sigma}_{r\theta}(\theta)$ computed by using the Fourier approximation formula given by Bilek[5].

If we use the mean value of K_{1c}, the criterion (7) can be rewritten as

$$C_{cr} = \left[\frac{\overline{K}_{1c}}{\Gamma(1+\frac{1}{u})\sigma_y}\right]^4 B_{cr}\sigma_y^m \iint \left[\frac{\sigma_1(u,0)}{\sigma_y}\right]^m \left[\frac{\tilde{\sigma}_1(\theta)}{\tilde{\sigma}_1(0)}\right]^m u\,du\,d\theta = \sigma_u^m V_u \tag{8}$$

Similarly, for notched specimens, we have criterion (6) rewritten in the form:

$$\sigma_f^m \frac{B_n\rho^2}{Q^m} \iint \left[\frac{\sigma_1(\xi,0)}{\sigma_y}\right]^m \phi_1^m(\theta)\xi\,d\xi\,d\theta = \sigma_u^m V_u \ln\frac{1}{1-P} \tag{9}$$

and

$$C_n = \left[\frac{\overline{\sigma}_f}{\Gamma(1+\frac{1}{m})}\right]^m \left[\frac{B_n\rho^2}{Q^m}\right] \iint \left[\frac{\sigma_1(\xi,0)}{\sigma_y}\right]^m \phi_1(\theta)\xi\,d\xi\,d\theta = \sigma_u^m V_u \tag{10}$$

where $\xi = 1+x/\rho$, and x is the distance from the notch-tip, $\sigma_1(\xi,0)$ is the maximum principal stress distribution along $\theta=0$ and had been calculated by Griffiths and Owen[6] using FEM. $\phi_1(\theta)$ is angular factor of $\sigma_1(r,\theta)$ and can be estimated by $\phi_1(\theta) = [\cos(\frac{\omega}{2}+\theta)/\cos(\frac{\omega}{2})]^n$ according to Bates[7], here ω is the flank angle of the notch, $Q = \sigma_1^{max}/\sigma_y$ is the stress intensification of notched specimen at fracture. B_n and ρ are the thickness and notch-root radius of the notched specimen.

Now, the Weibull parameter m can be determined by comparing the computing results of C_{cr} and C_n using (8) and (10) and the measured σ_f and K_{1c}, since m, $\sigma_u^m V_u$ are material constants. Thus we should have $C_{cr}=C_n$ at fracture. By computing C_{cr}/C_n for different m values, and finding the intersecting point of ($C_{cr}/C_n \sim m$) curve with the horizontal line $C_{cr}/C_n=1$, the value of m at the intersecting point is then its right value, and the value of parameter $\sigma_u^m V_u = C_{cr} = C_n$ is also determined.

The detail calculation procedures and results will be published elsewhere[8].

RESULTS AND DISCUSSION

Using the cleavage fracture σ_f and the cleavage fracture toughness K_{1c} measured for 4Si-Fe steel (4.10 wt.% Si, 0.067 wt. % C), and the yield strength at the crack tip region σ_y = 529 MPa under the strain rate $\dot{\varepsilon} = 2\varepsilon/t = 1\times10^{-2}s^{-1}$ and temperature T = 243°k, the computed results of C_{cr} and C_n for different values m are shown in table 1. Weibull

Table 1 The variations of the values of C_{cr} and C_n with m in 4Si-Fe

m	Cracked Specimen			Notched Specimen			
	$A = \int_0^{0.03} [\frac{\sigma_1(u,0)}{\sigma_y}]^m u du$	$B = \int_{-\pi}^{\pi} [\frac{\tilde{\sigma}_1(\theta)}{\tilde{\sigma}_1(0)}]^m d\theta$	$C_{cr} = B_{cr}\sigma_y^{m-4}[\frac{\bar{K}_{1c}}{\Gamma(1+\frac{1}{4})}]^4 AB$	$D = \int_1^{5.5} [\frac{\sigma_1(\xi,0)}{\sigma_y}]^m \xi d\xi$	$E = \int_{-\frac{3\pi}{8}}^{\frac{3\pi}{8}} \phi_1^m(\theta) d\theta$	$C_n = [\frac{\bar{\sigma}_f}{\Gamma(1+\frac{1}{m})}]^m \frac{B_n \rho^2}{Q^m} DE$	$\frac{C_{cr}}{C_n}$
21	2.017×10^7	2.35	1.21×10^{58}	1.422×10^9	0.946	2.98×10^{57}	4.06
20	5.603×10^6	2.35	6.34×10^{54}	5.736×10^8	0.971	2.33×10^{54}	2.72
17.4	2.096×10^5	2.36	1.98×10^{46}	5.458×10^7	1.042	1.94×10^{46}	1.02
15	1.055×10^4	2.37	2.91×10^{38}	6.300×10^6	1.120	6.97×10^{38}	0.42
12	2.759×10^2	2.39	5.18×10^{28}	4.310×10^5	1.240	3.49×10^{29}	0.15

where $\bar{\sigma}_f = 1352$ MPa Q = 2.55 $\sigma_y = 529$ MPa T = 243 K

$\rho = 0.25 \times 10^{-3}$ m, $B_n = 12.7 \times 10^{-3}$ m, $\bar{K}_{1c} = 30.5$ $MP_a\sqrt{m}$, $B_{cr} = 10 \times 10^{-3}$ m

parameter were obtained m = 17.4, $\sigma_u^m V_u = 1.96\ 10^{46} MPa^{17.4}\ m^3$

Based on this result, K_{1c} - T curves for different cumulative fracture probability P = 0.1, 0.6 and 0.9 are plotted using the Eqn. (8), the result is show in Fig. 2. The scatter band of K_{1c} with the temperature is obtained.

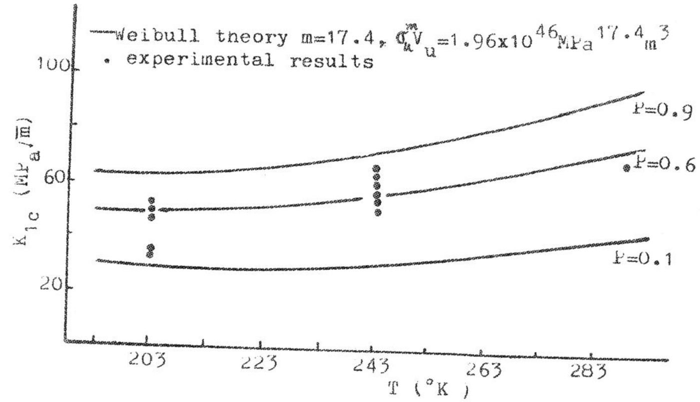

Fig. 2. Comparison of the Weibull model predictions cleavage K_{1c} for 4Si-Fe.

In addition if we define the cleavage stress $\sigma_{f,cr}$ measured by crack specimen as the maximum principal stress $\sigma_1^{max} = R\sigma_y$ in crack tip at fracture, Eqn. (7) becomes

$$\sigma_{f.cr}^m V_{eff.cr} = \sigma_u^m V_u \ln(\frac{1}{1-P}) \tag{11}$$

where

$$V_{eff.cr} \equiv B_{cr}(\frac{K_{1c}}{\sigma_y})^4 \frac{1}{R^m} \iint (\frac{\sigma_1(u,0)}{\sigma_y})^m [\frac{\tilde{\sigma}_1(\theta)}{\tilde{\sigma}_1(0)}]^m u du d\theta \tag{12}$$

Similarly for notched specimens we have:

$$\sigma_{f.n}^m V_{eff.n} = \sigma_u^m V_u \ln(\frac{1}{1-P}) \tag{13}$$

where

$$V_{eff.n} \equiv \frac{B_n \rho^2}{Q^m} \iint [\frac{\sigma_1(\xi,0)}{\sigma_y}]^m \phi_1^m(\theta) \xi d\xi d\theta \tag{14}$$

The computed result is shown in Fig. 3, where the straight lines of V_{eff} versus σ_f for different cumulative fracture probability (P=0.1, 0.6, 0.9) were drawn using the Weibull parameters m and $\sigma_u^m V_u$ determined by the experimental results of notched and cracked spcimens (Table 1).

In Fig. 3. the values of cleavage stress σ_f measured on smooth specimens of different volume $V=V_{eff}$ were also dotted. It is evident, although some deviation of experimental points for smooth specimen from the predicted scatter band can not be neglected, the remarkable size or volume effect and scatter band of cleavage stress measured by cracked, notched and smooth specimens can be successfully interpreted by using the above statistical cleavage frecture criterion. The reason of the diviation of the locations of points for smooth specimens from the straight line scatter band is not clear and needs further investigation. However, it is suggested that the difference in stress state of smooth and notched or cracked specimens, and also the error of mathematical approximation used in Weibull statistics especially for cracked specimen may have some effect on the above deviation.

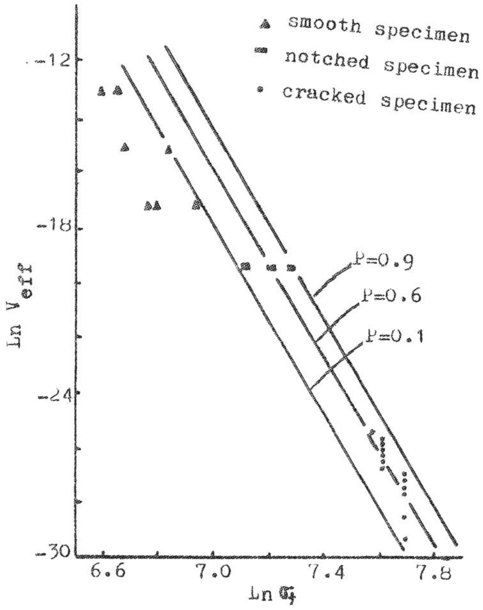

Fig.3. Ln σ_f v.s. Ln V_{eff} for different specimens, in 4Si-Fe.

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