A SIMPLE AND DIRECT METHOD FOR CALCULATING FATIGUE CRACK PROPAGATION RATE USING LOW CYCLE FATIGUE PROPERTIES

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INTRODUCTION

In recent years, many investigators have devoted themselves to the study of the relationship between fatigue crack propagation rate (FCPR) and low cycle fatigue (LCF). The original work of relating FCPR to LCF using cyclic stress-strain was Ref. [1] by B. Tomkins. The further analysis was carried out in Ref. [2]. In Refs. [3] and [4], the effect of the microstructural parameter ρ^* on the FCPR was introduced based on Liu and Iino's model [5]. Using Liu and Iino approach, Chakrabortty[6] introduced a concept of microstructural deformation zone (MDZ) and assumed a distribution for calculating plastic strain range in the MDZ. Refs. [7] and [8] expressed the FCPR by J-integral and calculated the FCPR by using the LCF properties; several fracture mechanics models of LCF specimen were compared in Ref.[8].

Following Ref. [6], this paper modifies Liu and Iino's model by introducing the microstructural size. Instead of the plastic strain the life of the fatigue failure of each fatigue element ahead of the orack tip is calculated directly. Then, according to the cumulative damage law, an expression for FCPR can be written as follows

$$\frac{da}{dN} = \sum_{i=1}^{n} \lambda_i \left[\frac{1}{N_f(x_i)}\right]$$

In this paper, the results for steel, Al-alloy and Ti-V alloy etc. have been calculated. The comparison between experimental and calculated results shown excellent agreement.

RELATIONSHIP BETWEEN DISTANCE X FROM CRACK TIP AND LIFE OF FATIGUE FAILURE

In LCF, the plastic strain dominates over the life of fatigue failure. However, in general case, the fatigue resistance is usually considered to consist of elastic and plastic parts, so it is necessary to consider the effect of both parts on the life of fatigue failure. The relationship of elastic and plastic parts to the life of fatigue failure are

$$\Delta\varepsilon_e/2 = \left(\frac{\sigma_f'}{E}\right)(2N_f)^b \qquad (1)$$

$$\Delta\varepsilon_p/2 = \varepsilon_f'(2N_f)^c \qquad (2)$$

where $\Delta\varepsilon_e/2$ is the elastic strain amplitude, $\Delta\varepsilon_p/2$ is the plastic amplitude, E is the elastic modulus and b, c, ε_f', σ_f' are material constants, N_f is the number of cycles to failure.

Assuming that the relationship between the total strain amplitude and life can be obtained by linear summation of eq. (1) and eq. (2), it then follows

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \qquad (3)$$

that is

$$\Delta\varepsilon = 2\left[\frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c\right] \qquad (4)$$

Noting

$$\Delta\sigma\cdot\Delta\varepsilon = \frac{\Delta K^2}{(1+n')\pi E X} \qquad (5)$$

and

$$\frac{\Delta\sigma}{2} = \sigma_f'(2N_f)^b \qquad (6)$$

multiplying the both sides of eq. (4) by $\Delta\sigma$, and substituting (5) and (6) into (4), we get

$$\frac{\Delta K^2}{(1+n')\pi E X} = 4\left[\frac{\sigma_f'^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}\right] \qquad (7)$$

From (7), the distance X from the crack tip can be expressed as follows

$$X = \frac{\Delta K^2}{4\pi(1+n')} \Big/ \left[\sigma_f'^2(2N_f)^{2b} + \sigma_f'\varepsilon_f' E(2N_f)^{b+c}\right] \qquad (8)$$

Under the cyclic loading the reversal plastic zone (RPZ)[3,4] is

$$R_p = \frac{\Delta K^2}{4\pi(1+n')\sigma_y'^2} \qquad (9)$$

Then the eq. (8) can be written as follows

$$X = R_p\cdot\sigma_y'^2 \Big/ \left[\sigma_f'^2(2N_f)^{2b} + \sigma_f'\varepsilon_f' E(2N_f)^{b+c}\right] \qquad (10)$$

DESCRIPTION OF FCPR MODEL

It is assumed that the RPZ ahead of a propagation crack tip is composed of a row of small strips of material called fatigue elements which undergo uniaxial cyclic plastic deformation.

The damage of elements depends on the distance from the crack tip. It is assumed that after N cycles the crack tip advances a distance Δx which is just equal to the spacing between two elements. This means that the cumulative damage required to cause fatigue failure of an element is equivalent to sum of the damage experienced during one cycle by each element lying within the RPZ. Let $N_f(x_i)$ be the fatigue life of an element and use linear fatigue damage summation, the failure of the element will occur when

$$\sum_{i=1}^{n} \frac{N}{N_f(x_i)} = 1 \qquad (11)$$

where n is the number of fatigue elements.

Assuming that ΔK is a constant in the propagation process, the crack advances a constant amount per cycle, that is, $\Delta x/\Delta N = da/dN$. Noting that $N=\Delta N$, $\Delta x = x_i - x_{i-1} = \lambda_i$ as shown in Fig. 1, and multiplying the right side of eq. (11) by da/dN and the left side by $\Delta x/\Delta N$, we obtain the following expression

$$\frac{da}{dN} = \sum_{i=1}^{n} \left(\frac{da}{dN}\right)_i = \sum_{i=1}^{n} \lambda_i \left[\frac{1}{N_f(x_i)}\right] \tag{12}$$

where λ_i represent the size of each of fatigue elements. For simplicity, let a_0 be average size of each fatigue element which divides the RPZ into a number of fatigue elements. The first fatigue element starts at the crack tip and ends at the boundary of the next element. The average λ_i size may be written as (Fig. 1)

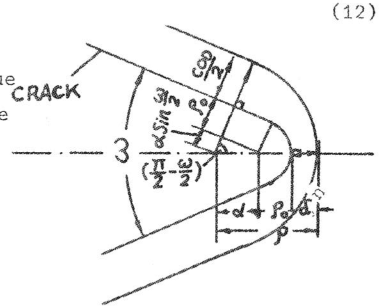

Fig. 1 Model for crack propagation

$$\lambda_i = \begin{cases} a_0/2 & i = 1 \\ a_0 & i = 2,3,\ldots \text{etc.} \end{cases}$$

where $a_0 = C_m \cdot E/(\sigma_y')^2$[7]; $C_m=0.24$ when units of E and σ_y' are in lb/in^2, and $C_m=4.29\times10^{-3}$ when units of them are in kg/mm^2.

In order to estimate the values of $N_f(x_i)$, following assumptions are made:

1. The crack opening displacement (COD) is calculated by using K_{max} which is equal to ΔK, when R=0. When $R \neq 0$, it will be written as

$$COD = \frac{\Delta K^2}{M\sigma_y' E}\left(\frac{1}{1-R}\right) \tag{13}$$

It is assumed that under the cyclic loading, only possitive part can contribute to COD, i.e., in the case of R= -1, the COD is the same as that in the case of R=0. Here M is a factor which depends on the strain state. According to [10], we take M=1.7 for plate specimen with central crack; M=2.0 for bar specimen or others.

2. Let the radius of the crack tip be ρ_0, and ρ for the blunted end of the crack, the relation between ρ, ρ_0 and COD is given as follows[11]

$$\begin{aligned} \rho &= \alpha + \rho_0 + \delta \\ &= \alpha \sin\left(\frac{\omega}{2}\right) + \rho_0 + \frac{COD}{2} \end{aligned} \tag{14}$$

where α. δ and ω are defined in Fig. 2. The ideal sharp crack is the crack

with $\omega=0$ and $\rho_0=0$; i.e., $\rho=COD/2$.

In general, either ρ_0 or ω is not equal to zero, that is

$$\rho = C_\rho . COD \tag{15}$$

where C_ρ is a constant which should be determined by experiment. For simplicity of calculation, $C_\rho=1$ has been taken in this paper.

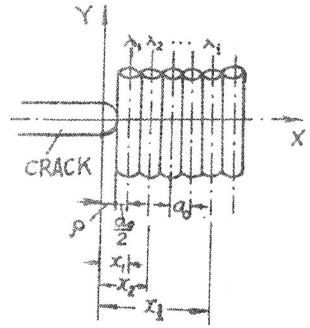

Fig.2 Scheme for COD analysis

Therefore the distance of each fatigue element from the crack tip is

$$X_i = COD + \sum_{i=1}^{n} \lambda_i \tag{16}$$

PROCEDURE OF CALCULATION AND RESULTS

The calculations for annealed M300, 2024–T351, 7075–T6, Al-6Zn-2Mg and Ti-24V have been carried out. The LCF properties used in analysis for these materials are listed in Table 1.

Table 1 Fatigue properties of materials*

Property	M300 (annealed)	2024–T351	7075–T6	Al-6Zn-2Mg	Ti-24V
E ksi(GPa)	26000(179)	10200(70.4)	10300(71.1)	10200(70.4)	16000(110)
a_0 in(μm)	.001(25.4)	.00072(18.3)	.00044(11.2)	.00028(7.11)	.00025(6.4)
b	-0.053	-0.047	-0.0451	-0.049	-0.048
c	-0.83	-0.52	-0.52	-0.70	-0.80
σ_f' ksi(MPa)	190(1311)	98.6(681)	113.3(782)	131(904)	248(1712)
ε_f'	2.0	0.21	0.19	0.55	0.65
n'	0.09	0.098	0.088	0.07	0.06
K'ksi(MPa)	200(1380)	117.1(808)	113.2(781)	185(1277)	181(1249)
σ_y'ksi(MPa)	85(587)	60(414)	75(518)	94(649)	124.6(860)

* Data of the first three materials are taken from Ref. [7], the others from Ref. [9] and [6].

The procedure of calculation is as follows:

1. According to eq. (10) and giving a series of values of $2N_f$ a priori, calculate X_i which corresponds to $2N_f$ and plot the X_i-$2N_f$ curves with different given values of ΔK. As an example, a X_i-$2N_f$ curve for M300 is shown in Fig. 3.

2. Using eq. (9) and size of a_0, estimate the number of fatigue elements lying within RPZ.

3. From eq. (13), evaluate the value of COD.

4. Following eq. (16), calculate the distance X_i from crack tip.

5. From X_i-$2N_f$ curves, find the life of fatigue failure $N_f(X_i)$.

6. Finally, according to eq. (12) and predicting $(da/dN)_i$ for each fatigue element, summarize them to obtain (da/dN).

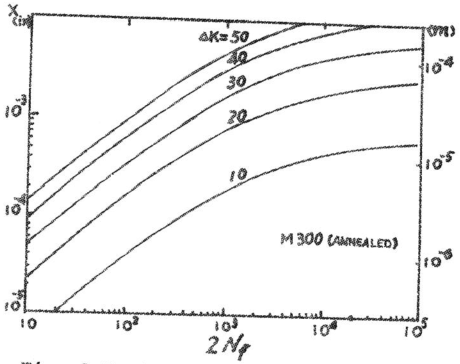

Fig. 3 X_i-$2N_f$ curves for M300

The comparison between calculated and experimental results for five materials are shown in Fig. 4(a)–4(e).

CONCLUSIONS

1. Only the elements near the crack tip make the main contributions to FCPR.

2. The model proposed in this paper has been applied to steel, Al-alloy and Ti-alloy. The agreement between experimental data and calculated results is excellent.

3. The method of calculation in this paper is very simple and direct.

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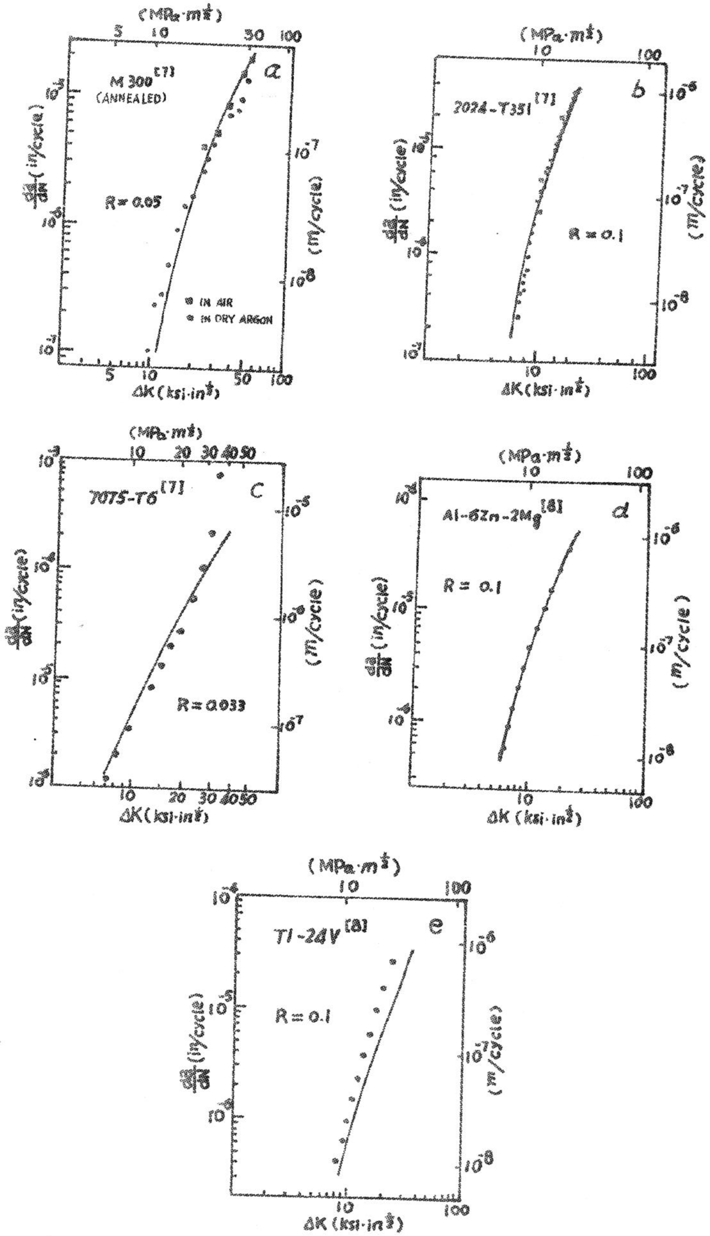

Fig. 4 Comparison of calculated and experimental results for (a) M300 (annealed), (b) 2024-T351, (c) 7075-T6, (d) Al-6Zn-2Mg, (e) Ti-24v

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