ECM-Based Statistical Simulation of Progressive Failure in Symmetric Laminates Damaged by Transverse Ply Cracking

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Abstract The purpose of the present paper is to propose an analytical model that enables the evolution of transverse ply cracking to be investigated in fiber reinforced laminated composites subjected to in-plane tensile loading and thermal residual stress. The strain energy release rate associated with matrix cracking in transverse layer is derived using a 2-D shear-lag stress analysis and the equivalent constraint model (ECM). An energy criterion, based upon the fracture mechanics approach, is proposed to describe the initiation and propagation of the transverse ply cracking. This work is also concerned with statistical distributions of the defects with due consideration given to the scale size effect. Monte-Carlo simulation technique is implemented to predict mechanical behaviors of composite structures by taking into account the statistic of critical fracture toughness G_c of the transverse ply. The results deduced from the numerical procedure are in good agreement with the experimental results obtained for laminated composites formed by unidirectional fiber reinforced laminae with different orientations.

Keywords Laminated composites, Transverse cracking, Fracture mechanics, Statistical analysis, Modelling

1. Introduction

Composite laminates have gained widespread acceptance in many structural applications on account of their high specific strength and stiffness. There is a strong need of comprehensively understanding damage mechanisms to ensure the structural integrity of composite materials.

Performance in service of a composite structure is influenced by the progressive occurrence and interaction of some or all of the many multi-mechanisms of damage such as matrix cracking, delamination, fiber fracture and fiber/matrix debonding under loading and environment conditions. The early stage of damage is dominated is matrix cracking parallel to the fibers in the off-axis piles. These matrix cracks develop in the fiber direction and extend across the laminates from the free edges of test specimens. The number of cracks increases with increasing the load until a saturation crack density. Matrix cracking is usually referred as transverse cracking because the crack plane is commonly transverse to the laminate middle-plane. It can result in significant deterioration of the residual stiffness and loading-bearing capacity of laminates and triggers development of other more harmful damage mechanisms [1].

The detailed mechanisms of transverse ply cracking in laminates have been a subject of active research effort. A large variety of analyses have been conducted to investigate the damage phenomena of composite laminates: different modifications of the shear-lag model [2], variational approach based on minimization of complementary energy [3], continuum damage mechanics approach [4] and numerical methods, for example, based on finite element analysis [5]. The overwhelming majority of studies investigating behavior and properties of composite laminates with matrix cracks assume that cracks are equally spaced [6]. Such a deterministic approach, which ignores the fact that transverse cracking is a progressive damage mode, predicts the appearance of many transverse cracks simultaneously when the first transverse cracking strain is reached [7]. In fact, the transverse matrix cracking is an inherently stochastic process due to the random variations of local material properties of the plies caused by inner original defects [8]. The random distribution of these defects is the major factor that affects both crack initiation and propagation. Prediction on the durability of these materials tends to be probabilistic in character. Modelling of transverse cracking in laminated composites should, therefore, consider the details of its construction and the use of statistical analysis.

In the present study we derive an analytical model which has the capacity of representing and predicting the fracture of laminated composite structures damaged by transverse cracking, appropriate for the mechanical analysis of fiber reinforced composite laminates. The energy release rate associated with matrix cracking in the transverse layer of general symmetric composite laminates is presented using a 2-D shear lag stress analysis and the equivalent constraint model (ECM) [9,10]. An energy criterion, based on fracture mechanics approach, is proposed to predict and describe the ignition and propagation in symmetrical composite laminates subjected to in-plane loading. This work is also concerned with a stochastic description of the inner defects with due consideration given to the scale size effect. The random properties are introduced with the use of Weibull two-parameter probability density function. A numerical procedure is developed to investigate the multiplication of transverse cracking and predict the statistical failure strengths of composite structures by taking into account the statistic of critical fracture toughness. The results are also compared with those obtained from experiments and other existing models.

2. Formulation and algorithm

2.1. Analytical model

Micro-cracking has been a mechanism of failure extensively studied in many investigations. This mechanism occurs under transverse tensile or longitudinal shear loading conditions. A known analytical modeling of mechanical properties due to transverse cracking was performed by using the suggested “equivalent constraint model” (ECM) to determine the in-plane stress distributions in the damaged layers [11, 12]. The reduction in stiffness properties due to transverse ply crack was described by two in-situ damage effective functions introduced in Refs. [9, 11, 12]. It was shown that a crack lamina behaves within a laminate in a different manner compared to an infinite effective medium containing many cracks [13].

Our simulation reproduces the initial crack and its propagation in the thickness direction using the energy criterion. Indeed, the potential energy method has been presented by Zhang et al.[12, 14] for the evaluation of the energy release rate for damage initiation and propagation due to transverse ply cracking. The required energy release rate, G, for damage growth can be calculated using energy approach and then predicting that damage will grow when G exceed the critical energy release rate, G_c , or toughness of the material.

Energy release rate G associated to the appearance of a crack for a given stress state according to fracture mechanics is defined by the following expression,

$$G = \frac{\partial}{\partial A}\Pi(\sigma, A). \tag{1}$$

where Π is the strain energy of the whole laminate. A is crack surface area. σ is the applied laminate stress.

An opening displacement of transverse crack (COD) solution for a crack is introduced to obtain the strain energy release rate due to matrix cracking, such that

$$G_n(\sigma, D_n) = g_n(L, D_n)\left(\frac{\sigma_x^{0(2)}}{Q_{22}^0}\right)^2. \tag{2}$$

where $\sigma_x^{0(2)}$ is the x-axis normal stress in the xy-plane and superscript 0 is used to indicate the quantities that belongs to the undamaged state. Q_{ij}^0 are the in-plane stiffness components of an undamaged lamina, and $g_n(L, D_n)$ is the normalized energy release rate, which is independent of the applied load. A complete description of the strain energy based model was presented by Zhang [10, 15].

A damage criterion is proposed to predict the evolution of matrix cracking. Estimated quantity is compared with the associated critical value of the stain energy release rate

$$G(\sigma, D_n) = G_c. \tag{3}$$

where G_c denotes the fracture toughness of the composite materials.

2.2. Modelling of progressive cracking

The origins of transverse cracking are the inherent material defects such as the microcracks, voids, debonded fibers, areas of high fiber volume fraction, etc [8]. This causes the transverse layer to have a statistical nature of the fracture toughness along its length. Thus the transverse cracking propagation can be investigated by assigning a random distribution of the fracture toughness along the transverse layers.

In the present paper, the crack multiplication can be simulated by dividing the initial gage length into equal elements along the direction of the length $2L$ and randomly assigning fracture toughness G_c to each of them in accordance with two-parameter Weibull distribution.

Based on the Weibull statistics, the fracture toughness is given as

$$G_c = G_0 \cdot \left[\ln\left(\frac{1}{1-F}\right)\right]^{\frac{1}{m}}. \tag{4}$$

where G_0 represents the scale parameter, or characteristic quantity of the material. The Weibull modulus or shape parameter m controls the degree of disorder in the distribution, experimentally found to describe a variety of materials. $F \in [0,1]$ is a random number. The parameters m and G_0 can be calculated by statistical method

$$\overline{G}_c = E(G_c) = G_0 \Gamma\left(1 + \frac{1}{m}\right), \tag{5}$$

$$D(G_c) = G_0^2\left\{\Gamma\left(1 + \frac{2}{m}\right) - \left[\Gamma\left(1 + \frac{1}{m}\right)\right]^2\right\}. \tag{6}$$

Where $E(G_c)$ and $D(G_c)$ are the mean and variance of random variable, respectively. In the approach we use here, transverse cracking at some location takes place when the energy released by a cracking event becomes equal to the critical strain energy release rate, G_c , at this position.

In fiber-reinforced composites, the largest portion of the loads is resisted by the fibers. When matrix crackings occur, the internal loads must redistribute to other areas of the structures, and may cause a structural collapse [16]. For every state of stress, a simple criterion such as Hoffman criterion is expressed mathematically in the following fashion,

$$\frac{\sigma_1^2-\sigma_1\sigma_2}{X_tX_c}+\frac{\sigma_2^2}{Y_tY_c}+\frac{X_c-X_t}{X_tX_c}\sigma_1+\frac{Y_c-Y_t}{Y_tY_c}\sigma_2+\frac{\tau_{12}^2}{S^2}=1. \tag{7}$$

in which σ_1, σ_2 and τ_{12} are longitudinal stress, transverse stress and shear stress, respectively. X_t, X_c are the tensile strength and compressive strength of the unidirectional layer parallel to the fiber direction; Y_t, Y_c are the tensile strength and compressive strength one of the unidirectional layer transverse to the fiber direction; S is the shear strength of the unidirectional layer transverse and parallel to the fiber direction, respectively.

It should be mentioned that in the present study the ultimate failure of materials takes place when the stress state of the primary load-bearing lamina satisfies the condition mentioned [17].

2.3. Numerical algorithm

The simulation was conducted by controlling the stress loading of the model. Fig. 1 shows the progressive failure analysis algorithm for estimating damage growth:

Step 1. Ply material and ply orientation are selected to form a laminate. The critical strain energy release rate, G_c , is assigned to of each element. A virtual crack was introduced in all positions as possible sites for failure and the work performed to close the crack surfaces [18] was calculated. New transverse cracking happens in any location once the energy released by a cracking event is equal to the critical value. The virtual cracking procedure was repeated unceasingly until cracking is terminated under the same applied stress.

Step 2. The equivalent constraint model is employed to analyzing the damaged lamina. In the ECM, all the laminae below and above the damaged lamina under consideration are replaced with homogeneous layers [9]. The reduced stiffness properties of the damaged layer can be calculated by applying the laminated plate theory to the ECM. Thus, the in-plane microstresses of the primary loading-bearing lamina can be obtained by the constitutive relationship.

Step 3. With the stresses calculated, they are substituted in the failure criteria Eq. (7) to check for failure. When the loads are increased monotonically, the matching strains are computed and the resulting stresses are substituted in the failure criteria until they are satisfied. The ultimate load of the laminate is thus determined.

4. Results and discussion

A detailed analysis is conducted to assess the predictive capabilities of the present methodology. The mechanical properties of each type of unidirectional laminate used in this work are given in Ref [17]. Each set of Monte-Carlo simulation consists of 200 data points. The element length of 2.5×10^{-2} mm is chosen for the sake of efficiency [7].

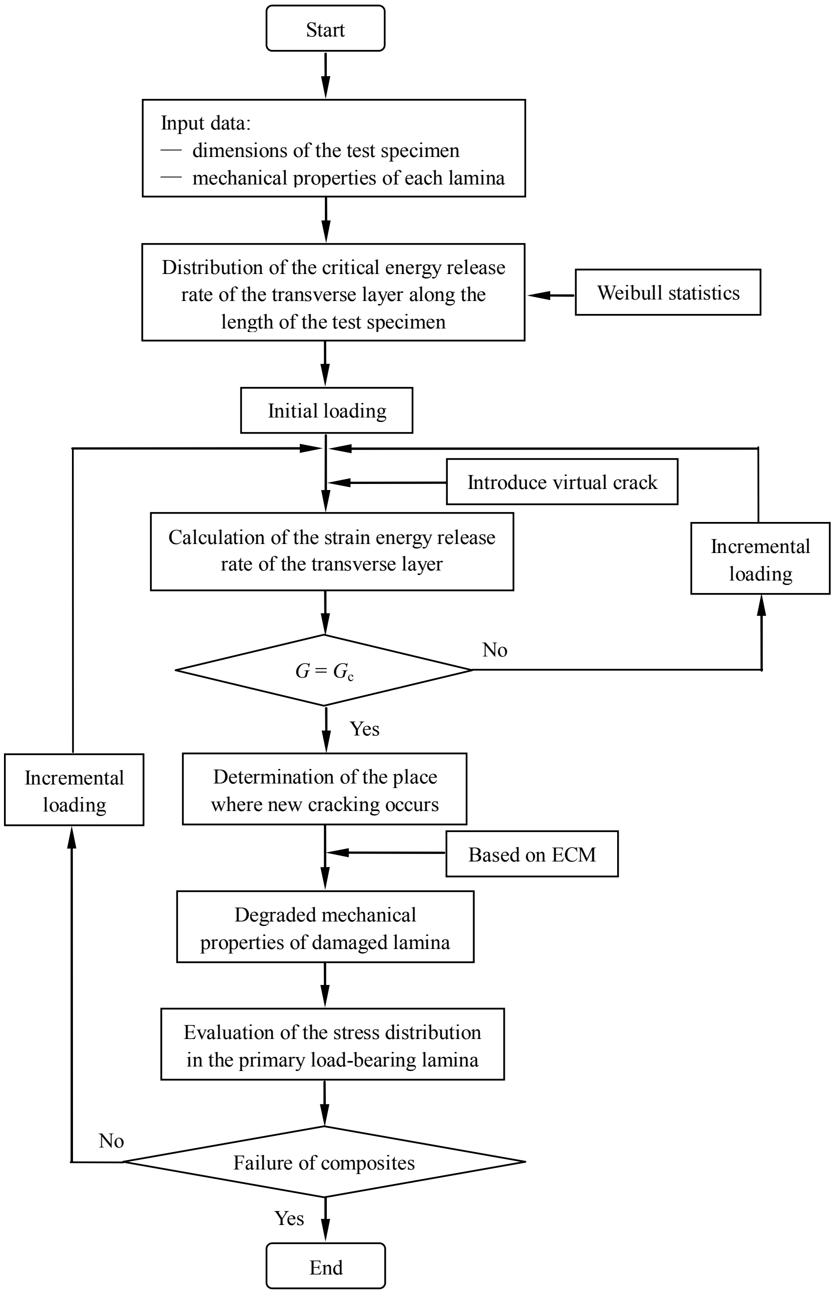

Figure 1. Schematic diagram of the procedure implemented for the analysis of the progression of transverse cracking

4.1. Prediction of stiffness reduction

It is well understood that transverse matrix cracking may occur long before final failure. Matrix cracking causes stiffness degradation and induces other failure modes. Figs. 2 and 3 shows the evolution of the longitudinal Young's modulus and Poisson's ratio for $[0_2/90_4]_s$ glass-fibre/epoxy laminate as a function of matrix crack density. It must be pointed out that the numerical result of each case is the average value of 200 data. The figures exhibit the comparison between the predictions provided by the analytical approach and experimental values. It is shown that the degradations in Poisson's ratio are much higher than the ones observed for the longitudinal Young's modulus. This can be explained by the fact that the Poisson's ratio is a transverse property as compared to the longitudinal nature of the Young's modulus.

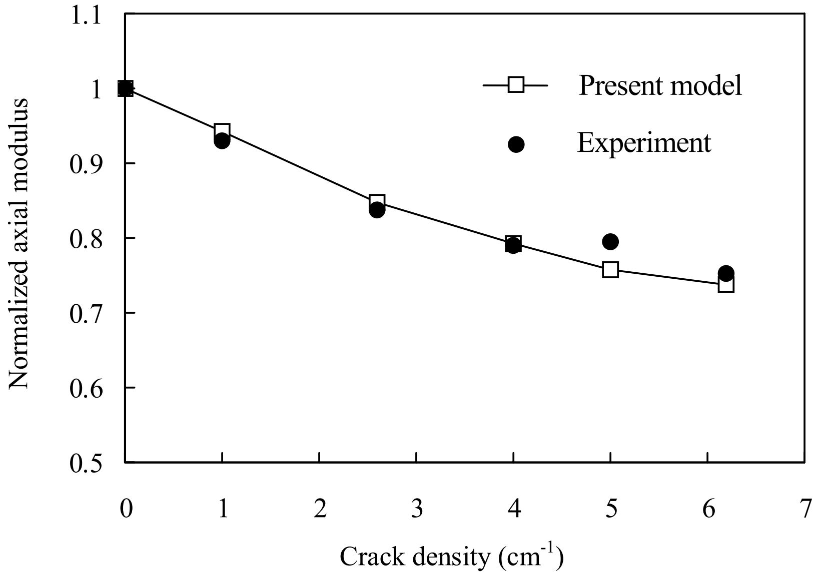

Figure 2. Change of normalized axial modulus as a function of crack density for $[0_2/90_4]_s$ laminate

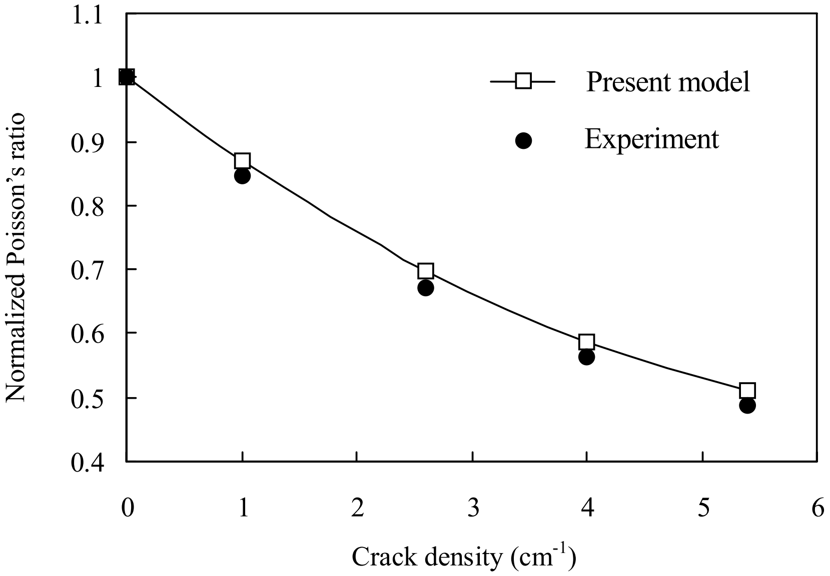

Figure 3. Change of normalized Poisson's ratio as a function of crack density for $[0_2/90_4]_s$ laminate

4.2. Analysis of progressive cracking

Transverse cracking is a progressive damage mode which evolves with increasing in the applied load and a measure of accumulated damage is the crack density. $G_0 = 750\ J/m^2$ and $\beta = 10$ are used as the scale parameters in the Weibull distribution for the glass-epoxy laminates with a stacking sequence of $[\pm 15/90_4]_s$. The simulation results are shown in Fig. 4 compared with experimental values. Since the fracture toughness in transverse layer follows Weibull distribution, the transverse cracking procedure is related to statistical aspect instead of deterministic one. Indeed, the distribution of cracking space is determined by load level and transverse fracture toughness distribution. It is found that the energy criterion can predict and describe the initiation and propagation of matrix cracking.

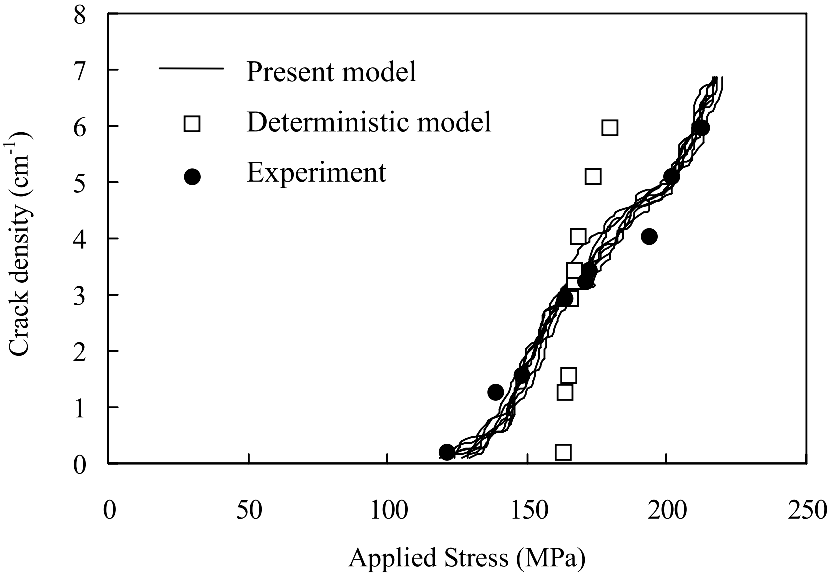

Figure 4. The crack density as a function of the stress applied to $[\pm 15/90_4]_s$ glass-fibre/epoxy laminates

4.3. Estimation of ultimate strength

In Fig. 5, the uniaxial strengths predicted by the current analysis are compared with experimental measurements for the CFRP $[0_2/90_m]_s$ laminates. The prediction is in good agreement with experimental data. The observed offset of ultimate strength is due to the lack of the micro-delamination mechanism in this approach. Especially for thicker layers, delamination at the interface is the dominant damage mode. One feature worthy of note, which is demonstrated by the deterministic model predictions [17], is that the current theory correctly takes account of the effects of the variability in fracture toughness due to microflaws distributed randomly along the length.

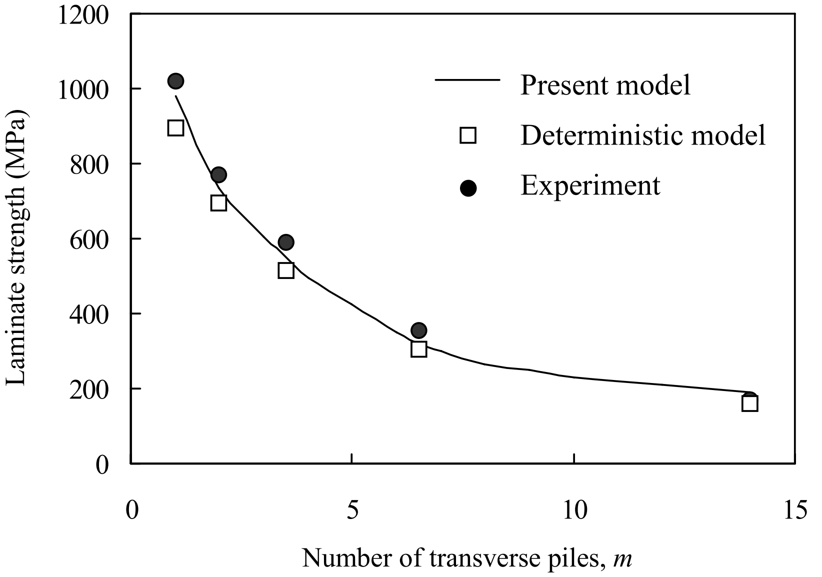

Figure 5. Influence of the 90° lamina thickness on the ultimate strength of the $[0_2/90_m]_s$ CFRP laminates

4.4. Investigation of faliure envelopes

Fig. 6 is biaxial failure envelopes predicted for a $[90/\pm45/0]_s$ AS4/3501-6 epoxy laminate under combined σ_x and σ_y stresses. It is clearly seen that numerical predictions exhibit good agreement with the experimental results present by Soden et al. [19]. It is interesting to note that locating the 90° ply adjacent to the laminate mid-plane will lower the laminate strength. This is because in such case the length of ply cracks in the through-thickness direction is double that for the other laminates, leading to a larger stress concentration in the 0° plies and to fiber failures at lower stresses [20].

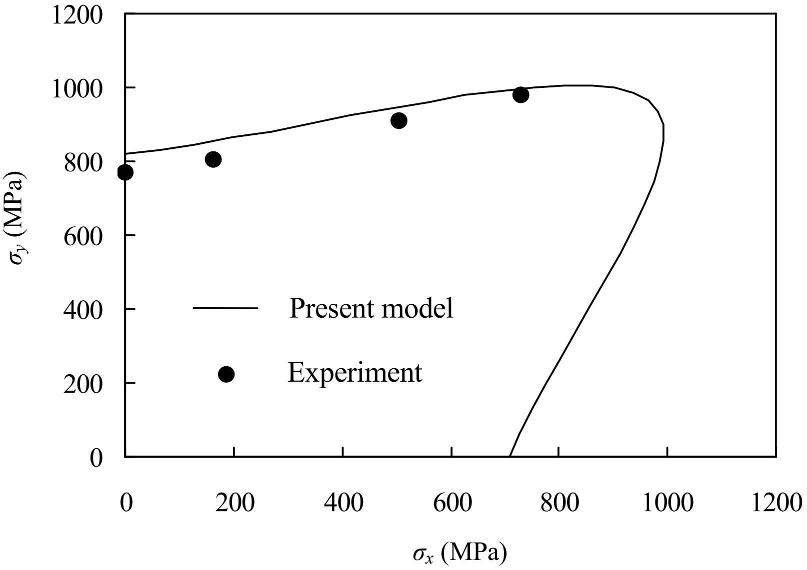

Figure 6. Biaxial failure envelope for $[90/\pm45/0]_s$ laminate made of AS4-3501/6 under combined loading

5. Conclusions

The objective of this exercise is to approach the modelling of progressive damage and failure in composite laminates. A statistical model based on the computation of the strain energy release rate associated with matrix cracking is proposed to study the stiffness reduction and predict ultimate strength of in symmetric laminated composite. The statistical distribution of the critical energy release rate, G_c, in the transverse layer is described by a two-parameter Weibull function in this article. Several sets of application examples and comparison with experimental results show that the present numerical model is able to reproduce the mechanical behavior of laminated composite formed by unidirectional fiber reinforced laminae with different orientations.

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