AN INVESTIGATION ON THE NONLINEAR STRESS FIELDS NEAR CRACK TIP USING PHOTOELASTOPLASTIC ANALYSIS

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In this paper a general parameter, elastoplastic geometry factor α_{Iep}, by which either elastic or plastic behavior near a crack tip is shown and its determination by means of photoelastoplastic analysis is presented. The solution herein provided for the near-field to reflect SIF K_I is examined by an illustration using photoelastic analysis. A study on the nonlinear stress fields near the crack tip using photoelastoplastic analysis in a power hardening material is made, and several useful results are obtained.

MODE-I ELASTOPLASTIC GEOMETRY FACTOR α_{IEP} AND STRESS INTENSITY FACTOR K_{IEP}

The local stress field at a crack tip front consists of a singular field and a finite one, which can be expressed in terms of the non-dimensionalized form by

$$\frac{\sigma_{ij}}{\sigma_\infty} = \alpha_{Iep}\,\sigma_\infty^{\frac{1-n}{1+n}}\,\rho^{-\frac{1}{1+n}}\,\tilde{\sigma}_{ij}(\theta) + \frac{\sigma_{ij0}}{\sigma_\infty} \tag{1}$$

where α_{Iep} is referred to as elastoplastic geometry factor, $\rho = \frac{2r}{a}$, r is the polar coordinates measured from a crack tip, a, the characteristic crack depth, σ_∞ is the far-field stress, n is the strain power hardening coefficient and the product of α_{Iep} and $\sigma_\infty^{(1-n)/(1+n)}$ is the non-dimensional parameter. The power hardening relation between the plastic strain ε_{ijp} and the stress σ_{ij} is assumed the same as it is in the simple tension $\varepsilon_p = \sigma_0\,\sigma^n$, in which σ_0 is the material constant.

The dominant term of Eq.(1) has HRR singularity, angle functions $\tilde{\sigma}_{ij}(\theta)$ are the same as defined in Ref. [1]. In Ref. [1] the stress field

associated with the dominant singularity is given as

$$\sigma_{ij} = \left(\frac{J}{\sigma_0 I}\right)^{\frac{1}{1+n}} r^{-\frac{1}{1+n}}\tilde{\sigma}_{ij}(\theta) \tag{2}$$

From Eq.(1) and (2), we can easily get the relationship

$$\alpha_{Iep}\,\sigma_\infty^{\frac{2}{1+n}}\,(a\pi)^{\frac{1}{1+n}} = \left(\frac{2\pi J}{\sigma_0 I}\right)^{\frac{1}{1+n}} \tag{3}$$

For small-scale yielding, utilizing familiar relations between J and K_I, we can express the I integral as

$$I = \frac{2\pi(1-\nu^2)}{\sigma_0 E}\,\frac{\alpha_I^2}{\alpha_{Iep}^{1+n}} \quad \text{(for plane strain)},$$

$$I = \frac{2\pi}{\sigma.E}\,\frac{\alpha_I^2}{\alpha_{Iep}^{1+n}} \quad \text{(for plane stress)} \tag{4}$$

where ν is the Poisson's ratio, E is the modulus of elasticity and α_I and K_I are Mode-I geometry factor and SIF respectively. In Eq.(3) and (4), both integrals I and J are as defined in Ref. [1]. Here the term K_{Iep}, called elastoplastic Mode I-SIF, is introduced, and it is expressed as

$$K_{Iep} = \alpha_{Iep}\,\sigma_\infty^{\frac{2}{1+n}}\,(a\pi)^{\frac{1}{1+n}} \tag{5}$$

From Eq. (3) it is found

$$K_{Iep} = \left(\frac{2\pi J}{\sigma_0 I}\right)^{\frac{1}{1+n}} \tag{6}$$

For the elastic case n = 1, $\sigma_0 E = 1$, from Eq.(4) I_e is obtained to be

$$I_e = 2\pi(1-\nu^2)\left(\frac{\alpha_I}{\alpha_{Ie}}\right)^2 \ \text{(for plane strain)},\quad I_e = 2\pi\left(\frac{\alpha_I}{\alpha_{Ie}}\right)^2 \ \text{(for plane stress)} \tag{7}$$

From Eq.(5) K_{Ie} is obtained to be

$$K_{Ie} = \alpha_{Ie}\sigma_\infty\sqrt{\pi a} \tag{8}$$

If the angle functions $\tilde{\sigma}_{ij}(\theta)$ in the dominant singularity term of Eq.(1) are taken as in Ref. [2], the familiar relationship between K_I and

α_I will be

$$K_I = \alpha_I \sigma_\infty \sqrt{\pi a} \tag{9}$$

This is identical with Eq.(8) in form. Thus elastoplastic SIF K_{Iep} can be considered as a natural extension of SIF K_I in plastic zone. By means of both Eq.(5) and (8) two SIF K_{Iep} and K_{Ie} can be determined if α_{Iep} and α_{Ie} are known. Therefore, α_{Iep} and α_{Ie} are two goal parameters for elastoplastic analysis.

K_I REFLECTED BY THE NEAR-FIELD ANALYSIS[4] AND DETERMINATION OF α_I WITH A CRACK IN STRESS CONCENTRATION REGION

The following discussion is restricted within an elastic zone. Several dimensionless parameters N, T, ψ, e, b and ζ are introduced. They are

$$N = \frac{\sigma_1 - \sigma_2}{\sigma_\infty}, \quad T = \rho^{-\frac{1}{2}}, \quad \sin 2\psi = \frac{2\sigma_{xy}}{\sigma_1 - \sigma_2},$$

$$e = \frac{2(\sigma_{yy0} - \sigma_{xx0})\sin\frac{3}{2}\theta + 4\sigma_{xy0}\cos\frac{3}{2}\theta}{\alpha_I \sigma_\infty \sin\theta}, \quad b = \frac{(\sigma_{yy0} - \sigma_{xx0})^2 + (2\sigma_{xy0})^2}{(\alpha_I \sigma_\infty \sin\theta)^2}$$

$$\zeta = \frac{\sin(2\psi \pm \frac{3}{2}\theta)}{\sin^2 2\psi - \sin^2\frac{3}{2}\theta}$$

By means of Williams series solution[2], the following equation is obtained from Eq.(1)

$$\frac{N}{\sin\theta} = \alpha_I T \sqrt{1 + \frac{e}{T} + \frac{b}{T^2}} \tag{10}$$

If $T \gg 1$, the terms containing T^{-3} and T^{-4} are negligible, then we can get

$$\alpha_I = \frac{\varepsilon}{\sin\theta} \frac{\partial N}{\partial T} \tag{11}$$

Eq. (10) shows that the relation between N and T is linear when r approaches zero, α_I is its slope. However, because N possesses singularity as r approaches zero, it is difficult to determine α_I with this analysis, yet Eq.(11) shows that α_I in independent of r and can be reflected by mechanical parameters in the near-field. The reflecting relative error is

$$\eta = -\frac{2r}{2T + e}\left(\frac{\partial e}{\partial r} + \frac{1}{T}\frac{\partial b}{\partial r}\right) \tag{12}$$

Eq.(12) shows that η approaches to zero in the order of $r^{3/2}$ as r decreases. Here the near-field is in $r \ll \frac{a}{2}$ range.

To provide a further examination of the analysis mentioned above, the photoelastic analysis was performed in the epoxy plate specimens with the notch crack in the interference region between two holes. The test results are shown in Fig. 1.

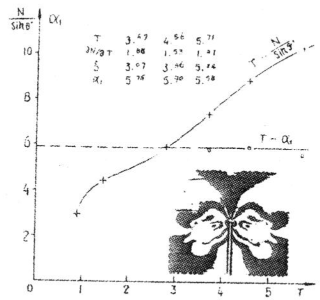

Fig. 1 N/sinθ VS. T and α_I VS. T curves

Usually only the dominant singularity term is considered in photoelastic analysis, in which, $\zeta = 1$. This method is not applicable here, because in our test $\zeta \doteq 5$.

A PHOTOELASTOPLASTIC ANALYSIS FOR NONLINEAR STRESS FIELDS NEAR CRACK TIP

The basic assumptions in the analysis are: 1) the material obeys Mises yield criterion and possesses a power hardening behavior, 2) the small strain formulation of plasticity can be applied, and 3) the stress field is in the state of plane stress. The following four differential equations for the principal stresses σ_1, σ_2, the effective stress σ_e in the plastic zone and α_{Iep} are thus found

$$\sigma_{{1 \atop 2}p} = \frac{1}{2}\left(\hat{\sigma}_{ep} \cdot \frac{p}{\hat{p}} \pm \frac{n_0 f_\sigma}{t}\right), \quad \sigma_{ep} = \frac{1}{2}\sqrt{\left(\hat{\sigma}_{pp}\frac{p}{\hat{p}}\right)^2 + 3\left(\frac{n_0 f_\sigma}{t}\right)^2}, \quad \alpha_{Iep} = \left.\frac{N}{\tilde{N}(\theta) T}\right|_{r \to 0}$$

where

$$\hat{\sigma}_{pp} = \hat{\sigma}_1 + \hat{\sigma}_2 = \sqrt{4\sigma_y^2 - 3\left(\frac{\hat{n}_0 f_\sigma}{t}\right)^2}, \qquad \tilde{N}(\theta) = \sqrt{(\tilde{\sigma}_{rr} - \tilde{\sigma}_{\theta\theta})^2 + (2\tilde{\sigma}_{r\theta})^2} \tag{13}$$

and

$$T = \rho^{-\frac{1}{1+n}}, \qquad N = \frac{\sigma_1 - \sigma_2}{\sigma_\infty^{2/(1+n)}} = \frac{n_0 f_\sigma}{t\, \sigma_\infty^{2/(1+n)}} \tag{14}$$

where p is the load, n_0 is the isochromatic fringe order, f_σ is the material fringe value, t is the thickness of the plate, σ_y is the yield stress and the parameter with symbol " $\wedge$ " refers to the value at the original yield surface. $N/\tilde{N}(\theta)$ vs. T curve is found by n_0 from photoelastoplastic analysis in the fracture specimens. α_{Iep} is the slope of the curve. Then the yield surface is obtained through linking all "knee" points at $N/\tilde{N}(\theta)$ vs. T curve with respective angles θ.

A set of simple tension and compact tension specimens were made form polycarbonate plate about 3mm-thick which possesses the power hardening behavior. Before the experiments, these specimens had been annealed, so that they have no initial stress. Measurement of material behavior is made through the experiment for a uniaxial mechanical and optical characterization using simple tension specimens, and several values of parameters: n, f_σ, σ_0 and E are obtained and then the fracture specimens put in a polarized light field are loaded by stages. After the stabilization of the isochromatic-fringe resulted from loading, the photo-elastoplastic-fringe patterns recorded with a camera provide the data necessary for the determination of a set of $N\sigma^{(1-n)/(1+n)}/\tilde{N}(\theta)$ vs. T, θ VS. $\hat{r}/2$ and σ_{ijp} vs. r/a curves which are shown in Fig. (2)-(4). The slope of the curves from Fig.2 $\beta=N\sigma_\infty^{(1-n)/(1+n)}/\tilde{N}(\theta)$ T is equal to the non-dimensional parameter $\alpha_{Iep}\sigma_\infty^{(1-n)/(1+n)}$ from which both α_{Iep} and K_{Iep} are obtained. This experiment gives both values $\alpha_I = 8.15$ and $\alpha_{Iep} = 54.43\ (kg\cdot cm^{-2})^{(1-n)/(1+\hat{n})}$, which are independent of load intensity within the experimental accuracy. For the specimen size the numerical calculation of boundary integrals[3] gives value α_{IN} equal to 8.23. Thus, we can see the value of α_{Iep} determined by the photoelastoplastic analysis is in good agreement with the value of α_{IN} determined by the numerical calculation of boundary integrals. The discrepancy is only about 1 percent. The curves shown in Fig. 2 is linear

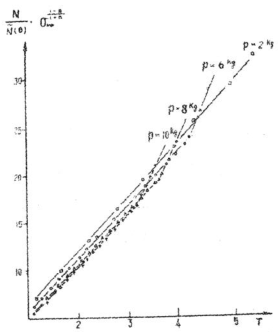

Fig. 2 Typical set of raw data illustrating α_I and α_{Iep} determination

in the plastic region near the crack tip, which shows that the dominant term singularity (HRR) of Eq.(1) is verified by this experimental results. The geometry of the yield surface is shown in Fig.3, two yield points located at $\theta = 0$ and $\theta = \pi$ are obtained according to the results of the isochromatic pattern of the residual stress under unloading. The distribution of plastic stresses σ_{ijp} are shown in Fig.4, in which the singularity near the crack tip is obvious and Mises yield condition is satisfied at yield point $\hat{r}$.

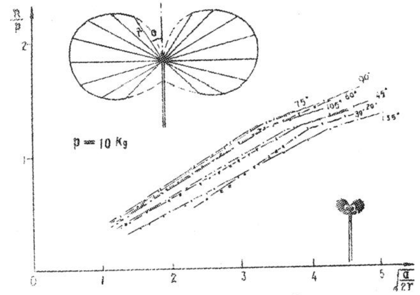

Fig.3 Geometry of an yield surface and isochromatic pattern of residual stress

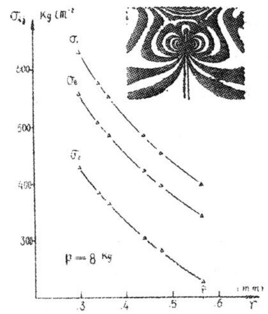

Fig.4 The distribution of plastic stress σ_{ijp} and the isochromatic-fringe pattern at the crack tip

CONCLUSIONS

In this paper are introduced two parameters i.e. Mode-I elastoplastic geometry factor α_{Iep}, which stands for the amplitude of the dominant singularity of stress fields near a crack tip in non-dimensional coordinate system, and stress intensity factor K_{Iep} associated with it. Here, the experimental results indicate that α_{Iep} is independent of load intensity, but dependent on the elastoplastic behavior near the crack tip. The analysis indicates that K_{Iep} is not only dependent on integral J which is associated with the elastic boundary conditions of far crack tip, but also controlled by integral I which is associated with the plastic stress and strain fields near crack tip, and it has a direct bearing on α_{Iep}. Therefore, it is expected that the K_{Iep} may be a parameter of driving force of crack extension. Thus α_{Iep} is a goal parameter for elastoplastic analysis. The use of near-field to reflect α_I can give results for determining SIF K_I for a crack. The test results show that it is necessary for near-field to reflect SIF K_I.

The fundamental equations applied to photoelastoplastic analysis on nonlinear stress fields near a crack tip of a plate with a power hardening material are herein developed and an appropriate procedure of experimental analysis is provided. Compact tension specimens made from a polycarbonate plate are used in the experiment. The results indicate that the stress component separation in a plastic region and the determination of α_{Iep} are simple and convenient, the yield surface can be clearly shown, and isochromatic-fringe are so dense and clear that enough number of data points can be obtained with good accuracy.

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