CRITERION OF "MAXIMUM FAILURE RATIO" AND FAILURE AT THE CORNER OF THE SQUARE VOID

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Many studies have been made on the damage and fracture of materials, and various involved criterions have been proposed[1,2].

In actual, the damage and fracture of materials depend on the size, quantity and orientation of various defects which are microcrack, void and the dislocation of crystal, etc. In such a study the growth of the defects contained in a solid body was considered. Now this theory can be used to consider not only the elastoplastic damage caused by both creep and fatigue, but also the damage of brittle materials[3,4].

Based on experiments[5], in this paper it is described that since the nonlinearity and nonsymmetry of the stress-strain relation of brittle material result from the presence of various internal defects, they therefore can be used to represent the degree of the damage and failure of material. Based on such a characteristic, in 1981 we proposed a new fracture criterion[6] in terms of "maximum fracture factor or failure ratio". In this paper, it will be described that the criterion based on the "maximum failure ratio" is able to represent the degree of the damage or failure induced in material and the form and size of local fracture region. It can also determine the fracture behavior and the direction. Applying this new criterion, the failure at the corner of square void under a compression load was analyzed, and the result agreed with the experiments well.

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NONLINEARITY AND NONSYMMETRY OF THE STRESS-STRAIN RESPONSE

Usually, the constitutive equation of metals, σ—ε relation, is expressed symmetrically with regard to the origin and hold the linearity in the elastic range.but there are many materials that show nonlinear σ-ε relation and do not show the symmetry in actual. In such a case, the constitutive equation can be expressed as $\sigma/\varepsilon = E_p$ by introducing a new elastoplastic modulus, E_p.

The experimental results of marble specimens under the uniaxial compression and tension[5] showed that: (1) first, with the increase of compressive stress from 0 to a certain extent, E_p gradually increased mainly owing to the change of voids. Then the trend changed to decrease until failure. (2) When a tension loading increased from no load, accordingly, E_p decreased gradually until failure of a specimen.

In general, both E_p and strength under tension are less than that under compression. This is considered to imply the defects such as voids and microcracks have much stronger influence on the strength of materials under tension than under compression.

The plastic deformation of crystalline materials is mainly connected with the motion of dislocations caused by shear stress. In the plastic deformation region, constitutive equation includes strainhardening during the plastic deformation and can be expressed as $d\sigma/d\varepsilon_p=H'$, where ε_p is plastic strain. However brittle materials are broken out without clear yield point. In this paper, the maximum of E_p is defined as pseudoyielding point, where tangent elastic modulus E is approximately considered as the elastic constant of perfectly elastic materials.

Furthermore, E_t and E_c are used as secant elastoplastic modulus under the maximum tension and compression respectively. Now the tension defect factor and compression defect factor are defined as following.

$$D_t = E_t/E \tag{1}$$

$$D_c = E_c/E \tag{2}$$

If E can be considered as the elastic modulus of perfectly elastic materials, D_t and D_c would express the degree of nonlinearity of $\sigma-\varepsilon$ relation in the case of tension and compression respectively, which may be treated as the degree of the effect combining defects. Accordingly, the difference between D_t and D_c would express the nonsymmetery difference in the $\sigma - \varepsilon$ relation for the cases of tension and compression, i.e. the difference in defect-combining effect for solids $D_t = D_c = 1$, when materials with defects $D_t < 1$, and $D_t \leq D_c$.

A $\sigma - \varepsilon$ relation under three axial stress can be expressed as following

$$\varepsilon_i = \sigma_i/E_i - (\nu_j\sigma_j/E_j + \nu_k\sigma_k/E_k) \tag{3}$$

where $i = 1,2,3$, $j = 1,2,3$, $k = 1,2,3$, but $i \neq j \neq k$

Elastoplastic modulus E and Poisson ratio ν are not the simple constant but are variables depending on the stress state. Therefore the stress analysis is very difficult. For reducing three constant E, D_t and D_c have been used to calculate the value close to the ultimate failure.

CRITERION OF "MAXIMUM FAILURE RATIO"

The failure of materials can be classified into three types, that is separation fracture due to tension, compression damage due to compression and shear failure due to shear. Now the strength of tension, compression and shear or pseudo-yield point are expressed by σ_t, σ_c and τ_s or σ_y respectively. And then the pseudo-yield ratio F_y is defined by the ratio of the principal strain (or stress) to the strain (or stress) at the pseudo-yield point, and the fracture ratio F is defined by the ratio of the principal strain (or stress) to the critical strain (or strength) for failure. These F_y and F are named failure ratio.

The principal strain is $\varepsilon_i = (\sigma_i - \nu\sigma_j - \nu\sigma_k)$ below the elastic limit. The strain at the pseudo-yield point is $\varepsilon_y = \sigma_y/E$. Thus the pseudo-yield ratio is

$$F_y = \varepsilon_i/\varepsilon_y = (\sigma_i - \nu\sigma_j - \nu\sigma_k)/\sigma_y \tag{4}$$

Now the major principal strain is:

$\varepsilon_1 = (\sigma_1 - \nu\sigma_2 - \nu\sigma_3)/E$ for $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$. The critical strain for failure under tension is given by $\varepsilon_t = \sigma_t/E \cdot D_t$. Thus the fracture ratio in the case of tension is expressed as

$$F_t = \varepsilon_1/\varepsilon_t = (\sigma_1 - \nu\sigma_2 - \nu\sigma_3)D_t/\sigma_t \tag{5}$$

Similarly for $\varepsilon_3 < 0$ the compression fracture ratio is expressed

$$F_c = \varepsilon_3/\varepsilon_c = (\sigma_3 - \nu\sigma_1 - \nu\sigma_2)D_c/\sigma_c \tag{6}$$

Shear fracture ratio is as following,

$$F_s = [D_t/(D_t+1) \sim D_c/(D_c+1)](\sigma_1 - \sigma_3)/\tau_s \tag{7}$$

If the fracture of a solid strongly depend on the mechanical condition at a crack tip, we can apply fracture mechanics and the toughness fracture ratio whcih is given by

$$F_k = K_i/K_{ic} \tag{8}$$

where K_i is stress intensity factor,

$K_i = Y\sqrt{a}$ and K_{ic} is fracture toughness, where i = I, II, III

In summary, we obtained the criterion named Maximum Failure Ratio as following:

1) If $F_t \geq 1$ and $F_t > F_c$, $F_t > F_s$, then the maximum tension would cause the separation fracture and the orientation of which would be normal to the direction of the maximum principal stress.

2) If $F_c \geq 1$ and $F_c > F_t$, $F_c > F_s$, then the maximum compression would cause the compression damage.

3) If $F_s \geq 1$ and $F_s > F_t$, $F_s > F_c$, then the shear stress would cause shear failure of which orientation would be in the direction along shear stress.

4) If $F_t \approx F_c \approx F_s \approx F_k \geq 1$, then possibly all kinds of failure would happen at the same time.

In short, the failure would initiate wherever the failure ratio is the maximum no matter what kind of failure ratio it is. Therefore the Criterion is named Maximum Failure Ratio or "F_{max} Criterion" in short.

FAILURE ANALYSIS AT THE CORNER OF SQUARE VOID

According to the experimental results, generally, failure initiated from corners of a square void under compression. Now we can apply the F_{max} criterion to analyze the failure and damage of PMMA, plaster and marble specimens with a square void.

Table 1 show the mechanical properties of specimens used in this study. The distribution of principal stress near the corners is shown in the Reference[8].

Table 1. Mechanical properties of specimens

Index of strength	PMMA	Plaster	Marble
Pseudo-yielding point σ_y(MPa)	120	2	43.5
Compressive strength σ_c(MPa)	340	4	47.5
Tensile strength σ_t(MPa)	76	2	3.5
Shear strength τ_s(MPa)	150	3	3.5
Tangent-elastic modulus E(GPa)	2	4	59
Secant-elastoplastic modulus under the tension E_t(GPa)	1.2	2.67	10
Secant-elastoplastic modulus under the compression E_c(GPa)	0.68	2	49.5
Strain-hardening rate H'(GPa)	0.5	2	25
Tension defect factor D_t	0.6	0.67	0.17
Compression defect factor D_c	0.34	0.67	0.83
Poisson's ratio	0.25	0.2	0.1-0.2

Fig. 1 shows the contour of F, which combining F_t, F_c and F_s in the cases of PMMA, plaster and marble specimens respectively. The numerical values on the contour line implies the dangerous degree and local fracture will be caused in the region at $F \geq 1$.

According to our calculations, we found that for the PMMA specimens F_{max} is F_t, then the separation fracture would happen; for plaster F_{max} is F_c, then the compression damage would happen; for marble F_{max} is $F_c \approx F_t \approx F_s$, therefore all kinds of failure may happen at the same time. In addition to this, Fig. 1 also represents the sizes and forms of the local fracture region, and the orientation of the failure caused by the principal stress.

DISCUSSION AND CONCLUSIONS

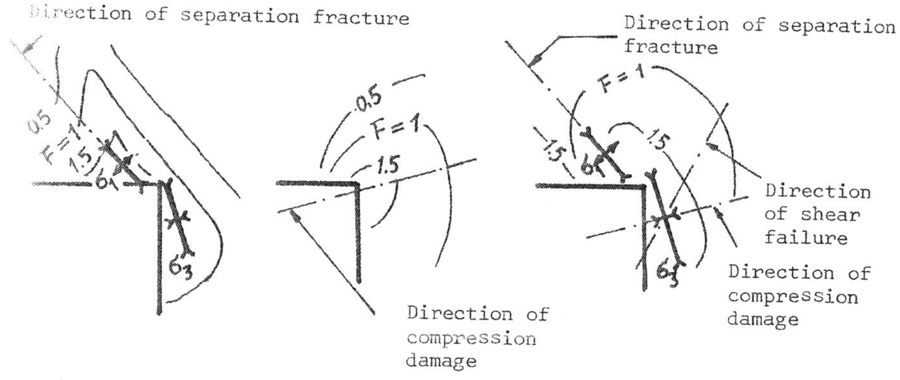
The results of our calculations using F_{max} criterion agree with that from experiments [7] well, which can be concluded as following.

1) When strength characteristic of materials is not governed by a single well-developed crack or with materials not sensitive to the crack initiation or under the condition of compression, it is of great significance to treat the nonlinearity and nonsymmetry of σ-ε relation as the effects combining all kind of defects.

2) There are no rateable differences among the stress distributions and the F_y distributions of PMMA, plaster and marble. So distinctions in material fracture can not be described only by stress distributions and yield criterion. The notable distinction among F distributions can be used to describe the distinctions in material fracture well.

3) Newly proposed "F_{max} criterion" has considered the change of σ-ε response owing to various defects and the change of strength under different stress. F_{max} Criterion implied that the failure would initiate at the point wherever the failure ratio takes the maximum, F_{max}, among all kinds of F_t, F_c, F_s and F_k.

F_{max} criterion can be used to describe failure degree and form and determine the size and form of local failure region. The new criterion has advantages of simplicity and conformity to experiments.



(a) PMMA (b) Plaster (c) Marble

Fig. 1 Contour of F

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