THE ANALYSIS OF STABLE CRACK GROWTH WITH ISOTROPIC AND KINEMATIC HARDENING

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ABSTRACT

The stable crack growth has been analyzed with a finite element method based on an incremental theory of isotropic hardening and kinematic hardening in this paper. The stress field near the crack-tip and the plastic energy during the process of crack growth have been obtained. The models of isotropic and kinematic hardening were compared with each other. Also, the crack growth of a combined-mode crack in plane stress has been determined.

INTRODUCTION

The stable crack growth, which has the practical application and the academic significance, has been studied by a number of investigators in the field of fracture mechanics in different ways[1–10].

Isotropic hardening of multilinear stress-strain relation and kinematic hardening of bilinear stress-strain relation were used in this paper. The load-crack size curve was taken as supplementary equation instead of the linear relation between plastic energy and crack size which was used in Reference[10]. A combined-mode crack with crack angle of 15° has been analyzed and its growth direction was assumed to follow the effective stress criterion in this paper. The load-crack curve based on experimental data of the crack of mode I can be used as a supplementary equation because the crack angle is very small. The relation between plastic energy and crack size, the stress field near crack-tip and the crack path during the process of crack growth have been obtained with these two models.

THE BASIC EQUATIONS

The equations of equilibrium in the incremental form can be written as

$$\delta\sigma_{ij,j} = 0 \tag{1}$$

The strain-displacement based on small strain theory in the incremental form are written as

$$\delta\varepsilon_{ij} = \delta\varepsilon'_{ij} + \delta\varepsilon''_{ij} = \tfrac{1}{2}(\delta u_{i,j} + \delta u_{j,i}) \tag{2}$$

where ε'_{ij} and ε''_{ij} are elastic and plastic strain, respectively. The stress deviator S_{ij} is

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} \tag{3}$$

and the effective stress is written as

$$\sigma_e^2 = \frac{3}{2} S_{ij}S_{ij} \tag{4}$$

In isotropic hardening, the yield function is taken as

$$f = f(\sigma_{ij}, \sigma^*) = \sigma_e^2 - \sigma^{*2} \tag{5}$$

where σ^*, depending on the loading history, is an ever increasing quantity and its initial value is equal to the yield stress, σ_s. It can be found that loading means

$$f = 0, \quad \delta\sigma_e = \delta\sigma^* > 0 \tag{6}$$

unloading means

$$f < 0, \quad \delta\sigma^* < 0 \tag{7}$$

and neutral loading means

$$f = 0, \quad \delta\sigma_e = \delta\sigma^* = 0 \tag{8}$$

The constitutive equation is as in Reference [8].

In kinematic hardening model, the yield function can be taken as

$$f = f(\sigma_{ij}, \varepsilon_{ij}, x) \tag{9}$$

where x is a hardening parameter. In this paper, f is taken as

$$f = \tfrac{1}{2}(S_{ij} - C\varepsilon''_{ij})(S_{ij} - C\varepsilon''_{ij}) - \frac{1}{3}\sigma_s^2 \tag{10}$$

For the case of simple tension, C is as

$$C = 2S/3(1 - S/E) \tag{11}$$

S is the slope of stress-strain curve in the plastic stage. It should be noticed that Eqn. (10) is just Von Mises yield criterion in initial yield stage. The constitutive equation was given in Reference[10].

THE ANALYSIS OF CRACKED SPECIMEN

The cracked specimen is a rectangular one of length 2L, width 2W, thickness B and a centered line crack of initial crack size 2a. The material is 2024-T3 aluminum alloy with Young's modulus $E=7.118\times10^4$ MPa, yield stress $\sigma_s = 3.73\times10^2$ Mpa and $E/S = 44$. The transient crack size becomes an unknown variable during the process of crack growth. Therefore, the load-crack size curve based on the experimental data was taken as a supplementary equation of crack growth. For a combined-mode crack, one more equation, which can govern the direction of crack growth, is needed. A criterion which can be called the effective maximum stress criterion was taken as a supplementary equation in this paper. Each step of crack growth was assumed from a nodal point of an element to the next nodal point. Then the modified orientation of crack growth is obtained as

$$\theta_m = tg^{-1}(F_x/F_y) \tag{12}$$

where θ_m is the angle measured clockwise from the x-axis, F_x and F_y are the two components of nodal force at the crack-tip. After that, the subroutine REMESH is called to modify the finite element mesh accordingly.

Because of symmetry for the crack of mode I, only the quadrant of the plate needs to be analyzed[1,8]. There is a periodicity of π about z-axis for a combined-mode crack, only half of the plate, $R=R_1+R_2=[x,y|0\leq x\leq W, -L\leq y\leq L]$, was analyzed. R_1 and R_2 are the upper part and lower part of R, respectively. The incremental boundary conditions were given in Reference[10].

THE ANALYTICAL RESULTS AND CONCLUSIONS

Two kinds of elements, i.e. the trigonometrical elements for kinematic hardening model and the quadrilateral elements for isotropic hardening model, are used. The area of the smallest element is 0.25×10^{-4} WL for the trigonometrical element and 0.14×10^{-4} WL for the quadrilateral element. The applied stress-crack size curve obtained on experimental data was shown as in Fig. 1. The relation between σ_y and σ_e of crack-tip and crack size was shown in Fog. 2. In case of a crack angle of 15°, it was presented in Fig.3. The relations, in both cases of isotropic and kinematic hardening, between plastic energy of the half plate and crack size for the case of mode I were shown in Fig. 4. Fig. 5 showed the plastic energy for the case of crack angle of 15°. In the case of crack angle of 15°, the path of crack growth was shown in Fig. 6. The solid line shows the path of crack growth based on the effective maximum stress criterion and the dash line shows the path of crack growth based on the maximum stress criterion.

From the previous results, it can be seen that the stresses near crack-tip were almost constant during the process of crack growth and the results based on two models were almost identical. The linear relation between plastic energy and crack size was shown again in Fig. 4. The slopes and initial values of plastic energy of crack growth are also approximately equal to each other. The analytical method and computer program of the combined-mode crack have been developed. The result showed that there are some discrepancies between the path obtained in this paper and the path based on the maximum stress criterion.

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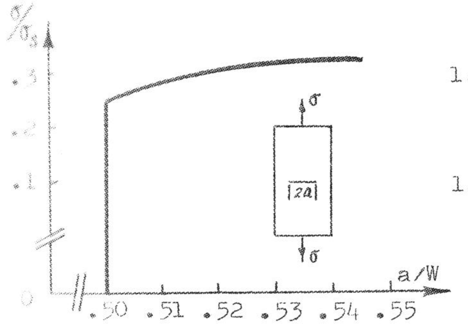


Fig.1

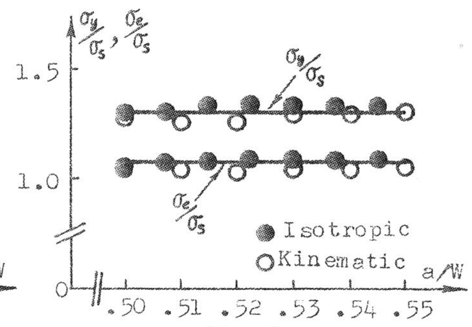
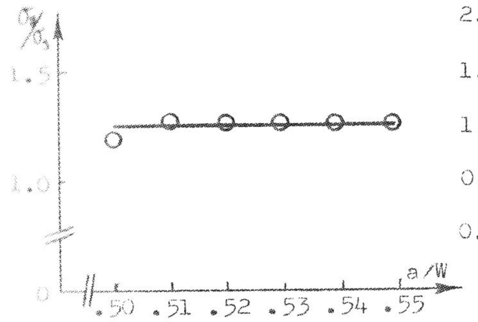


Fig.2



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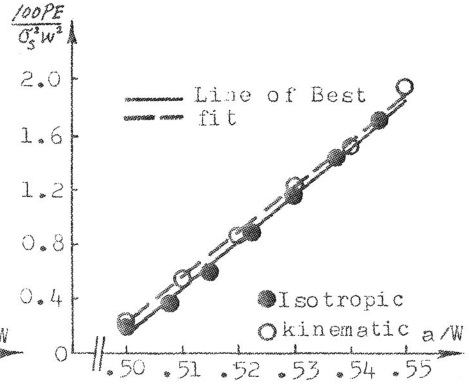

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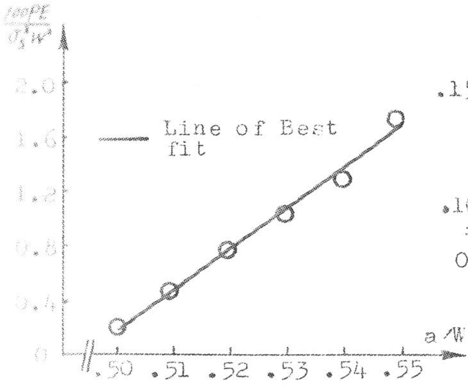

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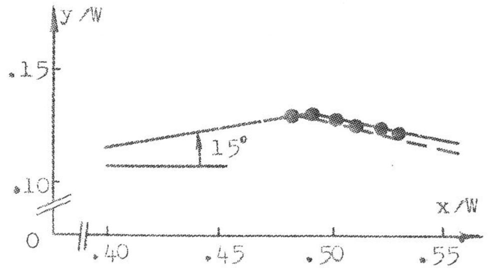

Fig.6