Application of Boundary Element Method to

Stress Intensity Analysis for Surface Cracks

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Abstract

Surface cracks are the most common flaws in many structural components. Accurate analyses for stress intensity factors for surface crack are necessary for fracture mechanics prediction of the crack growth and the fracture strength. Many solutions for the surface crack problems have been obtained by a finite element method, an alternating method, a body force method and a photo-elasticity method. There still remain some problems in the accuracy or the efficiency. Recently, boundary element method (BEM) has attracted special interest as a powerful method for solving three dimensional crack problems, especially since the sophisticated numerical techniques have been employed to the BEM.

In this study, three-dimensional elastic static BEM analysis were conducted on the stress intensity factors for a semi-circular and a semi-elliptical surface crack in a finite plate under uniform tension or bending stress. The BEM programs are developed essentially after Lachat and Watson, who first proposed the sophisticated numerical techniques in BEM. Eight node quadrilateral elements are used on the

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boundary surface and singular elements are placed at both elements adjacent to the crack front. Kelvin's solutions for infinite body and also Mindlin's solutions for semi-infinite body are employed as a fundamental solutions in three-dimensional BEM analysis. The stress intensity K_I along the crack front are determined by various techniques, using the solutions for the stresses and the displacements in the vicinity of the crack front.

Comprehensive analyses for semi-elliptical surface crack with various aspect ratio and depth ratio are carried out. The solutions obtained are compared with the Raju-Newman's solutions by FEM, Isida's solutions by body force method, and other solutions.

Figs.1 a)and b) show the computed variation of K_I along the crack front for semi-circular crack under uniform tension and bending, respectively. Our results are in good agreement with Raju-Newman's results and also Isida's results for various configurations of surface crack except for the case that the surface crack is extremely shallow one.

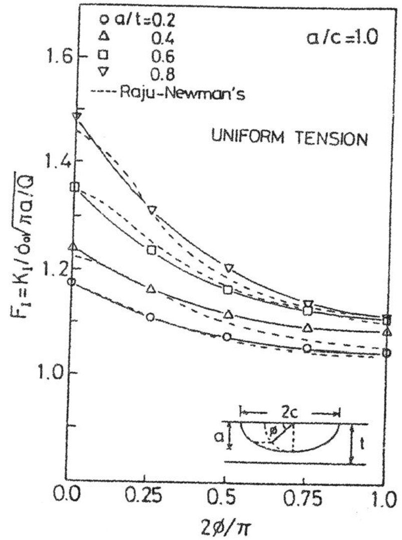

Fig.1 a)

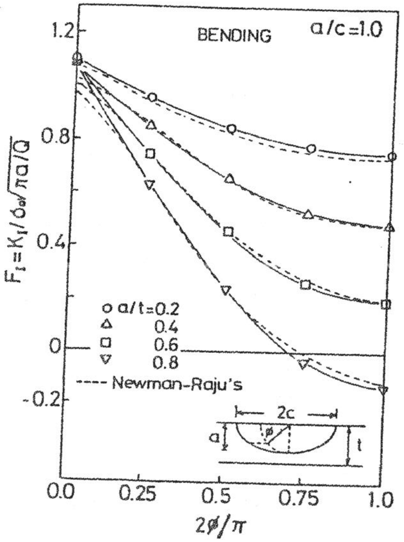

Fig.1 b)

It is confirmed that the BEM is one of the most useful methods to solve the stress intensity for surface crack and accurate solutions can be obtained by some devices in the determination of K_I.

FRACTURE OF JOURNAL IN HUB-ARMS OF HELICOPTERS

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THE STRESS INTENSITY FACTORS OF CYLINDER AND SHAFT SPECIMENS WITH SURFACE CRACKS

According to the experimental results and with the crack area S_0 as a main parameter, the compliance and the stress intensity factor of the cylinder under the bending condition can be determined as

$$\lambda = \frac{l^3}{48EJ}\left(1+\frac{12\alpha_s EJ}{G'Sl^2}\right) + \frac{l^2}{E'D^3}\left[1+3.19\,\frac{d}{D-d}\left(\frac{S_0}{S}\right)^2\right] \times\left[1.34tg^{5/4}\left(\frac{\pi S_0}{2S}\right) + 2.19tg^2\left(\frac{\pi S_0}{2S}\right)\right] \qquad (1)$$

where $E' = E$ for plane stress and $E/(1-\nu^2)$ for plane strain; G' the shear modulus; E the Young's modulus; ν the Poisson's ratio; S the cross-sectional area of the cylinder; J the moment of inertia ; l the span; α_s the shear factor and $\alpha_s = \frac{7+6\nu}{6(1+\nu)}$; d the inner diameter and D the outer diameter.

$$K_I = \frac{\pi}{8\sqrt{2}}\,\frac{D^4-d^4}{D^{5/2}S^{1/2}}\,\sigma f\left(\frac{S_0}{S}\right) \qquad (2)$$

where

$$f\left(\frac{S_0}{S}\right) = \left\{3.19\frac{2d}{D-d}\left(\frac{S_0}{S}\right)\left[1.34tg^{5/4}\left(\frac{\pi S_0}{2S}\right) + 2.19tg^2\left(\frac{\pi S_0}{2S}\right)\right] + \left[1+3.19\frac{d}{D-d}\left(\frac{S_0}{S}\right)^2\right]\right.$$

$$\times\left[1.67tg^{1/4}\left(\frac{\pi S_0}{2S}\right)+4.38tg\left(\frac{\pi S_0}{2S}\right)\right]\frac{\pi}{2}\sec^2\left(\frac{\pi S_0}{2S}\right)\Bigg\}^{1/2} \quad (3)$$

If d=0, then eqs.(1) and (2) lead to the compliance λ and the stress intensity factor K_I of the shaft can be expressed by eqs. (1) and (2) respectively.

THE RATE OF FATIGUE CRACK PROPAGATION

It has been found that the features of the fatigue crack propagation of the cylinder and shaft cannot be fully expressed by $\frac{da}{dN}$. It is better to use the area rate of fatigue crack propagation $\frac{dS_0}{dN}$, whose expression is as follows:

$$\frac{dS_0}{dN} = C(\Delta W_1)^n \quad (4)$$

where ΔW_1 is the energy that makes the crack propagate; c and n are material constants.

The experimental results show that the fatigue surface propagation of the cylinder and shaft specimens under bending stress can well be expressed by eq. (4). The test points of the cylinder and shaft specimens all fall within the same narrow scatterband, which shows that c and n are really material constants. During the experiments, the loading ratio $R = \frac{P_{min}}{P_{max}} = 0.1$. From the maximum and minimum load, we can calculate the energy release rate range ΔG. Then ΔW_1 can be calculated as follows:

$$\Delta W_1 = \int_{l_i} \Delta G dl_i \quad (5)$$

where l_i is the front line of the fatigue crack determined by the geometric features of the fatigue crack.

The expression for the area rate of fatigue crack propagation is

$$\frac{dS_0}{dN} = 1.043\times10^{-4}(\Delta W_1)^{1.288} \quad (6)$$