AN APPLICATION OF J INTEGRAL TO FINITE ELEMENT ANALYSIS OF STRESS INTENSITIES OF MIXED MODE CRACKS

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INTRODUCTION

The J integral [1] has been successfully adopted as an elastic-plastic fracture criterion in fracture mechanics [2,3]. Some numerical or experimental estimation procedures of the value of the J integral has been developed for its effective use [4,5,6,7,8]. In these works, the J integral has in most cases applied only to the case of the first mode crack. However, cracks in actual structures are very often subjected to the conditions of combined tensile and shear loading. For the case of the mixed mode crack, it is necessary to discuss how the fracture behavior of the crack, such as the initiation condition of a stable crack, is influenced by each deformation of mode I and II. Then, if we use the concept of the J integral to discuss the behavior of the mixed mode crack, it would be required to separate the J integral of the mixed mode crack into two components of mode I and mode II.

On the other hand, in the previous paper [9] an analytical method to estimate separately J_I related to mode I deformation and J_{II} related to mode II deformation from the J integral of a mixed mode crack was presented. Moreover, it was proved that both these J_I and J_{II} are path independent integrals, and that $J = J_I + J_{II}$.

In this paper, the separation method of J integral to J_I and J_{II} will be briefly introduced, and as an examle of applications of the method, finite element analyses of stress intensity factors of mixed mode cracks by means of these J_I and J_{II} will be presented. Since the integration of J integral may be done for any path far from a crack tip, the J integral can serve as a useful tool for estimating stress intensity factors by finite element method. To examine the accuracy and applicability of the present method, two types of rectangular cracked plates are analysed by the finite element method.

ANALYTICAL PROCEDURE OF SEPARATION OF J INTEGRAL TO J_I AND J_{II}

The J integral of a mixed mode crack can be separated into two path independent components, J_I of mode I and J_{II} of mode II, for the integration path in the linear elastic deformation field outside the plastic or non-linear elastic region near the tip of a crack [9]. The relationship between J and J_M (M=I,II) is given by

$$J = J_I + J_{II} \tag{1}$$

$$J_M = \int_\Gamma [W_M dx_2 - \vec{T}^M (\partial \vec{u}^M / \partial x_1) ds] \qquad (\ M = I,\ II\ ) \tag{2}$$

where W_M is the strain energy density function, and σ_{ij}^M, ε_{ij}^M, $\vec{u}^M$ and $\vec{T}^M$ are components of mode M (M = I or II) of stress, strain, displacement and traction vector, respectively.

Each component of mode I or II of stress σ_{ij} et al. can be obtained as follows: Because of the symmetricity and skew-symmetricity of crack deformations of mode I and II with respect to the X_1 axis, shown in Fig.1, stress σ_{ij} and displacement $\vec{u}$ at a point of coordinates (X_1, X_2) can be separated as

$$\sigma_{ij} = \sigma_{ij}^{I} + \sigma_{ij}^{II} \tag{3}$$

$$\begin{Bmatrix} \sigma_{11}^{I} \\ \sigma_{22}^{I} \\ \sigma_{12}^{I} \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} \sigma_{11} + \sigma'_{11} \\ \sigma_{22} + \sigma'_{22} \\ \sigma_{12} - \sigma'_{12} \end{Bmatrix} , \qquad \begin{Bmatrix} \sigma_{11}^{II} \\ \sigma_{22}^{II} \\ \sigma_{12}^{II} \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} \sigma_{11} - \sigma'_{11} \\ \sigma_{22} - \sigma'_{22} \\ \sigma_{12} + \sigma'_{12} \end{Bmatrix} \tag{4}$$

$$\vec{u} = \vec{u}^{I} + \vec{u}^{II} \tag{5}$$

$$\begin{Bmatrix} u_1^{I} \\ u_2^{I} \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} u_1 + u'_1 \\ u_2 - u'_2 \end{Bmatrix} , \qquad \begin{Bmatrix} u_1^{II} \\ u_2^{II} \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} u_1 - u'_1 \\ u_2 + u'_2 \end{Bmatrix} \tag{6}$$

where ( )' denotes the values of stress and displacement at the point P' that is a symmetric point of the point P with respect to X_1 axis. Strain components, ε_{ij}^{I} and ε_{ij}^{II}, may be obtained from the strain-displacement relationship, traction vector components, $\vec{T}^I$ and $\vec{T}^{II}$, from σ_{ij}^{I} and σ_{ij}^{II}, for a given integration path.

On the other hand, for the case of the linear elastic material, J_I and J_{II} are related to stress intensity factors, K_I and K_{II}, respectively, as

$$K_I = (8GJ_I/(1+\eta))^{1/2}, \qquad K_{II} = (8GJ_{II}/(1+\eta))^{1/2} \qquad (7)$$

where G is the modulus of shearing elasticity, and $\eta = (3-\nu)/(1+\nu)$ for plane stress and $\eta = 3-4\nu$ for plain strain, ν is Poisson's ratio. Therefore, using Eq.(7), stress intensity factors, K_I and K_{II}, of mixed mode cracks may be obtained from J_I and J_{II} of Eq.(2) which are given by the substitution of σ_{ij}^M and $\vec{u}^M$ of Eqs.(4) and (6) and ε_{ij}^M and $\vec{T}^M$ obtained from those into Eq.(2).

FINITE ELEMENT ANALYSIS OF K_I AND K_{II} OF MIXED MODE CRACKS

To examine the accuracy and applicability of the J integral method for estimating the values of stress intensity factors, K_I and K_{II}, of mixed mode cracks, two types of rectangular plates with edge-type bent cracks and an internal slant crack shown in Figs. 2 and 6 are analysed by the finite element method. The former is analysed by the finite element method of standard triangular elements. An example of the finite element mesh is shown in Fig. 3. Because of the symmetry of the problem, a quarter region is analysed. Numerical results are shown in Figs. 4 and 5. In these figures, F_I and F_{II} are non-dimensional stress intensity factors and the reference values are given by Higuchi et al.[10] who analysed the same problem by the finite element method using the substructure model. These figures show that if we take the numbers of element greater than about two hundred, we can get fairly accurate solutions, comparing with the results by Higuchi et al.[10]. However, since the solutions by Higuchi et al. are also obtained by the finite element method, we cannot explicitely discuss their errors.

Second example shown in Fig. 6 is analysed by the finite element method of standard eight-noded isoparametric elements, while for the crack tip region distorted isoparametric elements are used. Fig. 7 shows the finite element mesh, and numerical results are given in Fig. 8. This figure shows the path independence of J integral. It is found from Figs. 5 and 8 that the path independence of J_I (F_I) and J_{II} (F_{II}) are numerically proved. The average values of F_I and F_{II} for 26 paths of the integration in Fig. 7 are 0.8401 and 0.4489, respectively. This problem is also solved by the collocation method [11]. Comparing with the results of the collocation method, the present results are accurately obtained with errors less than

0.2 %.

It is concluded from the above-mentioned results that an accurate finite element analysis of stress intensity factors of mixed mode cracks can easily be done by the present method.

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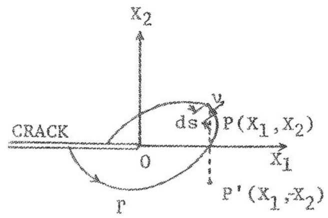


Fig. 1 Integration path of J integral

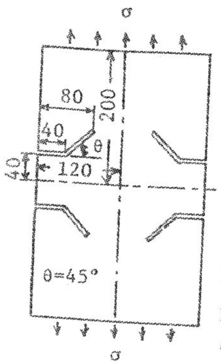


Fig. 2 Rectangular plate with four bent cracks

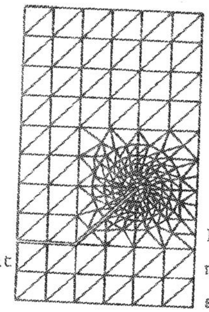


Fig.3 Finite element mesh for the problem shown in Fig. 2

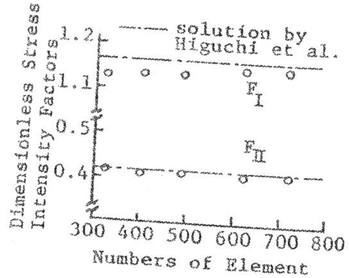


Fig. 4 Effect of numbers of elements on dimensionless stress intensity factors

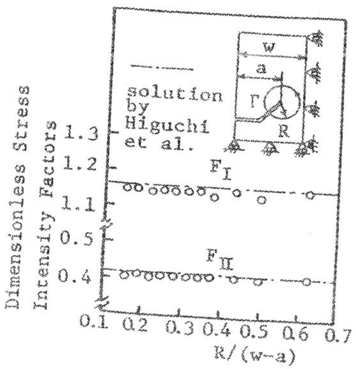


Fig. 5 Path independence of J_I (F_I) and J_{II} (F_{II})

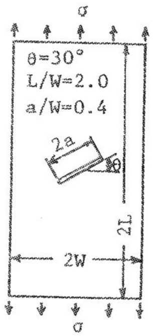


Fig. 6 Rectangular plate with an internal slant crack

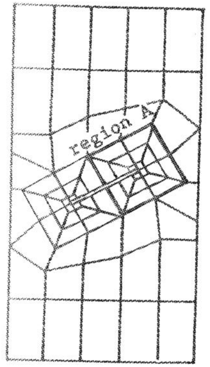
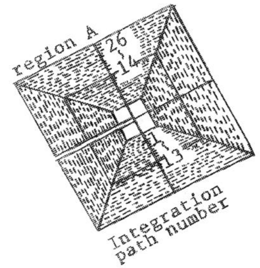


Fig. 7 Finite element mesh for the problem shown in Fig. 6 and integration paths



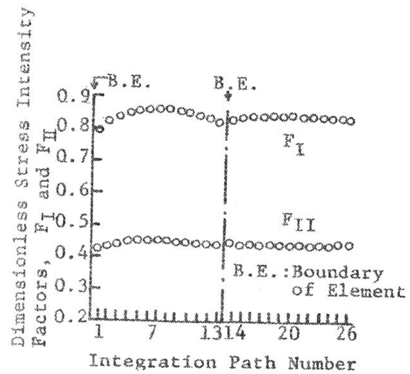


Fig. 8 Path independence of $J_I(F_I)$ and $J_{II}(F_{II})$