SOME THEORETICAL WORKS ON FRACTURE MECHANICS IN CHINA

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In this paper we give a review of the theoretical work on fracture mechanics carried out in Institute of Mechanics of the Academy of Sciences, Tsinghua University and Harbin Shipbuilding Engineering Institute.

I. STRESS ANALYSIS OF CRACKED BODIES

1. LEFM Analysis of Cracked Plates and Shells

The linear elastic fracture mechanics (LEFM) analysis of cracked plates and shells should be based on the higher-order theory of plates and shells, such as Reissner's theory. Using the eigen-function expansion, Liu [1] derived the near-tip solution and obtained the first terms of the expressions for generalized displacements. Based on these results, Liu in [2] calculated the stress intensity factors and attained higher precision owing to the use of higher-order singular finite element. Later the work was extended to problems in mixed mode I-II-III.

Yu et al.[4] decomposed the stress state in a cracked plate into several zones, namely, the exterior zone, Reissner's boundary layer zone and the near-tip singularity zone. Using matching technique of perturbation method, they analysed the Reissner's plate with crack of mixed mode I-II and obtained the relation, to the lowest order of approximation, between the stress intensity factors of Reissner's plate and the classical Kirchhoff's plate. The above results were extended in [5] to the plates with corner regions under various kinds of boundary conditions, and the problem was reduced to the solution of two uncoupled simpler problems: the shear problem and the biharmonic problem. In the study[6] of the stress intensity factor of cracked shallow shells by singular perturbation, the solution of

the tenth-order equations was reduced to the solution of three uncoupled problems: plane stress, classical plate bending and Reissner's boundary layer, and they were solved by successive iteration. The stress intensity factor was obtained by path-independent integral to the lowest two approximations.

2. Stress Analysis of Cracked Bodies with Hardening

He and Hutchinson[7,8] proposed a method for nonlinear cracked body problem —— the modified energy method, in which the principles of minimum potential energy and minimum complementary energy were extended to the infinite body with crack. They used this method for finding the near-tip displacement (or stress) fields and the upper and the lower limits of J-integral. The method was used to various crack problems, such as penny-shaped crack,[7] central or side crack in plane strain or plane stress,[8] the effect of loads in the direction parallel to the crack[9], and crack of mixed mode[10], etc.

In a study [11] of cylinder with a central penny-shaped crack, the J-integral was extended to axially symmetrical problems, and the near-tip stress and strain fields were calculated by finite element technique.

3. Study of Weight Functions

Referring to Bueckner's work(1973), T.C.Wang[12] discussed the basic properties of weight functions, and obtained their expressions for an infinite body with central crack, colinear cracks and circular crack. By superposing the finite element solution on a strong singularity term, X.T.Zhang[13] calculated the weight functions for an edge-cracked specimen. The results showed that the method is more efficient than the usual method used earlier by other authors. As an extension of the weight functions suggested by Bueckner, K.R.Wang[14] gave the definition of weight functions for the various coefficients in William's expansion for stresses and displacements in cracked bodies, and presented a method for finding these functions. In[15] are given the calculated results for weight functions for circular and rectangular cracked bodies.

4. Finite Element Analysis of Cracked Bodies

W.Z.Chien et al.[16] used a method for the calculation of stress intensity factor by superposing the finite element analysis on a field with singularity. J.F.He et al.[17, 18] used hybrid stress element and isoparametric element for calculating the J integral and stress intensity factors for plates with and without stiffeners.

II. RESEARCH ON CRACK CRITERION

T.C.Wang in [19] proposed a linear elastic crack criterion based on maximum circumferential stress on iso-strain-energy-density locus, and the results lie within those given by the criteria of maximum circumferential stress and minimum strain energy density. In the same paper, using the William's eigen function expansion near the tip of the branch crack and taking only the leading term, Wang calculated the strain energy release rate as the length of the branch crack approaches zero. Later Hwang et al.[20] have shown that, for the calculation of the strain energy release rate with vanishing branch crack, not only the leading term, but also the sum of the series of William's expansion should be taken into account, and they have given the critical curve of K_I-K_{II}, based on criterion of maximum strain energy release rate. In [21] it was shown that, in case of mode-III crack, this method leads to the well-known exact results in closed form, and the method was applied to the case of full mixed mode I-II-III.

The experimental results of H.Gao et al.[22] for mixed mode I-II crack on bend specimens made of high strength steel GC-4, medium strength rotor steel 30Cr2MoV and nodular cast iron showed that the critical K_{IIC} is much greater than the value expected by linear elastic criteria. This was attributed in [23] to greater effect of plasticity on mode II crack than on mode I. Finite element calculations made in [23] led to the fact that, under comparable loads, the size of plastic zone for mode-II crack is much greater than that for mode-I. The experimental results of K.R.Wang et al.[24] on four-point bend specimen of Al-alloy AU4G and AU2GN agree well with the results by two-criteria method of CEGB.

Xu et al.[25] proposed a new model for an elastic-plastic fracture for hardening material in plane stress. This model is an extension of Dugdale's, taking into account a diffuse plastic zone besides the strip

plastic necking zone. The results of finite element calculation based on elastic-plastic deformation theory and incremental theory agree pretty well with the results of tensile test on center-cracked wide plates. Taking into account of finite plastic deformation in finite element calculation, and using the criterion that, during crack extension, the relative separation of the necking strip equals the ultimate elongation of the material, Xu et al.[26] predicted very well the load crack growth relation obtained in Al-alloy wide plate test.

III. SINGULARITY FIELDS DURING CRACK GROWTH

The research on the near-tip singularity fields has played an important role in the development of fracture mechanics. In linear elastic fracture mechanics, the stress intensity factor k is taken as the measure of the intensity of stress-and strain-singularities. Similarly, in nonlinear fracture mechanics and under certain restricted conditions, the J integral can be adopted as the measure of the intensity of near-tip singularity (HRR singularity).

Chitaley and McClintock (1971) first obtained the near-tip stress- and strain-field for mode-III crack in steady growth in elastic perfectly-plastic material. The singularity behavior of strain ahead of the crack can be described as

$$\gamma_y^p = \frac{\tau_0}{G}\left\{\frac{1}{2}\left(\ln\frac{R_0}{r}\right)^2 + \ln\frac{R_0}{r}\right\}$$

in which G —— shear modulus, τ_0 —— shearing yield stress, $R_0 \approx k_{III}^2/\tau_0^2$ —— size of plastic zone and k_{III} —— mode-III stress intensity factor. This logarithmic singularity is weaker than the r^{-1} singularity for stationary crack. It is to this weaker singularity that McClintock et al. attributed the stable crack growth in ductile materials after crack initiation. Assuming a critical-strain type of crack-growth criterion, with the critical value $\eta\tau_0/G$ of shearing strain at an assigned distance ahead of crack-tip, we can calculate the ratio k_{IIISS}/k_{IIIC} (k_{IIISS} —— value of k_{III} necessary for driving the crack steadily and k_{IIIC} —— k_{III} at initiation) as a function of the material ductility η.

However, the opening displacement at the current tip of the propagating crack obtained by Chitaley et al. (1971) does not vanish, though it should. The corrected result by Hwang et al.[27] for the opening displacement of mode-III crack in steady growth is

$$2W\Big|_{\theta=\pi} = 2\,\frac{\tau_0}{G}\left\{(\sin\theta_p)\; r\ln\frac{R_0}{r} + O(r)\right\}$$

$$=0.6746\,\frac{\tau_0}{G}\left\{r\ln\frac{R_0}{r} + O(r)\right\} \tag{1}$$

where $\theta_p = 0.3440$ rad. is the half angle subtended by the plastic zone ahead of crack-tip. Based on their results of finite element calculation, Dean and Hutchinson (1980) obtained, through curve-fitting, the coefficient in (1) to be 0.83. In contrast to (1) for propagating crack, the crack opening displacement for stationary crack is

$$2W\Big|_{\theta=\pi} = 2\,k_{III}^2/\tau_0 G \tag{2}$$

If we assume a COD-type of crack growth criterion, with the critical COD (the same for both initiation and stable growth) at an assigned distance r_c behind the crack-tip, then from (1),(2) and taking only the leading singularity term, we obtain the relation between k_{IIISS} and k_{IIIC} as follows

$$\frac{k_{IIISS}}{k_{IIIC}} = \exp\left(\frac{k_{IIIC}^2}{2\tau_0^2 r_c \sin\theta_p}\right) \tag{3}$$

If we adopt the 45°-COD similar to that defined by Rice for mode-I crack, we will have from (1),(2)

$$\frac{k_{IIISS}}{k_{IIIC}} = \left(\frac{\tau_0}{G}\right)^{\frac{1}{2}} \exp\left(\frac{G}{2\tau_0 \sin\theta_p}\right) \tag{4}$$

For elastic perfectly-plastic material (for Poisson's radio $\nu=\frac{1}{2}$) Slepjan (1974), Gao[28] and Rice (1980 or 1981) obtained independently the singularity field for steadily growing mode-I crack in plane strain. It is much more difficult to find the singularity field of growing mode-I crack in elastic perfectly-plastic compressible material ($\nu<\frac{1}{2}$) and hardening material. It is necessary for this purpose to look into some properties of the basic equations and to study the contiguity conditions at the boundaries of neighbouring domains (elastic-plastic boundary, or boundary within plastic zone), and thereby to have a correct formulation of the problem.

1. Type of Basic Equations (Gao and Hwang[29,30])

Denote by $\sigma_{ij}, \varepsilon_{ij}, \varepsilon_{ij}^{*e}, \varepsilon_{ij}^{*p}$ the tensors of stress, strain, elastic strain and plastic strain, respectively. Then the constitutive

equations for the J_2-flow theory will be

$$\varepsilon_{ij} = \varepsilon_{ij}^{*e} + \varepsilon_{ij}^{*p}, \quad \varepsilon_{ij}^{*e} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_k^k g_{ij}$$

$$\varepsilon_{ij}^{*p} = \lambda S_{ij}, \qquad \lambda = \mu h(\sigma)\dot{\sigma} \tag{5}$$

Here E denotes Young's modulus, ν Poisson's ratio, g_{ij} the metric tensor, $h(\sigma)$ a material function and S_{ij} the stress deviator

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_k^k g_{ij}, \qquad \sigma = \left(\frac{3}{2} S_{ij} S_{ij}\right)^{\frac{1}{2}} \tag{6}$$

The time-derivatives of the equations of equilibrium and of compatibility equation are, respectively,

$$\nabla_i \dot{\sigma}^{ij} = 0 \tag{7}$$

$$\nabla_i\nabla_j\dot{\varepsilon}_{kl} + \nabla_k\nabla_l\dot{\varepsilon}_{ij} - \nabla_i\nabla_k\dot{\varepsilon}_{jl} - \nabla_j\nabla_l\dot{\varepsilon}_{ik} = 0 \tag{8}$$

Assume $\dot{\sigma}_{ij}$ and their first-order partial derivatives with respect to coordinates to be continuous. Choose the local cartesian coordinates such that $x=0$ be the surface of possible discontinuity of some of the second-order derivatives $\partial^2\dot{\sigma}_{ij}/\partial x^2$. Then from the three equation $\partial(7)/\partial x$ and the three equations among (8) containing $\partial^2/\partial x^2$ (i.e. for i,j =1,1 and k,l = 2,2; 3,3; 2,3), and substituting the constitutive equations into them, we can write them in the form

$$[A]\{\partial^2\dot{\sigma}/\partial x^2\} + \ldots = 0 \tag{9}$$

Here $[A]$ and $\{\partial^2\dot{\sigma}/\partial x^2\}$ are 6×6 and 6×1 matrices, respectively, and by... are denoted terms of lower-order derivatives,

$$\{\partial^2\dot{\sigma}/\partial x^2\}^T = \left\{\frac{\partial^2\dot{\sigma}_x}{\partial x^2}, \frac{\partial^2\dot{\sigma}_y}{\partial x^2}, \frac{\partial^2\dot{\sigma}_z}{\partial x^2}, \frac{\partial^2\dot{\sigma}_{xy}}{\partial x^2}, \frac{\partial^2\dot{\sigma}_{xz}}{\partial x^2}, \frac{\partial^2\dot{\sigma}_{yz}}{\partial x^2}\right\}$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ D_{11} & D_{12} & D_{13} & F_{11} & F_{12} & F_{13} \\ D_{21} & D_{22} & D_{23} & F_{21} & F_{22} & F_{23} \\ D_{31} & D_{32} & D_{33} & F_{31} & F_{32} & F_{33} \end{bmatrix} \tag{10}$$

$$[D_{ij}] = \frac{3h}{2\sigma}\begin{Bmatrix} S_{yz} \\ S_y \\ S_z \end{Bmatrix}(S_x, S_y, S_z) + \frac{1}{E}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{\nu}{E}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$[F_{ij}] = \frac{3h}{\sigma}\begin{Bmatrix} S_{yz} \\ S_y \\ S_z \end{Bmatrix}(S_{xy}, S_{xz}, S_{yz}) + \frac{1+\nu}{E}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

At the surface of discontinuity $x = 0$ should be fulfilled the condition that the determinant of [A] vanishes, det [A] = 0, which can be simplified to

$$\frac{1-\nu^2}{E} + \frac{3h}{2\sigma}\left\{ S_y^2 + S_z^2 + 2\nu S_y S_z + 2(1-\nu)\, S_{yz}^2 \right\} = 0 \qquad (11)$$

For elastic perfectly-plastic material, $h \to \infty$ in (5), λ is undetermined and we have an additional equation $\dot{\sigma}=0$. Then (11) becomes (see eq.(1.9) in[31])

$$S_y^2 + S_z^2 + 2\nu S_y S_z + 2\,(1-\nu)\, S_{yz}^2 = 0 \qquad (12)$$

It was pointed out in [30] that, since (11) cannot be satisfied for hardening materials, the basic equations are of elliptic type. The satisfaction of (11) is possible only when the elastic deformations could be neglected ($E\to\infty$) and (see eq. (1.15) in [30])

$$\sigma_{yz} = S_y = S_z = 0 \qquad (13)$$

It was noted in [30] that, generally speaking, the identity (13) throughout a surface or a region is impossible. As a special case, only at the crack-tip where the elastic deformations are negligibly small as compared to the plastic deformations, can (13) be satisfied in asymptotic sense. The situation is different for incompressible ($\nu = \frac{1}{2}$) elastic perfectly-plastic materials. In the latter case, we have for plane strain (in x, y plane) $\sigma_{yz} \equiv S_z \equiv 0$, and (13) becomes

$$\sigma_x = \sigma_y \qquad (14)$$

which can be satisfied along slip-lines. However, generally speaking, for plane strain problem (for nonhardening material with $\nu < \frac{1}{2}$ or for hardening material), eq. (13) can not be satisfied along a line in (x, y) plane. It can at most be satisfied at crack-tip in asymptotic sense along a direction.

Hence, for nonhardening material with $\nu < \frac{1}{2}$ and for hardening material the basic equations are of elliptic type, and the solution is sufficiently smooth inside the plastic domain (the same conclusion is reached in [29] for plane-strain problems). Only at crack-tip and in the asymptotic sense can discontinuities (even if weak discontinuities) take place.

Drugan and Rice (1982), using quite a different approach (without consideration of the type of equations), arrived recently at the same conclusion (13), as the condition of strong discontinuity for elastic perfectly-plastic materials.

2. Contiguity Conditions for Plane-Strain Problems (Gao and Hwang[29,30])

For plane-strain problems in (x,y) plane, let Γ be the boundary line of different zones (for example, elastic and plastic zones or two different plastic zones). The shape and location of Γ is, in general, changing with time. From the continuity of tractions on Γ follow the first and the second contiguity conditions

$$[\varphi]_\Gamma = [\partial\varphi/\partial n]_\Gamma = 0 \qquad (15)$$

Here φ is the Airy's stress function, $[P]_\Gamma$ denotes the jump of a quantity P across Γ, and n, s constitute a stationary orthogonal coordinate system with a family of curves parallel to Γ (at the instant under consideration) as s-lines and with the family of straight normals to Γ as n-lines. The third and the fourth contiguity conditions follow from the continuity of displacements $[u]_\Gamma = 0$, $[v]_\Gamma = 0$. Here we have to distinguish several cases:

1) Weak discontinuity, i.e. the stresses and strains are themselves continuous, but their time-rates or space-derivatives may suffer discontinuities across Γ. In [29] the third and the fourth contiguity conditions are given as

$$\left[\frac{\partial^2\varphi}{\partial n^2}\right]_\Gamma = 0$$

$$\frac{1-\nu^2}{E}\left[\frac{\partial^3\varphi}{\partial n^3}\right]_\Gamma - \frac{[\lambda]_\Gamma}{V}\left\{\frac{1}{2}(\sigma_s - \sigma_n) + \frac{2}{3}\left(\frac{1}{2}-\nu\right)^2(\sigma_n + \sigma_s - \sigma_p)\right\} = 0 \qquad (16)$$

Here V denotes the moving velocity (not material velocity!) of Γ normal to itself, and

$$\sigma_p = \frac{2}{3}E\exp\left(-\frac{2}{3}E\Lambda\right)\int_{t_0}^{t}\lambda\exp\left(\frac{2}{3}E\Lambda\right)(\sigma_n+\sigma_s)\,dt$$

$$\Lambda = \int_{t_0}^{t}\lambda\,dt \qquad (17)$$

where $t_0(x,y)$ denotes the instant when the material point (x,y) begins to enter the plastic zone.

2) Strong discontinuity, i.e. the stresses and the strains themselves suffer discontinuities across Γ. In [29] the surface of discontinuity Γ is replaced by a narrow transition zone $\bar{D}$, in which stresses and strains may change abruptly but continuously. Under the assumption that $\sigma_n+\sigma_s-\sigma_p$ and $\frac{1}{2}(\sigma_s-\sigma_n)+\frac{2}{3}(\frac{1}{2}-\nu)^2(\sigma_n+\sigma_s-\sigma_p)$ do not change sign in $\bar{D}$, it is shown in [29] that the strong discontinuity can occur only at a surface Γ where the two following conditions are satisfied

$$\frac{1}{2}(\sigma_s-\sigma_n)+\frac{2}{3}\left(\frac{1}{2}-\nu\right)^2(\sigma_n+\sigma_s-\sigma_p)=0 \qquad (18)$$

$$\left(\frac{1}{2}-\nu\right)(\sigma_n+\sigma_s-\sigma_p)=0 \qquad (19)$$

The latter condition (19) is equivalent to (see eq. (1-6) in [29])

$$\sigma_z = \frac{1}{2}(\sigma_n+\sigma_s) \qquad (19)'$$

The condition (19) can be satisfied in two ways:

i) for $\nu<\frac{1}{2}$,

$$\left.\begin{array}{l}\sigma_s-\sigma_n=0\\ \sigma_n+\sigma_s-\sigma_p=0\end{array}\right\} \qquad (20)$$

However, it is generally impossible that both conditions of (20) be satisfied everywhere on a curve Γ in (x,y) plane, because it is generally unrealistic that the curve $\sigma_n+\sigma_s-\sigma_p=0$ coincides completely with one of the curve-family $\sigma_s-\sigma_n=0$. That's why it is assumed in [29] that $\sigma_n+\sigma_s-\sigma_p\neq 0$, and this is criticized unjustly by Drugan and Rice (1982) as " a priori assumption ".

Therefore, generally speaking, the strong discontinuity takes place in another way, namely:

ii) for $\nu=\frac{1}{2}$

$$\sigma_s-\sigma_n=0 \qquad (21)$$

Herefrom follows the conclusion arrived at in [29] for plane-strain problem: only when $\nu=\frac{1}{2}$, the strong discontinuity can take place across a curve Γ(n=o), which is a slip line satisfying the condition $\sigma_s-\sigma_n=0$.

As to the elastic perfectly-plastic material with $\nu<\frac{1}{2}$ or the hardening material, the strong discontinuity satisfying the conditions (20) is possible only at the crack-tip and in the asymptotic sense. And this has already been taken into consideration in the paper [30] dealing with near-tip singularity solution for power-hardening materials.

In [29] are obtained the third and the fourth contiguity conditions in case of strong discontinuity (see eq. (3.23) in [29])

$$\left[\frac{\partial^2\varphi}{\partial n^2}\right]_\Gamma=0$$

$$\frac{1-\nu^2}{E}\left[\frac{\partial^3\varphi}{\partial n^3}\right]_\Gamma+\frac{2}{\rho}\left[\varepsilon^p_{ns}\right]_\Gamma-2\frac{d}{ds}\left[\varepsilon^p_{ns}\right]_\Gamma=0 \qquad (22)$$

where $1/\rho$ denotes the curvature of the curve orthogonal to the family of discontinuity curves (i.e. family composed of discontinuity curves at various instants).

3) At the unloading boundary Γ_B (i.e. the boundary curve between the plastic loading zone and the unloading zone), it can be proved that only weak discontinuity is possible. It is proved that, besides (15) and (16), the following additional condition should be fulfilled at Γ_B

$$[\dot{\sigma}]_\Gamma=0 \qquad (23)$$

3. Near-Tip Asymptotic Solutions for Cracks in Steady Growth

By use of the aforementioned basic equations and contiguity conditions (or, for the sake of convenience, contiguity conditions for $\dot{\varphi}$ derived from conditions for φ, as is done in [30]), the following results have been obtained.

In [32] Gao first pointed out the incorrectness for $\nu<\frac{1}{2}$ of modified Prandtl's field with four angular zones, which Rice et al.(1980) and Rice (1981) claimed to be the solution for mode-I crack steadily growing in elastic perfectly-plastic materials with $\nu<\frac{1}{2}$, because the unloading condition is violated in the unloading zone. And Gao gave in [32] a solution with five angular zones.

Later Drugan, Rice and Sham (1982) accordingly corrected their results and gave a solution with five angular zones, but differed in other respects from Gao's solution. Which of these two solution is correct, this is an issue to be clarified.

For mode-III crack in power-hardening material, Gao, Zhang and Hwang [33,34] obtained the near-tip stress- and strain-fields in steady growth as follows:

$$\tau_\alpha = (\ln \frac{A}{r})^{2/(n-1)} \; \tau_{\alpha 0}(\theta)$$

$$\gamma_\alpha^p = (\ln \frac{A}{r})^{2n/(n-1)} \; b_\alpha(\theta) \tag{24}$$

where A is the amplitude factor and n an exponent related to material hardening ($\gamma \sim \tau^n$). In [34] it was shown that, in order to satisfy the condition (23) at unloading boundary Γ_s, there appears an inner boundary near Γ_s, with its subtended angle approaching zero at the crack-tip.

For mode-I crack growing steadily in power-hardening materials ($\nu = \frac{1}{2}$), Gao and Hwang [30] obtained the near-tip stress- and strain-fields

$$\sigma_{ij} = (\ln \frac{A}{r})^{1/(n-1)} \sigma_{ijo}(\theta)$$

$$\varepsilon_{ij}^p = (\ln \frac{A}{r})^{n/(n-1)} \varepsilon_{ijo}^p(\theta) \tag{25}$$

However, there is one parameter not yet uniquely determined. Hence this solution remains in some respects to be refined.

For plane-stress problems, there is not yet analytical solution obtained. Luo, Zhang and Hwang [36] obtained by finite element calculations the stress- and strain-fields for power-hardening and elastic perfectly-plastic materials under small-scale yielding conditions.

In the above discussions, the material is assumed to obey the isotropic hardening rule, i.e. J_2-flow rule. However, most engineering materials exhibit anisotropic hardening effect, i.e. Bauschinger effect. By use of the constitutive equations of Kadaschevich and Novozhilov (1958), R.F.Zhang, X.T.Zhang and Hwang [37] obtained the asymptotic solution for steadily growing crack for mode-III as well as mode-I crack in plane stress and in plane strain [37] for linearly hardening materials; Xie and Hwang [38] obtained the corresponding solution for mode-III crack for power-hardening materials. All these results show that both the singularity and the structure of the near-tip field are dependent upon the

anisotropy of hardening.

Recently Gao and Nemat-Nasser [39-41] have obtained the near-tip dynamic singularity fields for crack growing in elastic perfectly-plastic and power-hardening materials.

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