Thermally induced creep rupture of fiber bundles

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Abstract Subcritical fracture driven by thermally activated crack nucleation is studied in the framework of a fiber bundle model. Based on analytic calculations and computer simulations, we show that, in the presence of stress inhomogeneity thermally activated cracking results in an anomalous size effect, i.e., the average lifetime of the system decreases as a power-law of the system size. We propose a modified Arrhenius law which provides a comprehensive description of the load, temperature, and size dependence of the lifetime of the system. On the microscopic level, thermal fluctuations trigger bursts of breaking events which proved to have a power-law size distribution. The waiting times between consecutive bursts are also power-law distributed with an exponent switching between 1 and 2 as the load and temperature are varied. Analyzing the structural entropy and the location of consecutive bursts, we show that, in the presence of stress concentration, the acceleration of the rupture process close to failure is the consequence of damage localization.

Keywords Subcritical fracture, Fiber bundle model

1. Introduction

Sub-critical rupture, occurring under a constant load below the fracture strength of materials, is of fundamental importance in a wide range of physical, biological, and geological systems. Depending on the type of materials, creep rupture can have a wide variety of microscopic origins from the existence of frictional interfaces through the viscoelasticity of the constituents, to thermally activated aging processes. Recent experimental and theoretical investigations revealed the high importance of thermally activated micro-crack nucleation in creep phenomena with consequences reaching even to geological scales [1–7]. Under creep loading failure often occurs as a sudden unexpected event following a short acceleration period which addresses safety problems for e.g. components of engineering constructions. Additionally, creep rupture underlies natural catastrophes such as landslides, stone and snow avalanches and it is also involved in the emergence of earthquakes.

On the macroscopic scale the rupture process is characterized by the strain-time diagram and by the lifetime of the system, which both have a complex dependence on the external load and on the temperature. In spite of the smooth macroscopic evolution, thermally activated breakdown proceeds in bursts on the microscopic scale. They may be exploited to gain information about the approach of the system to failure. In the present paper we investigate this problem in the framework of a fiber bundle model of thermally activated breakdown. In order to reveal the effect of the range of load redistribution we carry out computer simulations considering strongly localized stress redistribution after failure events and compare the outcomes to the analytic results obtained in the mean field limit [10–12].

2. Model

Our approach is based on the fiber bundle model (FBM), which has proven very successful during the past decades for the investigation of fracture phenomena [1–15]. In the model we consider N parallel fibers having a brittle response with identical Young modulus E. The bundle is subject to a constant external stress σ parallel to the fibers' direction. During the past decades several ways have been proposed to introduce time dependent rupture in stochastic fracture models. Following the pioneering works of Coleman on time dependent FBM [5], the models were further extended to a broad class of time dependent damage accumulation laws and fiber strength by Phoenix and Curtin [6–8]. In our work we apply the approach of Guarino et al. [1–4], i.e. we assume that the local load σ_i of fibers has time-dependent fluctuations $\xi_i(t)$ due to the presence of thermal noise so that the actual load of fiber i at time t reads as

$$\sigma_i(t) = \sigma_i^0(t) + \xi_i(t). \tag{1}$$

Here $\sigma_i^0(t)$ denotes the deterministic part of the stress, i.e., the local stress arising due to the external load and to load transfer following breaking events. The fibers have a finite strength characterized by a failure threshold σ_{th}^i, which is, in general, a random variable. A fiber fails during the time evolution of the bundle when the total load on it $\sigma_i(t)$ exceeds the respective threshold value σ_{th}^i. For simplicity, we assume that the system consists of homogeneous fibers, i.e. all the breaking thresholds are the same $\sigma_{th}^i = \sigma_{th}$, $i = 1, \ldots, N$, where $\sigma_{th} = 1$ is set. The assumption of homogeneity implies that there is no quenched disorder in the system. Thermally induced stress fluctuations $\xi(t)$ have a Gaussian distribution with zero mean and a variance controlled by the temperature T of the system

$$p(\xi) = \frac{1}{\sqrt{2\pi T}} \exp\left(-\frac{\xi^2}{2T}\right), \tag{2}$$

from which the complementary cumulative distribution follows as $P(\xi) = \int_{\xi}^{+\infty} p(x)dx$. The strength of thermal fluctuations is controlled by the value of T which can be scaled to the absolute temperature of the system. After fiber breakings, the load of broken fibers has to be redistributed over the remaining intact ones. In order to understand the effect of the range of load transfer on the process of thermally enhanced creep, we consider two limiting cases for the load redistribution: in the case of equal load sharing (ELS) all surviving fibers overtake equal fraction of the load. ELS ensures that the stress distribution remains homogenous in the bundle until the end of time evolution which also facilitates to perform analytical calculations. To study the effect of stress inhomogeneity on thermally activated breakdown, in our model the fibers are organized on a square lattice of size $L \times L$ and localized load sharing (LLS) is considered: the load of broken fibers is redistributed over their nearest intact neighbors, giving rise to high stress concentration around failed regions. Since the LLS case cannot be investigated by analytical means, computer simulations were carried out varying the load σ, temperature T, and the lattice size L in broad ranges.

3. Results and Discussions

Subjecting the bundle to a constant external load σ, two competing physical mechanisms contribute to the failure of fibers: When the load is small enough even a single fiber can sustain the entire load and the load increments arising in the vicinity of failed fibers are not sufficient to trigger further breakings. Hence, in this load regime, the failure process is dominated by the thermal fluctuations and there is practically no difference between ELS and LLS calculations since the range of interaction is irrelevant. However, at high load values $\sigma \to \sigma_{th}$, the load redistributions give rise to considerable increments of the local load on intact fibers leading to additional breakings. In the initial state of the system all the fibers have the same load $\sigma_i^0 = \sigma$, $i = 1, \ldots, N$. When a fiber breaks due to thermal noise $\sigma_i^0 + \xi_i \geq \sigma_{th}$, the deterministic part of the load σ_i^0 is transferred to its four intact neighbors resulting in the increment $\Delta\sigma^0 = \sigma/4$. If the updated load exceeds the breaking threshold $5\sigma/4 > \sigma_{\rm th}$ the fibers break again transferring the load further to their intact neighbors. Once this breaking sequence starts, removing all four neighbors of the initial one, it does not stop until all fibers break leading to macroscopic fracture. It follows that due to the localized stress transfer, the system has a critical load $\sigma_c = 4\sigma_{\rm th}/5$ above which even a single fiber breaking triggers the immediate collapse [10–12].

3.1. Scaling behavior of lifetime

The most important macroscopic characteristic quantity of the system is the average lifetime $\langle t_f \rangle$ which has a finite value even at zero external stress $\sigma = 0$ in the model if the temperature is finite $T > 0$. Under the assumption of equal load sharing it has been shown analytically in FBMs with a fixed breaking threshold σ_{th} that $\langle t_f \rangle$ follows the Arrhenius law $\langle t_f \rangle \propto \left(\sqrt{2\pi T}/\sigma\right)\exp\left((\sigma_{th}-\sigma)^2/2T\right)$ without any dependence on the system size N [1,2].

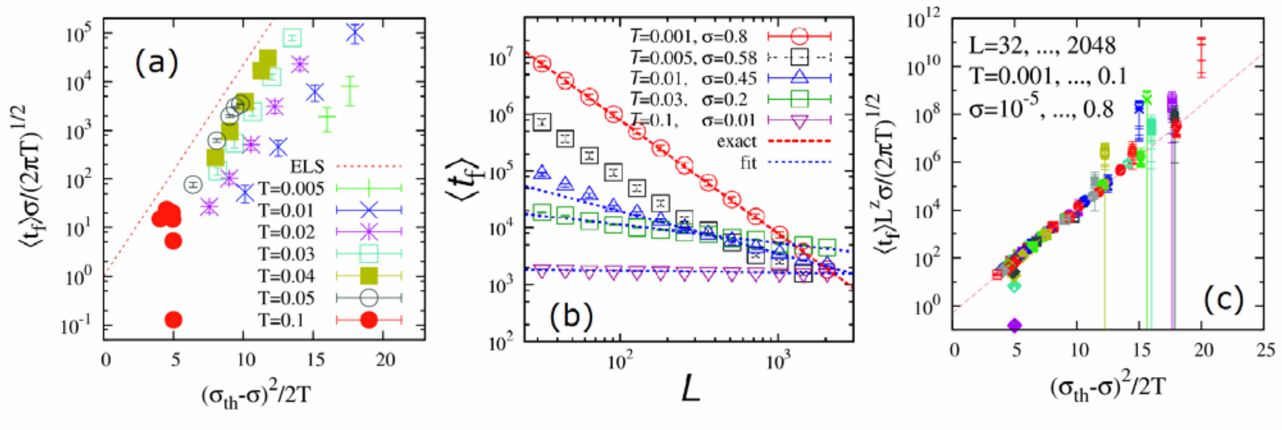

Figure 1. (a) Scaling plot of lifetime $\langle t_f \rangle$ obtained at different load σ and temperature T values by LLS simulations on a square lattice of size $L = 1024$. No data collapse is obtained. The simple Arrhenius law is indicated by the straight line of ELS. (b) Size scaling of lifetime. When stress concentration is dominated, power-law dependence is obtained with a high precision. Note that $\sigma_c = 0.8$ in the model. (c) Correcting the Arrhenius scaling form with the size dependence of lifetime a high quality data collapse is obtained.

Figure 1(a) presents the scaling plot of lifetime obtained by our computer simulations with the LLS FBM at the system size $L = 1024$ varying the load σ and the temperature T. No data collapse is obtained in the figure, which implies that the simple Arrhenius law does not hold when stress concentrations are present [10]. Our analytical and numerical calculations revealed that the interplay of stress concentrations and annealed disorder results in an anomalous size effect of the lifetime of the system, which is responsible for the discrepancy observed above [10]. In order to clarify the size scaling of the lifetime we carried out computer simulations varying the system size in a broad range $L = 32-2048$. These simulations showed that at any finite load value σ the average lifetime of the system decreases as a power law of the lattice size $\langle t_f \rangle \propto L^{-z(T,\sigma)}$. See Fig. 1(b). For the limiting cases of low ($\sigma \to 0$) and high ($\sigma \to \sigma_c$) loads the exponent z can be obtained analytically to be 0 and 2, respectively. Numerical calculations showed that varying the temperature and external load the scaling exponent z takes values between the two limits $0 \le z(T,\sigma) \le 2$ [10]. We propose a modified form of the Arrhenius law which takes into account the size scaling of lifetime

$$\langle t_f \rangle \cong \frac{L^{-z(T,\sigma)}}{2} \frac{\sqrt{2\pi T}}{\sigma} \exp\left(\frac{(\sigma_{th} - \sigma)^2}{2T} \right). \tag{3}$$

Figure 1(c) demonstrates that the modified Arrhenius law provides an excellent scaling of the numerical data obtained by computer simulations of LLS FBM varying the system size L, the temperature T, and the external load σ in the ranges $L = 32 - 2048$, $T = 0.001 - 0.1$, and $\sigma = 10^{-5} - 0.8$, respectively.

3.2. Microscopic time evolution

On the microscopic level, the fibers primarily break due to thermal fluctuations when their actual load exceeds the fixed breaking threshold $\sigma_i(t) > \sigma_{th}$. Of course, depending on the temperature in a given time step more than one fiber can break at the same time. The load of broken fibers is then redistributed over the intact ones according to the selected load sharing rule. The load increments on intact fibers can trigger additional breakings and eventually generate an entire breaking burst. Hence, irrespective of the range of load sharing, the failure of the bundle proceeds in bursts which are separated by silent periods with no breakings. The size of the burst Δ is simply the number of fibers breaking in a correlated trail of failure events, while the waiting time t_w is defined as the number of iteration steps without breaking events between two consecutive avalanches.

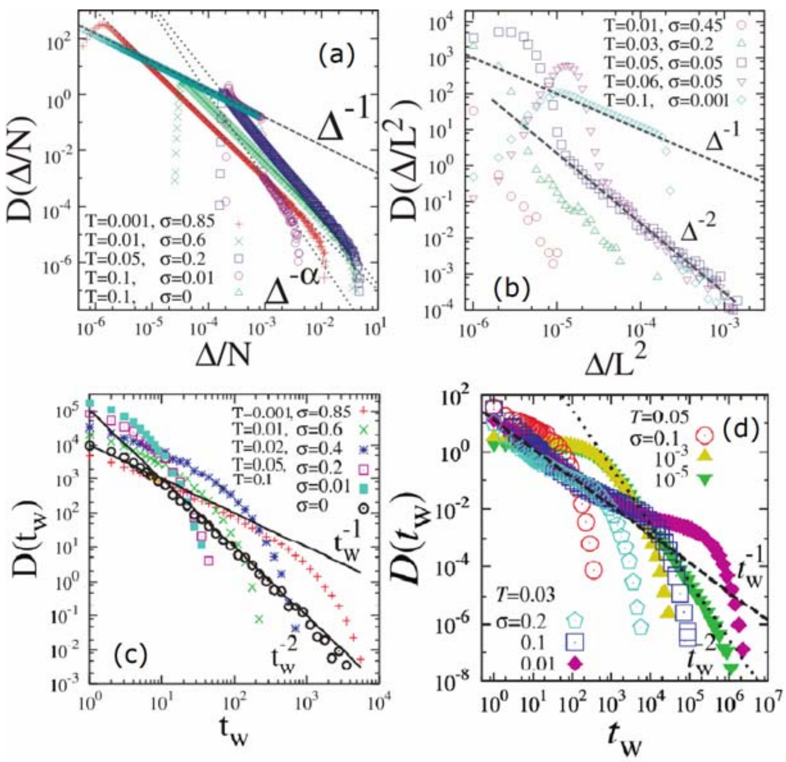

Figure 2. Size distribution of bursts for ELS (a) and LLS (b), and that of waiting time for ELS (c) and LLS (d). Power law functional forms are obtained followed by an exponential cutoff.

Bursts of breaking events generate acoustic waves so that they are responsible for the crackling noise accompanying the process of creep rupture [3]. In order to characterize the statistics of crackling events of our model, we analyzed the probability distribution of burst sizes $P(\Delta)$ and waiting times $P(t_w)$. For the ELS case the burst size distribution $P(\Delta)$ proved to have a power law functional form $P(\Delta) \propto \Delta^{-\alpha}$ with an exponential cutoff. See Fig. 2(a). The value of the exponent has a complex dependence on the load and temperature [10]. For localized load sharing computer simulations revealed that $P(\Delta)$ has a Gaussian form for small bursts followed by a power law regime over a broad range. See Fig. 2(b). It is important to emphasize that the power law exponent α of the LLS case does not vary continuously with the model parameters. Instead, it suddenly switches from $\alpha = 1$ to $\alpha = 2$ when the external load approaches the critical value $\sigma \to \sigma_c$, which is accompanied by the shrinking of the Gaussian regime. In the vicinity of σ_c, the system becomes very sensitive to the thermal fluctuations and cannot tolerate large bursts, which is expressed by the higher value of the exponent α (Figs. 2(a) and (b)). This is an important unique feature of thermally driven creep rupture; when quenched disorder dominates the rupture process the opposite effect occurs, i.e. the burst exponent decreases when approaching catastrophic failure [13–15]. The probability distribution of waiting times $P(t_w)$ shows the same qualitative behavior as the distribution of burst sizes, i.e. $P(t_w)$ has a power law functional form $P(t_w) \propto t_w^{-\beta}$ with an exponential cutoff, where the value of the exponent β depends both on the load and on the temperature [12]. See Figs. (c) and (d)

3.2. Acceleration due to localization

The overall time evolution of the rupture process can be characterized by studying the average waiting time $\langle t_w \rangle$ between consecutive bursts as a function of the fraction of broken fibers ϕ, where $0 \le \phi \le 1$ holds. For ELS $\langle t_w \rangle(\phi)$ can be cast into a closed analytical form, while for LLS we determined it numerically [12]. Calculations showed that at zero load the breaking process continuously slows down in such a way that the average waiting time has a power law divergence $\langle t_w \rangle \propto (1-\phi)^{-1}$ when approaching macroscopic failure $\phi \to 1$. It can be observed in Fig. 3(a) that for finite load values $\sigma > 0$ the slow-down is followed by acceleration such that the accelerating regime starts earlier when the load increases. It is interesting to note that when the load is high enough, acceleration is obtained right from the beginning of the process, i.e. in this parameter regime the rupture process continuously accelerates towards failure [12]. The most remarkable feature of the results is that in the case localized load sharing the qualitative behavior remains the same, however, the acceleration sets on earlier (see Fig. 3(a)).

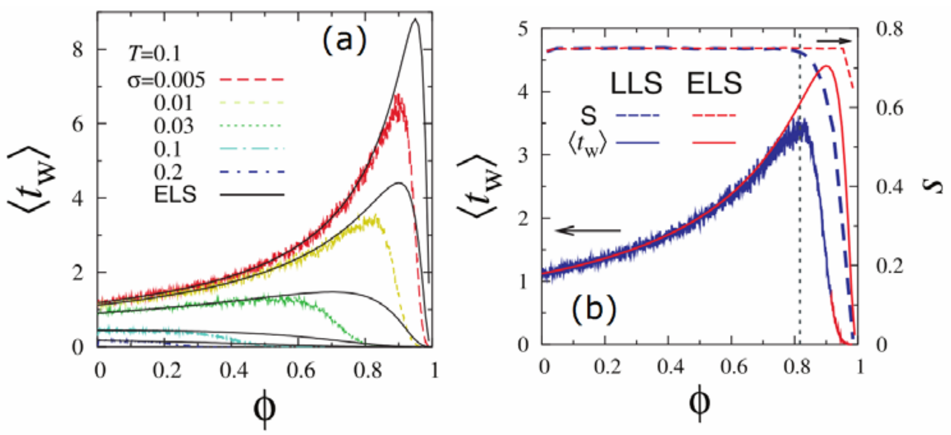

Figure 3. (a) Average waiting time for ELS and LLS as function of the fraction of broken fibers ϕ. (b) Comparison of the curves of average waiting time and entropy as a function of ϕ. For LLS the

entropy starts to decrease when the acceleration sets on.

In order to understand the background of early acceleration for LLS systems, Figs. 4(a)–(d) presents snapshots of the time evolution of an LLS bundle. Two regimes can easily be distinguished: at the beginning of the process cracks occur randomly all over the bundle (Fig. 4(a)). As time elapses more cracks nucleate and some of the previous cracks extend their size (Fig. 4(b)). Along the perimeter of growing cracks large stress is concentrated on the intact fibers, which increases the probability of further crack growth. As a consequence, one of the cracks gets selected and starts to grow rapidly, i.e. all bursts get localized along the front of a growing crack which then accelerates the process and leads to global failure (Figs. 4(c) and (d)) [12]. To quantify the degree of localization we introduced a so-called structural entropy S, which measures how scattered the new breaking events are in the bundle [12]. Large value of the entropy $S \to 1$ implies random cracking, while the small one $S \to 0$ marks the onset of localization. It can be observed in Fig. 3(b) that in the ELS case where there is no stress concentration and spatial correlation in the system the entropy is always high $S \approx 0.8$, even during the acceleration phase.

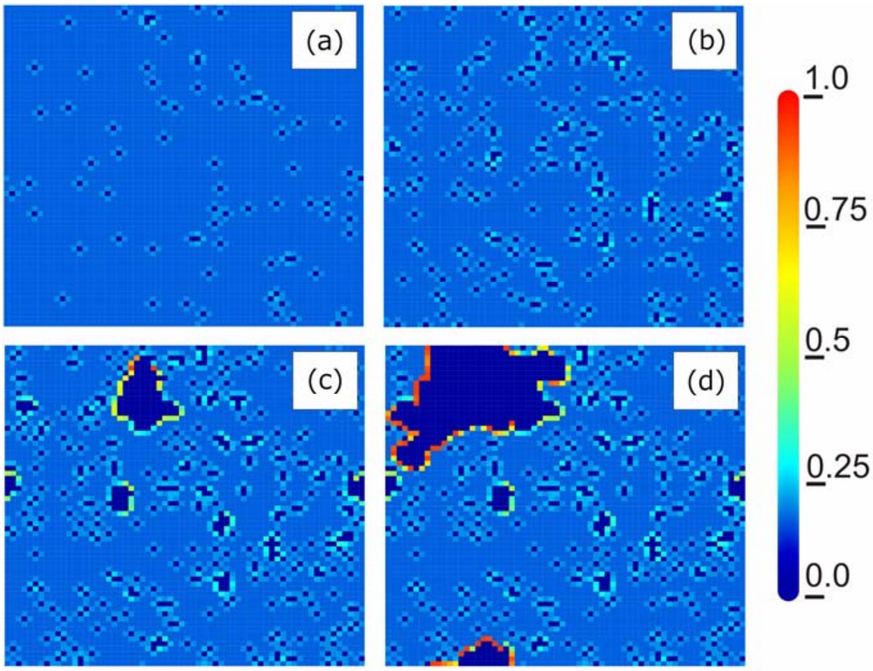

Figure 4. Snapshots of an evolving system, where fibers are colored according to their load. Deep blue represents zero load hence indicating cracks in the system.

On the contrary, in the presence of stress concentration, the entropy is high during the slow-down phase, however, it rapidly decreases to zero as soon as acceleration sets on (see Fig. 3(b)). Our results give a quantitative proof that the acceleration towards failure occurs due to the spatial localization of breaking events to the front of a growing crack [12].

4. Conclusions

Based on a fiber bundle model we showed that stress inhomogeneity play a crucial role in the process of thermally activated subcritical rupture giving rise to a broad spectrum of novel behaviors. Stress concentrations, arising in the vicinity of failed regions of the material, make the system more sensitive to thermal fluctuations. As a consequence, an astonishing size effect emerges where the average time-to-failure of the model system decreases as a power law of the system size. The size scaling exponent depends both on the temperature and on the external load. We proposed a modified form of the Arrhenius law of lifetime which provides a comprehensive description of thermally activated breakdown phenomena [10–12].

On the micro-level, thermally driven breakdown proceeds in bursts of breakings which are separated by waiting times. The size distribution of bursts and the distribution of waiting times between consecutive events proved to have power law functional forms followed by an exponential cutoff. The power law exponents have a complex dependence on the load and temperature of the system [10,12]. To characterize the overall time evolution of the system, we analyzed the average waiting time between bursts as a function of the fraction of broken fibers. Calculations showed that the thermally induced creep process has two phases: at low loads and high temperatures the process slows down after the load is set, which is then followed by an accelerating period. However, when the load is high enough the system continuously accelerates towards failure. We demonstrated that in the case of localized load sharing, the stress concentration around cracks leads to spatial correlation of breaking events and to an enhanced breaking probability which in turn is responsible for the early acceleration [10–12].

In order to quantify the effect of spatial correlation on the time evolution of the creep rupture process, we evaluated the structural entropy of avalanches and their consecutive positioning. As a very important outcome, our calculations revealed that the decreasing extension and the spatial localization of avalanches to a bounded region of the specimen are responsible for the acceleration towards macroscopic failure. Final failure is driven by a single growing crack which becomes unstable as the avalanches localize to its perimeter [12].

Acknowledgements

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