# On the blunting line in the ${\rm J}_{\mbox{\scriptsize 1c}}$ test - the comparison of theories and experiments -

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## INTRODUCTION

For an ideal crack, such as a saw cut crack or a fatigue pre-crack where the previous fatigue loading effect can be considered negligible compared with the following monotonic load, a relation between the crack tip opening displacement,  $\delta$ , and the stress intensity factor, K, or the J-integral, J, of the form

$$\delta = (1 - v^2)K^2/\lambda E\sigma_{fs} \tag{1}$$

in the linear elastic fracture mechanics (LEFM) case or

$$\delta = J/\lambda \sigma_{fs} \tag{2}$$

in the elastic-plastic fracture mechanics (EPFM) case under plane-strain conditions has been found, where  $\nu$  is Poisson's ratio, E is Young's modulus,  $\sigma_{\mbox{fs}}$  is the yield stress for ideally plastic materials or the flow stress for hardening materials and  $\lambda$  is about 2.

A simple model for crack tip plastic blunting is shown in Fig. 1. The geometric relation between the crack extension,  $\Delta \alpha$ , or the sub-critical stretched zone width, SZW, and  $\delta$  is given by the following expression



Fig. 1 A simple model for crack tip plastic blunting.

 $\Delta\alpha = SZW = \delta/2\tan\beta = J/2\lambda\sigma_{\rm fs}\tan\beta \eqno(3)$  where 2 $\beta$  is the crack tip blunting angle and the quantity tan $\beta$  has a value between 0.7 to 1.

In the ASTM Standard for the elastic-plastic fracture toughness  $J_{\rm IC}$  test method, E813-81, the  $J-\Delta\alpha$  blunting line has been given as follows

 $\Delta \alpha = J/2\sigma_{\rm fs} \eqno(4)$  assuming that  $\lambda = 1$  and  $\tan \beta = 1$  in Eq. (3). The  $J_{\rm IC}$  value has been defined as a J value at the intersection of the blunting line and the fracture resistance curve (R-curve).

On the other hand, Kobayashi et al. [1  $\circ$  5] have examined fractographically the J-SZW blunting line in the J<sub>IC</sub> test. They have shown that the experimentally determined values of  $\lambda \tan \beta$  in Eq. (3) for alloy steels and aluminum alloys show a tendency to become smaller as  $\sigma_{\tilde{\text{fs}}}$  becomes larger. For intermediate-strength materials, however, Eq. (3) becomes as follows.

SZW =  $J/4\sigma_{fs}$  (5) Equation (5) is plausible, since it can be obtained assuming that  $\lambda=2$  and  $\tan\beta=1$  in Eq. (3). Thus the experimentally determined blunting line does not obey the assumed one, Eq. (4), in the ASTM Standard, where  $\Delta\alpha=$  SZW before the initiation of ductile tearing. Furthermore, for a wide range of metallic materials from high- to low-strength, the relation between J and SZW depends not on  $\sigma_{fs}$  but on E, and is given by the following expression.

$$SZW = 89J/E \tag{6}$$

So, a basic question on the blunting line still remains about the comparison of theories and experiments.

More recently, Weertman [6, 7] has extended the theory of the growth of a fatigue crack that is based on the crack tip shear sliding model. In this paper, Weertman's theory is extended to the blunting of the ideal crack. It is shown that the theory is in excellent agreement with the experiments for ductile metals.

## EXTENSION OF WEERTMAN'S THEORY TO IDEAL CRACK

Following Weertman [7], the atomically sharp crack begins to emit dislocations from the crack tip region when the conventional stress intensity factor, K, reaches the value

$$K = K_{cd}$$
 (7)

where  $\boldsymbol{\kappa}_{cd}$  is the critical stress intensity factor for a crack in an intrinsically ductile solid.

$$K_{cb} = [2E\gamma/(1 - v^2)]^{1/2}$$
 (8)

where  $\boldsymbol{\gamma}$  is the true surface energy of the solid. The term

$$g = K_{cd}/K_{cb} \tag{9}$$

is a constant whose value is a function of the ratio of the theoretical tensile strength,  $\boldsymbol{\sigma}_t$ , to the theoretical shear strength,  $\boldsymbol{\tau}_t$ . When g has a value smaller than one an atomically sharp crack would begin to growth in a ductile, shear sliding mode before it could growth in a brittle, cleavage mode.

As the value of K is increased to a level where K >  $K_{\rm cd}$ , many dislocations are emitted from the crack tip region and the crack tip becomes progressively more blunt. Let the radius of curvature of the crack tip at any instant be  $\rho$ . For an atomically sharp crack [8]

$$0 \le \rho \le 8b/\pi \tag{10}$$

where b is the interatomic distance. The reduced stress intensity factor due to blunting at the crack tip,  ${\rm K}_{_{\rm P}}$ , is

$$K_{r} = K(8b/\pi\rho)^{1/2}$$
 (11)

Dislocations can be emitted from the crack tip region so long as K  $_{\rm r}$  > K  $_{\rm cd}$  Hence the radius of curvature of the crack tip will continue to increase until

$$\rho = 8bK^2/\pi g^2 K_{\rm cb}^2 \tag{12}$$

The radius of curvature is also approximately equal to the distance by which the crack tip has advanced, as shown in Fig. 1. Thus

$$\Delta \alpha = \rho = 8bK^2/\pi g^2 K_{cb}^2 = 4b(1 - v^2)K^2/\pi g^2 E_{\gamma}$$
 (13)

The theoretical tensile strength of a solid in an inert environment is

$$\sigma_{\mathsf{t}} = (\mathsf{E}_{\mathsf{Y}}/b)^{1/2} \tag{14}$$

According to Kelly et al. [9], the value of  $\sigma_t$  is of the order of  $\alpha E$  where  $\alpha \simeq 1/5$ . Then the true surface energy is

$$\gamma = \alpha^2 E b \tag{15}$$

Hence Eq. (13) can be reduced further to the equation

$$\Delta \alpha = \frac{4(1-v^2)}{\pi\alpha^2 g^2} \left(\frac{K}{E}\right)^2 \tag{16}$$

Since

$$J = (1 - v^2)K^2/E$$
 (17)

in the LEFM case, Eq. (16) reduces to

$$\Delta \alpha = \frac{4}{\pi \alpha^2 g^2} \frac{J}{E} = C \frac{J}{E}$$
 (18)

where C is a constant for given values of  $\alpha$  and g.

## COMPARISON OF THEORIES AND EXPERIMENTS

The fractographically derived sub-critical stretched zone widths, SZW, as a measure of the crack tip plastic blunting have been examined quantitatively and a relation between J and SZW has been clarified for various metallic materials over a range from LEFM to EPFM cases [1  $\sim$  5]. The experimental values of C in Eq. (18) become as shown in Table 1 [5]. These values show a structure-insensitive property and may be considered as nearly constants for alloy steels and aluminum alloys, respectively.

Weertman [7] has estimated that g=1 for bcc metals and g=0.6 for fcc metals. The theoretical values of C are calculated from Eq. (18) for two different values of g on the assumption that  $\alpha=1/5$ . Table 2 lists the direct comparisons of the theoretical values and the experimental values for bcc and fcc metals, respectively. The theoretical values are in excellent agreement with the experimental ones, although there are some differences in quantities between the two values. However, such differences are thought to arise mainly from ambiguity for the assump-

Table 1 Experimental values of C.

Material		ofs	E	
		(MPa)	(GPa)	C
	304	496	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	88
Alloy steel	A533B-1	588		90
	4340	1068		57
	10B35 (873 K)	755		69
	(673 K)	1362		69
	(473 K)	1744		69
	Average		206	76
Aluminum alloy	2017-T3	384		48
	2024-T3	411		40
	5083-0	217		76
	7075-T6	537		58
	7N01-T6	321		55
	Average		71	55
Γ1-6A1-4V		970	113	94
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Table 2 Comparison of theoretical and experimental values of C.

	Theory	Experiment
bcc metals $(g = 1 \text{ for alloy steels})$	32	76
fcc metals $(g = 0.6 \text{ for aluminum alloys})$	89	55

tions of  $\alpha$  and g as well as possible experimental accuracy.

#### CONCLUSION

Weertman [7] has extended the theory of the growth of the fatigue crack that is based on the crack tip shear sliding model. Weertman's theory is extended to the blunting of the ideal crack. It is shown that the theory is in excellent agreement with the experiment for the ductile metals. According to Elber's concept [10], the residual plastic stretch left in the wake of the steadily advancing fatigue crack interacts with the plastic zone ahead of the crack tip and causes plasticity induced crack closure above zero load. So, for the growth of the fatigue crack, the influence of plasticity induced crack closure should be taken into consideration as follows

$$\frac{da}{dN} = A \frac{4(1 - v^2)}{\pi \alpha^2 g^2} \left( \frac{K_{eff}}{E} \right)^2$$
 (19)

where  $d\alpha/dN$  is the crack growth rate,  $K_{\mbox{eff}}$  is the effective stress intensity factor  $(K_{\mbox{eff}} = K_{\mbox{max}} - K_{\mbox{op}})$ ,  $K_{\mbox{max}}$  is the maximum value of K,  $K_{\mbox{op}}$  is about 0.58 [11] and A is about 3/4 [12] in the case that the stress ratio is about zero.

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