Asymptotic solutions for buckling delamination induced crack propagation in the thin film- compliant substrate system

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Abstract In a thin-film substrate system in-plane compressive stress is commonly generated in the film due to thermal mismatch in operation or fabrication process. If the stress exceeds a critical value, part of the film may buckle out of plane along the defective interface. After delamination buckling, the interface crack at the ends may propagate. In the whole process, the compliance of the substrate compared with the film plays an important role. In this work, we study a circular film subject to compressive stress on an infinitely thick substrate. We study the effects of compliance of the substrate by modeling the system as a plate on an elastic foundation. The critical buckling condition is formulated. The asymptotic solutions of post-buckling deformation and the corresponding energy release rate of the interface crack are obtained with perturbation methods. The results show that the more compliant the substrate is, the easier for the film to buckle and easier for the interface crack to propagate after buckling.

Keywords buckling delamination, interface crack, film-substrate system, post-buckling

1. Introduction

The thin film-substrate system is widely used in various of applications, such as thermal barrier coatings and micro-electronics [1-4]. In operation or fabrication process, in-plane compressive stress is commonly generated in the film due to thermal mismatch. If the stress exceeds a critical value, part of the film may buckle out of plane along the defective interface. Delamination buckling is considered as one of the most important failure modes in thin-film substrate system. Delamination buckling on a stiff substrate is well studied in the literature [5-7]. If the substrate is compliant, the critical buckling load and the energy release rate of the interface crack can be significantly affected[8]. Yu and Hutchinson analyzed the effects of compliance of substrate by introducing compliance coefficients [9]. After delamination buckling happens, the film and the substrate are detached. In this work we study the effects of compliance of the substrate by modeling the system as a plate on an elastic foundation. We use perturbation method to obtain the asymptotic solutions of post-buckling deformation and calculate the mode-adjusted energy release rate of the interface crack after buckling happens. Analysis of a plate on a foundation is a classical problem in mechanics, which can be an effective model of analyzing film-substrate system.

2. Governing equations

2.1. A circular plate on an elastic foundation

We study the delamination buckling of the thin film as buckling of a thin plate on an elastic foundation. To focus on the effects of the compliance of the substrate we study a simple configuration, a circular plate on an infinitely thick foundation, as shown in Fig. 1. We use the linear Wrinkler foundation model

$$F = Kw, \tag{1}$$

where w is the deflection, K the stiffness of the foundation, F the effective reaction force.

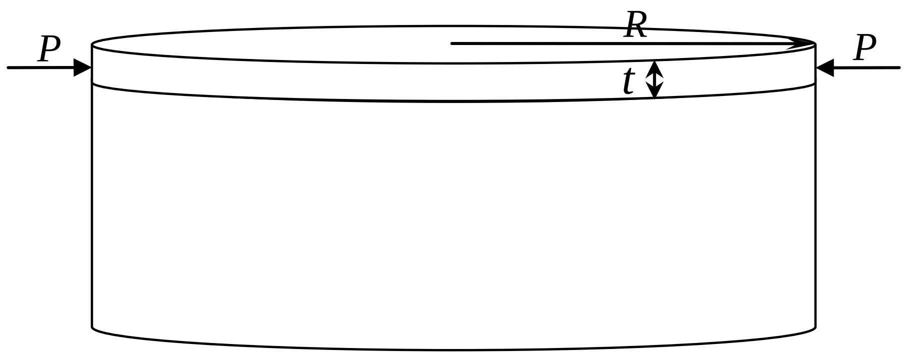

Fig.1. Schematics of a circular plate on an elastic foundation.

The equations of post-buckling of a thin plate on a linear Wrinkler foundation are [10]

$$\ddddot{w}+\frac{2\dddot{w}}{r}-\frac{\ddot{w}}{r^2}+\frac{\dot{w}}{r^3}-\frac{12}{t^2}\left(\dot{u}+\frac{\dot{w}^2}{2}+\nu\frac{u}{r}\right)\ddot{w}-\frac{12}{t^2}\left(\nu\dot{u}+\nu\frac{\dot{w}^2}{2}+\frac{u}{r}\right)\frac{\dot{w}}{r}+\frac{P}{D}\left(\ddot{w}+\frac{\dot{w}}{r}\right)+\frac{K}{D}w=0, \quad (2)$$

$$\ddot{u}+\frac{\dot{u}}{r}-\frac{u}{r^2}+\dot{w}\ddot{w}+\frac{(1-\nu)\dot{w}^2}{2r}=0, \quad (3)$$

where u is the in-plane displacement, ν the Poisson's ratio, t the thickness of the plate, D the bending stiffness of the plate, P the in-plane load, $\dot{w}$ denotes derivative with respect to r. The deformation is assumed to be axisymmetric. A material point at location r in deformed state is fully described by deflection $w(r)$ and in-plane displacement $u(r)$.

We normalize the quantities as the following: radius of a material point $x=r/R$, deflection $\bar{w}=w/t$, in-plane displacement $\bar{u}=uR/t^2$, in-plane load $\bar{P}=PR^2/D$, foundation stiffness $\bar{K}=KR^4/D$. The normalized governing equations are

$$\ddddot{\bar{w}}+\frac{2\dddot{\bar{w}}}{x}-\frac{\ddot{\bar{w}}}{x^2}+\frac{\dot{\bar{w}}}{x^3}-12\left(\dot{\bar{u}}+\frac{\dot{\bar{w}}^2}{2}+\nu\frac{\bar{u}}{x}\right)\ddot{\bar{w}}-12\left(\nu\dot{\bar{u}}+\nu\frac{\dot{\bar{w}}^2}{2}+\frac{\bar{u}}{x}\right)\frac{\dot{\bar{w}}}{x}+\bar{P}\left(\ddot{\bar{w}}+\frac{\dot{\bar{w}}}{x}\right)+\bar{K}\bar{w}=0, \quad (4)$$

$$\ddot{u}+\frac{\dot{u}}{r}-\frac{u}{r^2}+\dot{w}\ddot{w}+\frac{(1-\nu)\dot{w}^2}{2r}=0, \quad (5)$$

where $\dot{\bar{w}}$ denotes derivative with respect to x.

2.2. Strain energy release rate

In the film-substrate system, the interface crack may propagate after delamination buckling happens. The strain energy release rate is given in [11]

$$G=\frac{6(1-\nu^2)}{Et^3}\left(M^2+t^2\Delta N^2/12\right), \quad (6)$$

where E is the Young's modulus, M is the bending moment at the delamination edge, ΔN is the change of the in-plane force. The normalized form of these quantities are $\bar{M}=MR^2/(Dt)$, $\Delta\bar{N}=\Delta NR^2/D$, $\bar{G}=\bar{M}^2+\Delta\bar{N}^2/12$.

The crack is in a mixed mode of Mode I and Mode II. Define phase angle of mixture ψ. In this configuration the phase angle of mixture can be calculated as

$$\tan\psi=\frac{\sqrt{12}M\cos\omega+t\Delta N\sin\omega}{-\sqrt{12}M\sin\omega+t\Delta N\cos\omega}, \quad (7)$$

where $\omega=\omega(\phi,\varphi)$ is the phase factor, ϕ and φ are Dundur's elastic mismatch parameters. In the calculation, we set Poisson's ratio $\nu_{film}=\nu_{substrate}=1/3$. The details of value of ω can be found in the literature [11].

Considering the effects of mode mixture, the strain energy release rate is adjusted as

$$\bar{G}_{\psi}=\frac{\bar{G}}{f(\psi)}, \tag{8}$$

where the function $f(\psi)$ can be experimentally fitted. One of the common forms of $f(\psi)$ is

$$f(\psi)=1+(1-\lambda)\tan^2\psi , \tag{9}$$

where λ is the fitting parameter. In the calculation we set $\lambda=0.3$.

3. Asymptotic solutions

The perturbation method is an effective way of solving problems of plate undergoing large deflection[12]. We expand the displacement and load as

$$\bar{w}(x)=sw_1(x)+s^2w_2(x)+s^3w_3(x)+\dots, \tag{10}$$

$$\bar{u}(x)=su_1(x)+s^2u_2(x)+s^3u_3(x)+\dots, \tag{11}$$

$$\bar{P}=P_c+sP^{(1)}+\frac{1}{2}s^2P^{(2)}+\dots, \tag{12}$$

where s is the perturbation parameter.

Inserting Eqs. (10)-(12) into Eqs. (4) and (5) we obtain the decoupled equations of coefficients in the order of s, s^2, s^3. The out-of-plane equations are

$$\dddot{w}_1+\frac{2\ddot{w}_1}{x}-\frac{\ddot{w}_1}{x^2}+\frac{\dot{w}_1}{x^3}+P_c\left(\ddot{w}_1+\frac{\dot{w}_1}{x}\right)+\bar{K}w_1=0, \tag{13}$$

$$\dddot{w}_2+\frac{2\ddot{w}_2}{x}-\frac{\ddot{w}_2}{x^2}+\frac{\dot{w}_2}{x^3}-12\left(\dot{u}_1+\nu\frac{u_1}{x}\right)\ddot{w}_1-12\left(\nu\dot{u}_1+\frac{u_1}{x}\right)\frac{\dot{w}_1}{x}+P_c\ddot{w}_2+P^{(1)}\ddot{w}_1+\bar{K}w_2=0, \tag{14}$$

$$\begin{aligned}&\dddot{w}_3(x)+\frac{2\ddot{w}_3(x)}{x}-\frac{\ddot{w}_3(x)}{x^2}+\frac{\dot{w}_3(x)}{x^3}\\&-12\left(\dot{u}_1(x)\ddot{w}_2(x)+\dot{u}_2(x)\ddot{w}_1(x)+\frac{\dot{w}_1(x)^2\ddot{w}_1(x)}{2}+\nu\frac{u_1(x)\ddot{w}_2(x)+u_2(x)\ddot{w}_1(x)}{x}\right)\\&-12\left(\nu\frac{\dot{u}_1(x)\dot{w}_2(x)+\dot{u}_2(x)\dot{w}_1(x)}{x}+\nu\frac{\dot{w}_1(x)^2\dot{w}_1(x)}{2x}+\frac{u_1(x)\dot{w}_2(x)+u_2(x)\dot{w}_1(x)}{x^2}\right)\\&+P_c\left(\ddot{w}_3(x)+\frac{\dot{w}_3(x)}{x}\right)+P^{(1)}\left(\ddot{w}_2(x)+\frac{\dot{w}_2(x)}{x}\right)+\frac{1}{2}P^{(2)}\left(\ddot{w}_1(x)+\frac{\dot{w}_1(x)}{x}\right)+\bar{K}w_3(x)=0\end{aligned}, \tag{15}$$

The in-plane equations are

$$\ddot{u}_1+\frac{\dot{u}_1}{x}-\frac{u_1}{x^2}=0, \tag{16}$$

$$\ddot{u}_2+\frac{\dot{u}_2}{x}-\frac{u_2}{x^2}+\dot{w}_1\ddot{w}_1+\frac{(1-\nu)\dot{w}_1^{\,2}}{2x}=0, \tag{17}$$

$$\ddot{u}_3+\frac{\dot{u}_3}{x}-\frac{u_3}{x^2}+\dot{w}_1\ddot{w}_2+\dot{w}_2\ddot{w}_1+\frac{(1-\nu)\dot{w}_1\dot{w}_2}{x}=0, \tag{18}$$

The edge is clamped. The boundary conditions are

$$u\big|_{x=0}=0,\quad \dot{w}\big|_{x=0}=0,\quad \left(\ddot{w}-\frac{\dot{w}}{r}\right)\Big|_{x=0}=0, \tag{19}$$

$$u\big|_{x=1}=0,\quad w\big|_{x=1}=0,\quad \dot{w}\big|_{x=1}=0, \tag{20}$$

We use the central deflection w_0 as the perturbation parameter. We have additional condition $w_1(0)=1$; $w_j(0)=0 \quad j\neq 1$. Inspecting the six equations (13)-(18) together with boundary conditions (19) and (20), we can easily obtain solutions $u_1=0, w_2=0$ and $P^{(1)}=0$. Therefore, only three equations (13), (15) and (17) are to be solved. The general solution of Eq.(13) is

$$w_1(x)=C_1J_0(\alpha x)+C_2J_0(\beta x), \tag{21}$$

where $\alpha=\left(\dfrac{P_c-\sqrt{P_c^2-4\bar{K}}}{2}\right)^{1/2}, \beta=\left(\dfrac{P_c+\sqrt{P_c^2-4\bar{K}}}{2}\right)^{1/2}$.

Inserting boundary conditions into Eq. (21) we obtain

$$C_1J_0(\alpha)+(1-C_1)J_0(\beta)=0, \tag{22}$$

$$\alpha C_1J_1(\alpha x)+\beta(1-C_1)J_1(\beta x)=0, \tag{23}$$

Once the foundation stiffness $\bar{K}$ is given, we can obtain P_c and C_1 by solving Eqs. (22) and (23). Subsequently we use Eq. (17) to solve u_2. A simple way to obtain u_2 is expanding Bessel function into Taylor series. Once w_1 and u_2 are known, we use Eq. (15) to solve $P^{(2)}$. We multiply $w_1(x)x$ on both sides of Eq. (15) and integrate from 0 to 1. Using the feature of Bessel function, the solving process can be simplified. Similar process can be iterated to obtain more accurate solution by expanding displacement fields to higher order terms and solving higher order equations. Once the displacement fields are known, the bending moment at the edge, the change of in-plane force and the energy release rate can be obtained.

4. Results and discussions

The critical buckling load P_c at different value of foundation stiffness $\bar{K}$ is plotted in Fig. 2. It can be seen that as the foundation become stiffer, the critical buckling load increases. Using the way of normalization in this work, the relation between P_c and $\bar{K}$ is approximately linear. This result is consistent with the literature, where different normalized quantities are used to plot [13].

The post-buckling path of central deflection is plotted in Fig. 3. The horizontal axis σ/σ_c is the in-plane stress over the critical buckling stress. Note that, the critical buckling stress σ_c at different foundation stiffness is different. But for the aid of comparison, we plot them into the same figure. The curve with $\bar{K}=0$ reduces to classical post-buckling path of a thin circular plate without foundation. It is found that stiff foundation lowers the central deflection, especially in the initial stage of post-buckling.

The mode-adjusted energy release rate G_ψ given by Eqs. (6)-(9) is plotted in Fig. 4. The displacement fields solved with perturbation method are relatively accurate, while the bending moment and in-plane force are not especially when the post-buckling stress is high. Therefore, in

Fig. 4 we plot the post-buckling stress up to $\sigma / \sigma_c \approx 4$. It can be seen that the more compliant the substrate is, the easier for the interface crack to propagate after buckling.

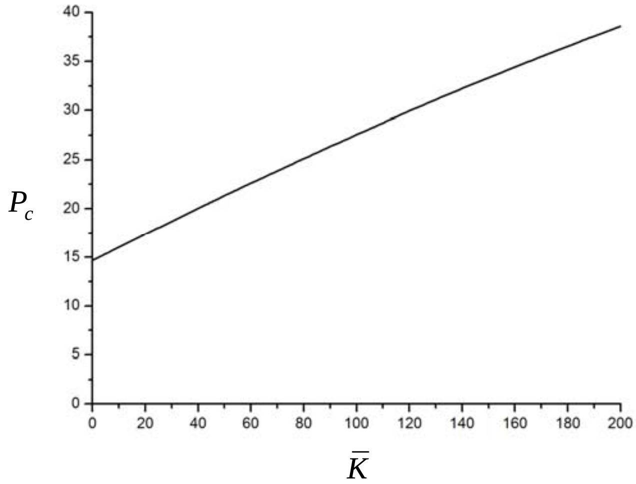

Fig. 2. Critical buckling load at different foundation stiffness.

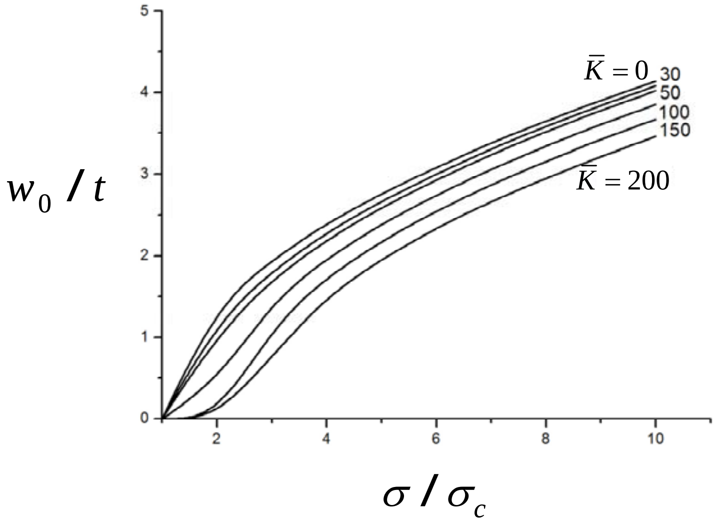

Fig. 3. Post-buckling path with different foundation stiffness.

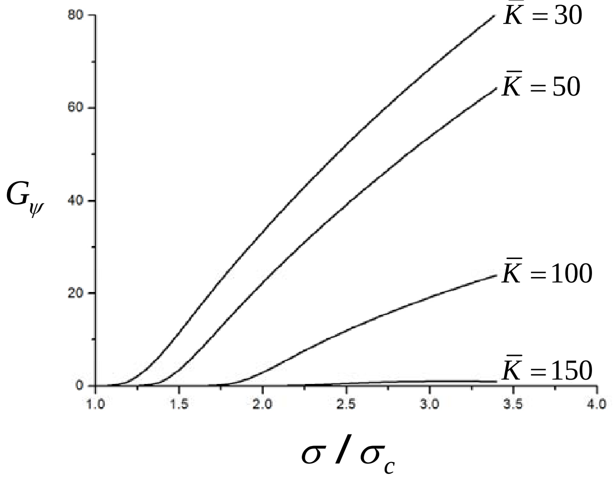

Fig. 4. Mode-adjusted energy release rate of interface crack with different foundation stiffness.

5. Conclusion

In the thin film-substrate system, compressive stress in the film can induce delamination buckling. We study the effects of compliance of the substrate by modeling the system as a plate on an elastic foundation. We use perturbation method to obtain the asymptotic solutions of post-buckling deformation and calculate the mode-adjusted energy release rate of the interface crack after buckling happens. The results show that the more compliant the substrate is, the easier for the film to buckle and easier for the interface crack to propagate after buckling.

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