A NEW METHOD OF CALCULATING FRACTURE PARAMETERS FOR A STIFFENED PANEL WITH A CRACK — A MIXED METHOD

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Stiffened panels are widely used in engineering structures. It consists of a relatively thin sheet and some stringers which are connected by rivets or other fasteners. The purpose of this paper is to improve the existing methods of calculating fracture parameters for cracked panels.

The two essential fracture parameters to be calculated are: 1) the reduced factor Cr of stress intensity factor for cracked sheet 2) the load concentration factor Ls of stringers. There are two methods for calculating Cr and Ls — the analytical and the finite element methods (FEM)[1,2]. For structures with relatively simple geometry and boundary conditions it is suitable to use the analytical methods whereas for structures with complex geometry and boundary conditions the FEM will be more convenient. Typical stiffened panels have in general such characteristics that the sheet is of simple shape but the cross section shape of stringers are rather complex. The stringer eccentricity effect cannot be accounted for by analytical methods while the FEM is not convenient for studying an optimal design problem of cracked panels. So a mixed method is proposed in this paper.

The essentials of the mixed method presented in this paper is that displacements of the sheet at centers of fastener holes are determined by analytical methods and displacements of the stringers at centers of fastener holes are obtained by FEM. These two displacements thus found must satisfy the compatibility equations of the structure. In the calculation of displacements of the sheet a relatively small plastic zone in the vicinity of the crack tip is assumed. In the calculation of displacements of the stringers and rivets an initial stress method based on the deformation theory of plasticity is adopted to account for plastic effects of stringers and rivets[3]. Then fastener forces will be used to calculate the fracture parameters. Residual strength of stiffened panels with cracks

are determined by the K_c criterion and compared with test results[4]. The calculation can be implemented in a mini- or micro- computer.

The continuity equations at points of rivets center can be written as

$$\Delta V_i^p = \Delta V_i^s + \Delta V_i^r \tag{1}$$

and

$$\Delta V_i^p = V_i^p - V_{i-1}^p$$

$$\Delta V_i^s = V_i^s - V_{i-1}^s$$

$$\Delta V_i^r = V_i^r - V_{i-1}^r$$

where V_i^p , V_i^s and V_i^r are displacement at the ith rivet, of the sheet, the stringers and the rivet respectively. They can be determined by the following equation.

$$\begin{aligned} V_i^p &= B_i\sigma - \sum_1^{mn} A_{ij}\, Q_j \\ V_i^s &= B_i^s\sigma \frac{E_s}{E} + \sum_{j=i}^{kn} A_{ij}^s\, Q_j \\ V_i^r &= \frac{Q_i}{G} \end{aligned} \tag{2}$$

where Q_i represents the ith rivet force, σ the external applied stress A_{ij}, and B_i the displacements of the sheet at the ith rivet due to unit Q_j and σ, respectively; A_{ij}^s and B_i^s the displacements of the stringer at the ith rivet due to unit Q_j and $\sigma \frac{E_s}{E}$, respectively. i and j refer to only those rivets in one quadrant of the sheet, k is the sequence number of stringers, m the number of stringers in a quadrant, n the number of fasteners in each stringer and E, E_s and G are elastic moduli of the sheet, stringers and rivets respectively.

Substituting (2) in (1), we obtain the following equations.

$$\sum_1^{mn} (A_{ij} - A_{i-1,j})Q_{j0} + \sum_{j=i}^{kn} (A_{ij}^s - A_{i-1,j}^s)Q_{j0} + \frac{Q_{i0} - Q_{i-1,0}}{G} - (B_i - B_{i-1})\sigma + (B_i^s - B_{i-1}^s)\sigma \frac{E_s}{E} = 0 \tag{3}$$

equation (3) can be used to solve the initial rivet force Q_{i0}.

The formulas of A_{ij} and B_i can be found in [1]. A_{ij} and B_i^s are determined by FEM.

In the kth iteration the above mentioned continuity condition are still valid, viz.

$$(V^p_i)_k = (V^s_i)_k + (V^r_i)_k \qquad (4)$$

It should be noted that there are two different cases. In the first case only a few rivets are working in the plastic range. In the second case many rivets and a segment of stringers are working in the plastic range. In the former case equation (5) should be used in place of equation (4)

$$(V^s_i)_k = (V^p_i)_k - (V^r_i)_k \qquad (5)$$

In both (5) and (4),

$$(V^r_i)_k = (Q_{i0} + (\Delta Q_i)_{k-i})/G \qquad (6)$$

where Q_{i0} and $(\Delta Q_i)_{k-1}$ are initial value and correction for nonlinearity to the fastener force. In (5),

$$(V^p_i)_k = B_i\sigma - \sum_{1}^{mn} A_{ij}Q_{j0} + \sum_{1}^{mn} A_{ij}(\Delta Q_j)_{k-1} \qquad (7)$$

In (4), $(V^s_i)_k$ is determined by the stiffness method, viz.

$$K(V^s_i)_k = P + (\Delta P)_{k-1} \qquad (8)$$

where P is the applied load including σ and Q,ΔP is the correction for nonlinearity and K is the stiffness matrix of a fictitious linear system.

The results of calculation by four different methods (the analytical method, the initial stress method and the incremental method based on deformation theory and incremetal theory of plasticity respectively[5,6] and the mixed method) and the experimental results are shown in table I and Fig. 1 for comparison. The reduced sectional area of Z type stringer was used in the analytical method[7]. The accuracy of the mixed method is as good as the other three methods and meets the engineering requirement. The mixed method consumes the least computer time as compared with the analytical method. It is simpler in computation, more general and more efficient in application. Therefore this method is better

than both analytical method and FEM for typical stiffened panels.

A flow chart for the algorithm is as follows.

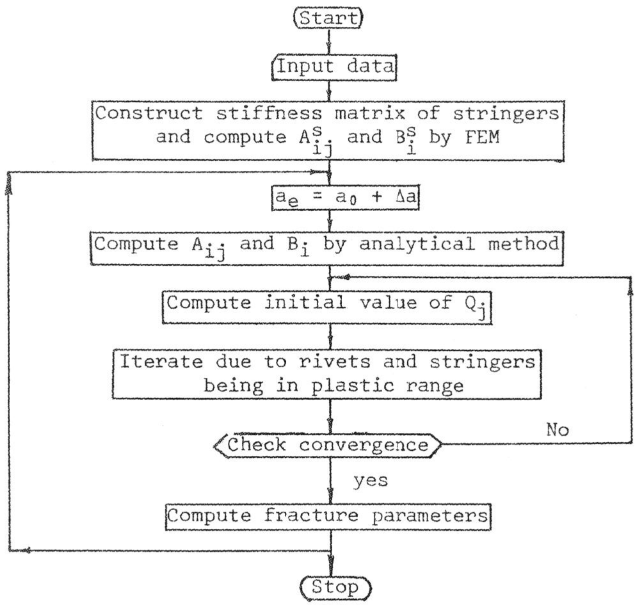

Table I

Method		Critical crack length a_c mm	Critical stress c kg/mm²	Relative error of a_c 0/0	Relative error of σ_c 0/0	Computer time
Test		189	21.3			
Analytical method		183	20.1	−3.2	−5.6	11'53"
FEM	Initial stress method	180.5	20.4	−4.5	−4.2	83'11"
FEM	Incremental method	180.5	20.8	−4.5	−2.4	175'20"
Mixed method		180.5	20.1	−4.5	−5.6	9'11"

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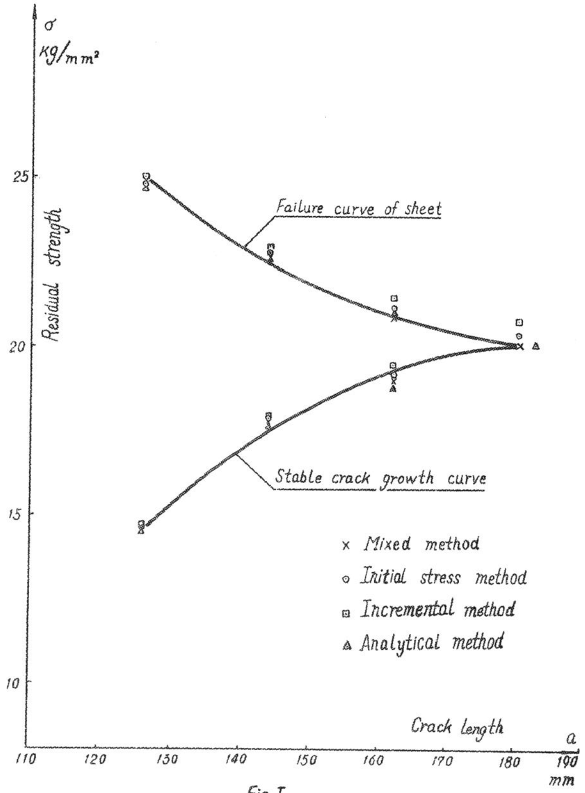

Fig.I