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INTRODUCTION

Fracture mechanics principles were introduced in the analysis of fatigue crack propagation relating the crack growth rate (da/dN) to the stress intensity factor range ΔK and the cycle ratio R (σ min/ σ max). Using these laws crack growth rates can be predicted accurately for some simple crack configurations subjected to constant amplitude loading. However, for more complex loading sequences such as flight simulation loading, the results are conservative by a factor 3 to 10 or more $\begin{bmatrix} 1 \end{bmatrix}$. In variable amplitude loading several interaction effects are to be considered to predict correctely the fatigue crack growth. Interaction effects are well manifested in the observed phenomena defined as retardation or acceleration of crack growth. This implies that the crack extension in a load cycle Δa will depend on what occurred in the preceding cycles. Δa will depend on such factors as crack tip blunting, shear lip development, crack closure, cyclic strain hardening and residual stresses around the crack tip, factors produced by a preceding load history [2, 3]. Schijve [2] has classified the various types of loading in five main groupes namely: overloads, step loading, programmed block loading, random loading and flight simulation loading. Moreover, he $\left[4\right]$ simplified these groups in two main categories : 1. Stationary variable amplitude loading where the sequence of load cycles is repeated exactly and regularly; 2. Non stationary variable amplitude loading with no sequencial repetition of a block of different load cycles. Elber [5] defined a short spectrum to be one where crack growth during one repeated interval is less than the plastic zone size created by the highest load in the spectrum. This study [6] was undertaken to investigate the crack growth rate behaviour in 2124 T

351 and 2618 AT 651 aluminium alloys under variable amplitude loading. Programmed block loading was chosen to simulate some simple flight types of the standardized load sequence for flight simulation tests on Transport Aircraft wing structure (TWIST [7]).

The fact that the chosen block loadings did not introduce any interaction effects (retardation) and consequently a linear damage accumulation, implies that the fatigue crack growth depends essentially on the stress intensity factor and the cycle ratio R. The good agreement between the experimental results and a cycle by cycle calculation has directed us to determine the parameters of an equivalent constant amplitude sequence that replaces the block loading in the crack growth calculations. The equivalent constant amplitude concept developed here is based on the physical aspect of damage and on crack closure phenomena.

TEST PROGRAM

The test program was designed so that the crack growth under different cycle ratios R would be investigated separately. The test matrix was defined in terms of three levels of loading A, B and S with R ratios 0.01, 0.63 and 0.45 respectively.

The patterns and loading values for the programmed block loading are shown in fig. 1. Type A is considered as a simple non disturbed flight and this represents the Ground Air Ground (G.A.G.) cycle. The types C/n (n takes values 2, 3, 6, 25 and 69) however, represent disturbing loads (gust, manoeuvre, etc...). The same definition is used for (S) and (B) cycles in types D and E.

EXPERIMENTAL PROCEDURE

The specimen geometries were: 1. Compact tension (C.T) specimens (12 mm thick, 75 mm wide); 2. Centre crack tension C.C.T. specimens (2 mm thick, 200 mm wide). The chemical compositions and mechanical properties of aluminium alloys are listed in tables 1, 2. All tests were run at a frequency of 10 Hz and in air at room temperature. The programmed block loadings were generated by a mini-computer PDP 11 which pilots the

testing machines continuously during the tests.

Table 1 : Materials' chemical composition

Alloy	Si	Fe	Cu	Mn	Mg	Cr	Ni	Zn	Ti
	0.09	0.21		0.63		0.01	1.14	0.04	
		~===							

Table 2 : Materials' tensile properties

Alloy	Sense	0.2 MPa	m MPa	A%
2124 T 351	TL.	328	472	15.16
2618 AT 851		410	450	6.03

TEST RESULTS AND ANALYSIS

Loading types C/n are characterised by a constant maximum loading level (P_1) and by one G.A.G cycle which occurs once per flight. This would enable us to compare types C/n (disturbed simulated flight) with type A (simple undisturbed flight), the comparison being based on the number of flights to failure and the rate of crack growth per flight. Having P_1 constant, the crack growth rate can be expressed as a function of K_{max} for different types of C/n to provide the possibility of investigating the influence of flight disturbances (E cycles) on crack propagation as shown in figures 2 - 5. The dashed lines in these figures represent the simple addition of crack growth by each level (A and B) to determine the crack growth rates as mm/flight for different types of C/n, (i.e), a non interaction summation of crack growth corresponding to basic data of types A and B: (da/dN) in mm/block = 1 x (da/dN) + (n - 1) x (da/dN) B for the block C/n, for the same Kmax.

It is interesting to find a good coincidence between these theoretical calculated growth rates and test data points for the different types of C/n. This agreement implies that it is only R ratio effects which caused these accelerations without any significant interaction effects [8, 9, 10].

ANALYSIS BASED ON CRACK CLOSURE CONCEPT

In attempt to understand the mechanism of crack propagation under spectrum loading, it was decided to measure the crack opening stress level on a 2 mm thick C.C.T specimen of 2124 T 351 under different loading types (A, C/3 and C/25). The same technique was applied as by Elber [11] by locating a displacement pick up on the surface of the specimen at the crack tip. It was difficult to measure Pop during the low ΔK cycles (B-cycles) in the block, so we considered the modification of Pop on the G.A.G. cycle as representing the crack opening load (Pop_{equiv.}) during the whole block. It was interesting to note in comparing the plot P=f (δ) that the level of Pop changes significantly with the number of B cycles in each block. The average values of α = Pop/Pmax are given in table 3.

Table 3 : Measured values of the crack opening ratio (Pop/Pmax)

Туре	А	c/3	c/25	В
Measured a	0.5	0.61 : 0.63	0.69	0.722

It is evident that increasing the number of B cycles per blocks tends to raise the relative crack opening level. It is logical to expect that for a high enough number of B cycles (thousands), the crack opening level will be stabilized and correspond to Pop of a constant amplitude loading of type B (R = 0.63). Based on Elber's relation [11]:

$$U = (Pmax - Pop)/(Pmax - Pmin) = (Pmax - Pop)/Pmax (1-R) = (1 - \alpha)/(1 - R)$$

 $\alpha = \text{Pop/Pmax} = 1 - U (1 - R)$

with U = 0.5 + 0.4 R for aluminium alloys

$$\alpha = 0.5 + 0.1 R + 0.4 R^2$$
 (1) $\alpha_{\rm p} = 0.5$ and $\alpha_{\rm B} = 0.722$

So as a first suggestion the ratio $\,$ for these types of spectra (C/n) will have values such that : $\alpha_{\rm A} < \alpha < \alpha_{\rm B}$ depending on the number of B cycles per block. Consequently, we can expect that the crack opening level under such sequences (Pmax is kept constant) will be stabilized after some crack growth and remain relatively constant throughout the crack propagation under these regular, short and stationary spectrum loadings.

Barsom [12] showed that for some random load distributions, the rate of crack growth was generally equivalent to the rate of crack growth under constant amplitude tests, with the same minimum load and an amplitude representing the root mean square (rms) of the random test: Elber [5] however, introduced this concept based on the crack closure phenomenon and on his test results.

Starting from Elber's definition [11] for a constant amplitude loading: da/dN = Ceff (Δ Keff)^m which can be rearranged as follows: da/dN = Ceff (ν Kmax)^m (with ν = 1 - α) (i.e) ν = Δ Keff/Kmax (normalized effective stress intensity factor range), and for aluminium alloys:

$$V = 0.5 - 0.1 R - 0.42 R^2$$
 (2)

Several observations based on above evidence and that now being collected on damage accumulation in stage II crack propagation, can be made:

- 1. Crack growth rates in mm/block for different types of loadings can be found easily from the relation da/dN = f (Kmax)
- 2. The programmed block loadings studied are stationary and short spectra
- 3. Measurements of (Pop) showed that it acquires a certain relative constant value between (Pop) of type A and (Pop) of type B depending on number of gust cycles per flight (n)
- 4. The greater is n, the nearer is $\ensuremath{(\text{Pop})}_{\ensuremath{\text{equiv}}}$ to the crack opening level of type B $\ensuremath{(\text{Pop})}_{\ensuremath{\text{B}}}$

On the basis of the above considerations, it is reasonable to expect that when dividing the growth rate (mm/block) by the number of maxima (n) per block we can find equal crack growth in (mm/cycle) based on equivalent damage accumulation due to equal effective stress intensities (Akeff) in the block. Of course, this holds good only as a long as (Pop) is higher than (Pmin) in the block. Having parallel Paris relations for the C.T specimens of 2618 AT 851, we have applied this assessment to find the equivalence Paris relation for each type of C/n as indicated in fig. 6. By a simple translation of the equivalent Paris relations given in (mm/cycle) in fig. 6 to Elber's line V equiv.

equation (2) the corresponding equivalent constant amplitude cycle ratio R eq can be found. Table 4 lists veq and Req corresponding to each spectrum studied. Thus Req and Pmax define the equivalent constant amplitude sequences that would replace each programmed block loading. Three basic remarks can be seen from table 4.

Table 4: Results of the equivalent constant amplitude concept

Туре	Veq	Req	αeq	Popeq
A* C/2 C/3 C/6 C/25 C/69 B*	0.5 0.445 0.41 0.355 0.305 0.29	0.01 0.25 0.365 0.489 0.58 0.61	0.5 0.55 0.59 0.645 0.695 0.71	Pop = (Pop) A Pop << (Pmin) B Pop eq (Pmin) B Pop eq > (Pmin) B

* Constant amplitude A et B

 $V_{eq} = (\Delta Keff) eq/Pmax$

Req = Pmin eq/Pmax

 $\alpha eq = 1 - Veq = Pop_{eq}/Pmax$

- 1. Veq for all spectrums C/n is greater than VB and less than ν_{A} ; ν_{B} < veq < ν_{A}
- 2. Req for all spectrums C/n is less than ${\rm R}_{\rm B}$ and greater than ${\rm R}_{\rm A}$; ${\rm R}_{\rm A}$ < Req < ${\rm R}_{\rm B}$
- 3. With an increasing number of maximum n per block, Veq decreases and Req increases resulting in a more severe flight type.

VALIDITY OF THE EQUIVALENT CONSTANT AMPLITUDE CONCEPT

From the evidence above, it is suggested that a block of C/n can be replaced by n cycles of an equivalent constant amplitude sequence defined by Req, Pmax and consequently $(Pmin)_{eq} \approx Req \times Pmax$.

Tests of constant amplitude loading corresponding to spectra C/3,

C/6 and C/25 were run on the same specimen geometry to verify the validity of this equivalence. Table 5 lists the ratio between the actual number of blocks and the number of equivalent sequences $(N_{\rm EX}/N_{\rm eq}(S))$. It ranges from 0.88 to 1.02, a range that is no greater than scatter that might be expected in fatigue crack propagation data. The comparison proves the applicability of the concept.

Table 5 : Validity of the equivalent constant amplitude concept

Type	Req	a o	a _f	N _{EX}	N _{E.C.A}	n	Neq(S)	N _{EX}
	•	mm	na m	block	cycle	max	sequen- ce	N eq(S)
c/3 c/6 c/25	0.365 0.489 0.58	26 26.5 26	40 40 40	38100 28000 12700	112500 190000 355000	3 6 25	37500 31666 14200	1.02 0.88 0.89

ANALYTICAL APPROACH

Based on linear damage accumulation observed under these types of blocks loadings it may be supposed that the equivalent effectiveness ratio \mathfrak{d} eq is the mean value of the effectiveness ratios of the different levels in the block loading. In a constant amplitude loading we have:

$$V = \frac{\Delta Keff}{Kmax} = \frac{U \Delta K (1-R)}{\Delta K}$$

and under controlled load we can write :

$$V = \frac{U \Delta P (1 - R)}{\Delta P}$$
 (3)

In stationary short spectra like types $\ensuremath{\text{C/n}}$, by similarity to equation (3) we can write :

$$V_{\text{eq}} = \frac{\sum \text{ni Ui } (1 - \text{Ri}) \Delta \text{Pi}}{\sum \text{ni } \Delta \text{Pi}}$$

which yields that :

$$veq = \frac{\sum ni \ vi \ \Delta Pi}{\sum ni \ \Delta Pi}$$
(4)

Table 6 lists the values of Veq corresponding to each type of C/n calculated from equation 4. There is very good agreement between calculated values and those determined from figure 6 (compare tables 4 and 6).

Table 6 : Calculated values of Veq for different spectra

Туре	А	C/2	c/3	c/6	C/25	c/69
√eq	0.5	0.441	0.406	0.355	0.300	0.286

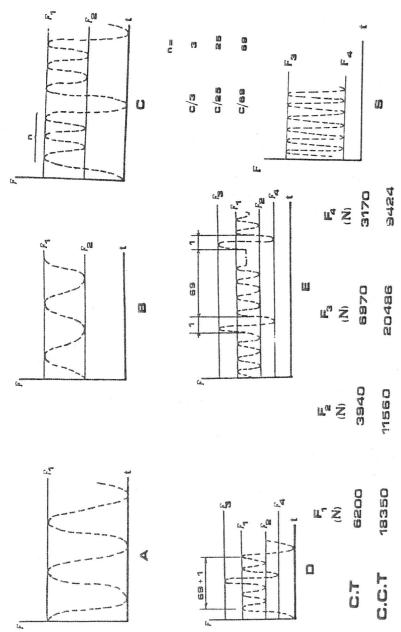
CONCLUSIONS

- 1. Aluminium alloys respond significantly to variation of the cycle ratio R (Pmin/Pmax). For a given value of ΔK the crack growth rate increases pronouncely with R. R ratio has a simultaneous effect on the translation of the Paris law curve.
- 2. Linear cumulative damage is established during crack propagation under the spectra studied where R ratio effects play the essential role.
- 3. Negative overloads added immediately after positive overloads can significantly reduce the crack growth delay of the latter.
- 4. The crack closure gives a significant contribution to the investigation of fatigue crack propagation under variable amplitude loadings.
- 5. The equivalent constant amplitude concept is proved to be applicable in cases of short stationary spectrum loadings.

REFERENCES

[1] De Koning, A.U., A simple crack closure model for prediction of fatigue crack growth rates under variable amplitude loading, ASTM. STP 743, pp. 63-85, (1981)

- [2] Schijve, J., Observations on the prediction of fatigue crack growth propagation under variable amplitude loading, ASTM.STP 595, pp.3-23, (1976)
- [3] Schijve, J., The accumulation of fatigue damage in Aircraft materials and structure, AGARD ograph n° 157, (1972)
- [4] Schijve, J., Four lectures on fatigue crack growth, Engineering Fracture Mech., Vol. 11 (1979), pp. 167-221
- [5] Elber, W., Equivalent constant amplitude concept for crack growth under spectrum loading, ASTM.STP 595, pp. 236-250, (1976)
- [6] Gabra, M., Contribution à la modélisation de l'endommagement des alliages d'aluminium en fatigue par blocs programmés, Thèse de Docteur Ingénieur, U.T. Compiègne, France, Octobre (1982)
- [7] De Jonge, J.B., and al, A standardized load sequence for flight simulation tests on transport, A/C wing structures, IBF BERICHT FB 106
 NLR TR 73029 N, March (1973)
- [8] Broek, D., and Schijve, J., The influence of the mean stress on the fatigue cracks in light alloy sheet, Aircraft Engineering, (1967) p.10
- [9] Gunn, N.J., Fatigue cracking rates and residual strength of eight Al alloys, R.A.E. Tech. Rept. 64024, October (1964)
- [10] Pearson, S., The effect of mean stress on fatigue crack propagation in half inch thick specimens of Al-alloys, R.A.E. Tech. Rept. 68297, (1968)
- [11] Elber, W., The significance of fatigue crack closure, ASTM.STP 486, pp. 230-242 (1972)
- [12] Barsom, J.M., In progress in flaw growth and fracture toughness testing, ASTM.STP, pp. 147-167 (1973)
- [13] Abelkis, P.R., Use of microfractography in the study of fatigue crack propagation under spectrum loading, ASTM.STP 645, pp. 213-234, (1978)



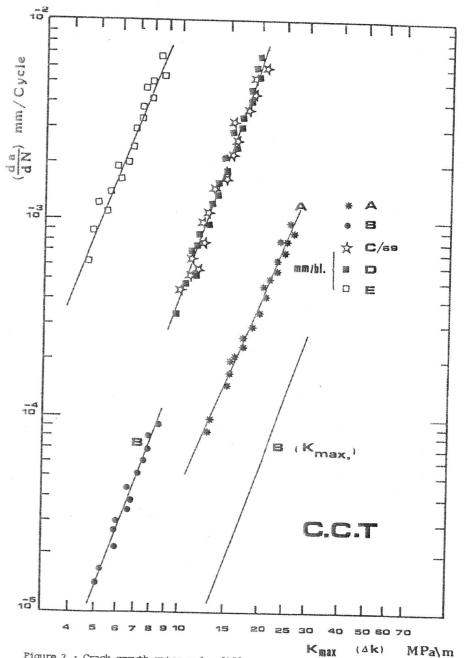


Figure 2 : Crack growth rates under different spectra (2124 T 351)

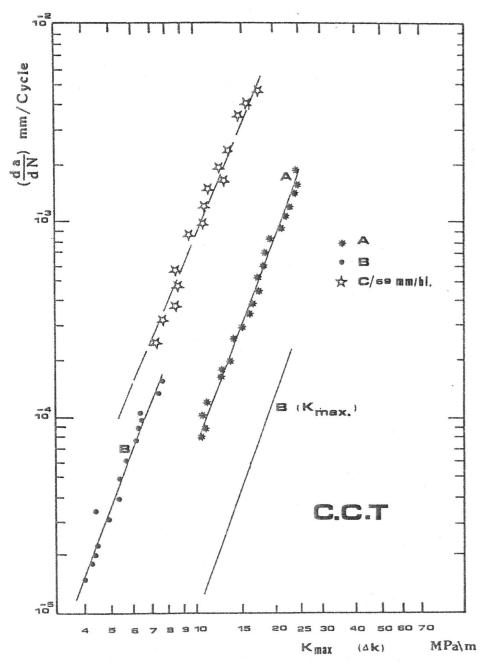


Figure 3 : Crack growth rates under different spectra (2618 AT 851)

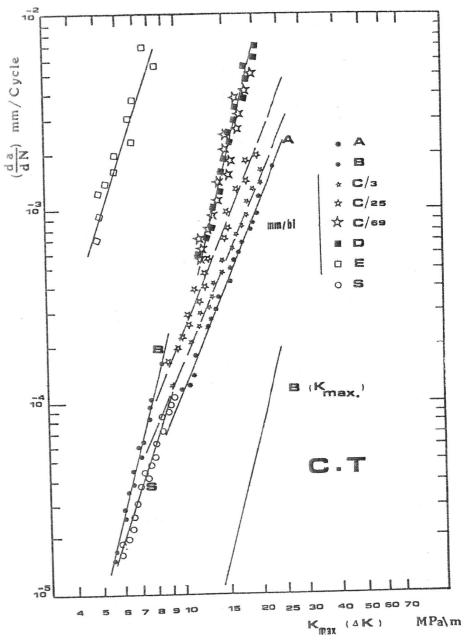


Figure 4 : Crack growth rates under different spectra (2124 T 351)

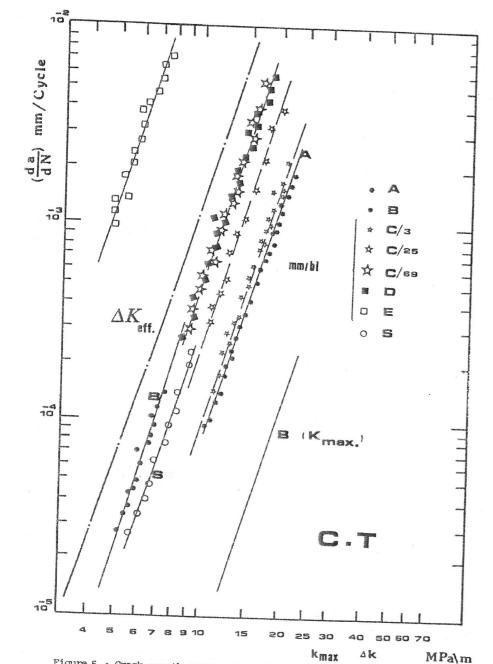


Figure 5 : Crack growth rates under different spectra (2618 AT 851)

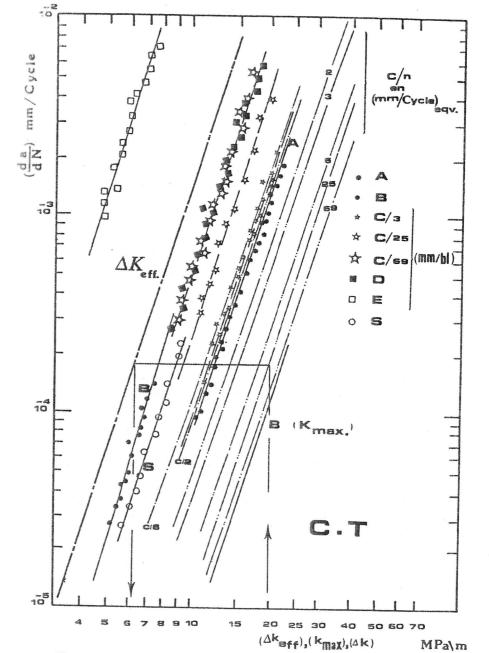


Figure 6 : Analysis of fatigue crack growth based on the crack closure concept