Electromechanical stability of cone dielectric elastomer actuator

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Abstract Dielectric elastomer actuators are susceptible to electromechanical instability, limiting operational voltages and attainable deformations. In this work, electromechanical stability is investigated at the example of a cone dielectric elastomer actuator. Under the preload and applied voltage, deformation of cone actuator is inhomogeneous. The electromechanical instability occurs when the Hessian of free-energy function of anyone particle in the membrane ceases to be positive definite. Critical voltage of the membrane decreases as the preload increases. The critical actuation stretches are proportional to the critical voltages. As the prestretch increases, the critical voltage decreases while the critical true electric field increases. These results can be used to guide the design of cone actuator.

Keywords dielectric elastomer actuator, electromechanical instability, inhomogeneous deformation

1. Introduction

Dielectric elastomer (DE) is a particularly promising class of EAP (Electroactive polymers) which can overcome many limitations of traditional smart material and transducer technologies. DE based actuators usually consist of elastomer membranes coated with compliant electrodes on both sides. Giant deformation of area expansion rates over 100% can be achieved by applying a voltage.[1] This kind of actuators have been rising as a potential candidate in diverse applications including medical device, soft robotics, energy harvesters and optics etc.[2-4]

Despite good laboratory performance, DEAs are susceptible to various modes of failure[5-6] which prevent their practical application. One of important failure modes is the electromechanical instability, limiting operational voltages and attainable deformations. The study on the failure and nonlinear electromechaniacl stability of DEAs has attracted intensive research [7-9]. Zhao and Suo[7] proposed a general method based on the nonlinear field theory of deformable dielectrics to study the stability. Along this line a series of works have been completed. Xu *et al.*[9] presented an explicit expression for critial stability electric fields by the concept of total stress. In these works, EMI is investigated at the example of parallelepiped actuators.

Cone actuators have a potential application as artifical muscles and were studied by Artificial Muscle Inc.(AMI) and MIT. The understanding of the electromechanical stability for cone actuators is important for their design. In this paper, we consider the definite positiveness of the Hessian matrix representing the electromechanical stability criteria. The critical voltages under different preload of the cone actuator are obtained. The actuation stretches of the actuator are related to the critical voltages. The effect of prestretch on the the critical voltage and the critical true electric field is also investigated.

2. State equations of cone actuators

Figure 1 gives a schematic illustration of cone actuator. A circular dielectric membrane in the undeformed state with thickness H and radius B is coated by two compliant electrodes on both sides [Fig.1(a)]. A general particle of the membrane is at a distance R from the center. Cone actuator is finished by sticking the membrane with a rigid inner frame of radius a and a rigid outer ring of radius b. When a preload F is applied to the inner frame and a voltage ϕ is applied between the two electrodes, the elastomer deforms into an out-of-plane axisymmetric shape with stretches

λ_1, λ_2 and λ_3 as well as gains a distance *u* of the inner frame relative to the ring and an amount of charge Q on either electrodes, and the particle *R* moves to a place with coordinate *r*(*R*) and *z*(*R*) [Fig.1(b)]. Where the longitudinal stretch λ_1 is defined by the distance between the two particles in the deformed state divided by that in the undeformed state, $\lambda_1 = dl/dR$ and the latitudinal stretch is $\lambda_2 = r/R$. Suppose the membrane is incompressible, the free energy density is a function of the three kinematic variables as

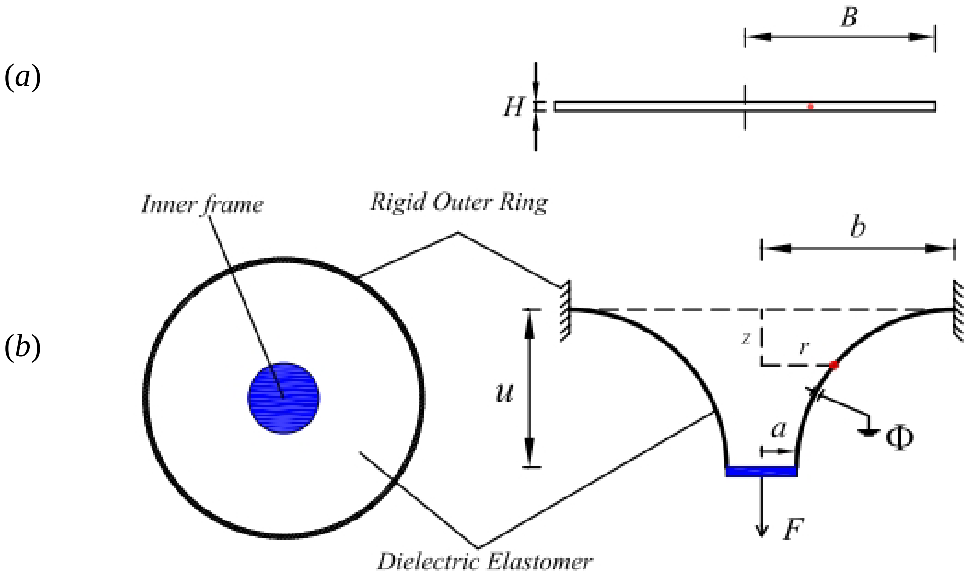

Figure 1. schematic illustration of a cone actuator

$W(\lambda_1, \lambda_2, \tilde{D})$, where $\tilde{D}$ is the nominal electric displacement defined as charge divided by the undeformed area of the membrane. Nominal stresses s and nominal electric field $\tilde{E} = \phi/H$ (the electric field in the undeformed state) can then be expressed as

$$s_1 = \frac{\partial W(\lambda_1, \lambda_2, \tilde{D})}{\partial \lambda_1}, \quad s_2 = \frac{\partial W(\lambda_1, \lambda_2, \tilde{D})}{\partial \lambda_2}, \quad \tilde{E} = \frac{\partial W(\lambda_1, \lambda_2, \tilde{D})}{\partial \tilde{D}} \tag{1}$$

The true stresses σ_1 and σ_2 relate to the nominal stress as $\sigma_1 = \lambda_1 s_1$ and $\sigma_2 = \lambda_2 s_2$. Adopting the Neo-Hookean strain energy model and the linear electrostatic energy function, the free energy function take the form[10]

$$W(\lambda_1, \lambda_2, \tilde{D}) = \frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \tag{2}$$

where μ is the shear modulus, ε is the permittivity.

Inserting Eq.(2) into Eq.(1), we get

$$s_1 = \mu(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \quad s_2 = \mu(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \tag{3}$$

$$\tilde{E} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \tag{4}$$

Equations of equilibrium can be obtained by balancing forces in the directions of *z* and *r* as

$$\frac{F}{2\pi HR\sin\theta} = s_1 \ , \qquad \frac{d(Rs_1\cos\theta)}{dR} = s_2 \tag{5}$$

For the actuator system, the thermodynamic stability requires that the following Hessian matrix

$$H=\begin{bmatrix} \mu(1+3\lambda_1^{-4}\lambda_2^{-2})+\frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2\mu\lambda_1^{-3}\lambda_2^{-3}+\frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2\mu\lambda_1^{-3}\lambda_2^{-3}+\frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \mu(1+3\lambda_2^{-4}\lambda_1^{-2})+\frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix} \tag{6}$$

must be definite positive at the equilibrium state. Eqs.(3～5) can determine the equilibrium values of the generalized coordinates $\lambda_1, \lambda_2, \tilde{D}$. It is noticeable that the deformation is inhomogeneous, so the λ_1 and λ_2 are different along coordinate r in the membrane. For a given dead mechanical preload F, we vary the voltage ϕ. When the voltage is small, the Hessians of all particles are positive definite. Once the Hessian of anyone particle ceases to be positive definite, the voltage reaches a critical value ϕ_c. The critical condition of EMI is set by $\det \mathrm{H}=0$. The condition $\det \mathrm{H}=0$, along with the equilibrium equations [Eqs.(3～5)], determine the critical values $\tilde{E}_c$, $\tilde{D}_c$ and the distrubtion of λ_1^c, λ_2^c in the membrane for any given dead preload F.

3. Electromechanical instability analysis and discussions

The stability criteria can be expressed as the positive sign for the Hessian matrix containing the second–order variations of the energy function as it has been formulated above, which we solve numerically. In presenting the results, we normalize the four variables into dimensionless forms: $\frac{F}{2\pi\mu Ha}$, $\frac{Q}{2\pi\sqrt{\varepsilon\mu}a^2}$, $\frac{\Phi}{H\sqrt{\mu/\varepsilon}}$, $\frac{D}{\sqrt{\varepsilon\mu}}$. We first fix the geometrical dimensionless parameters to special values, $b/B=1$, $b/a=4$. This condition means there is no prestress in the membrane.

Fig.2 shows the effect of the dead preload on the critical voltage ϕ_c. As the preload increases, the critical voltage of the membrane decreases. The result reveals a drop of the withstanding voltage ϕ_c of the membrane is part due to the decrease in the thinkness with increasing deformation of preload. Recall that the membrane deformation is inhomogeneous while the applied critical voltage ϕ_c is homogeneous in the membrane. the distribution of the critical true electric field E_c in the membrane is ploted in Fig.3. This figure shows that the distribution is more uneven and lower as the preload is bigger, but the maxima at the inner frame boundary are very close.

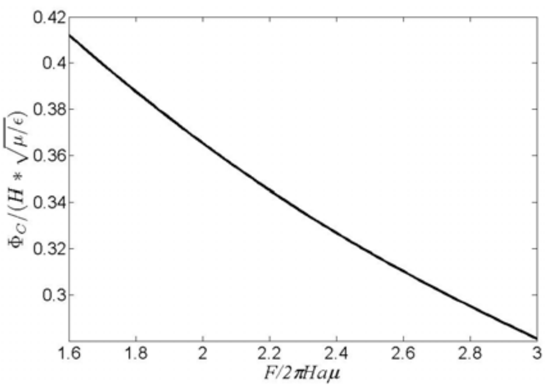

Figure 2. The effect of the dead preload on the critical voltage

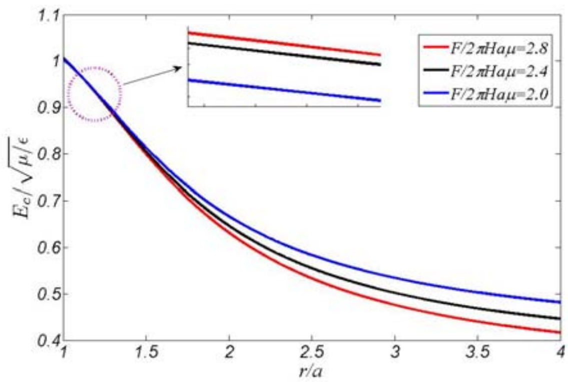

Figure 3. The distribution of the critical true electric field in the membrane

The longitudinal true stress σ_1 is always tensile in order to balance the preload. However, the latitudinal true stress σ_2 can become compressive when the applied voltage is large. The loss of tension may cause the membrane to buckle. Figure 4 shows the possible failure modes of the actuator. It is indicated that the loss of tension is expected to occur first and EMI comes next. EMI may finally result in electrical breakdown. Rupture will not occurr. This trend coincides with the experimental report.

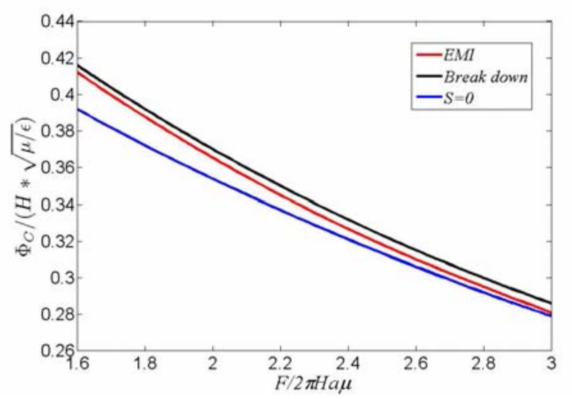

Figure 4. The possible failure modes of the actuator

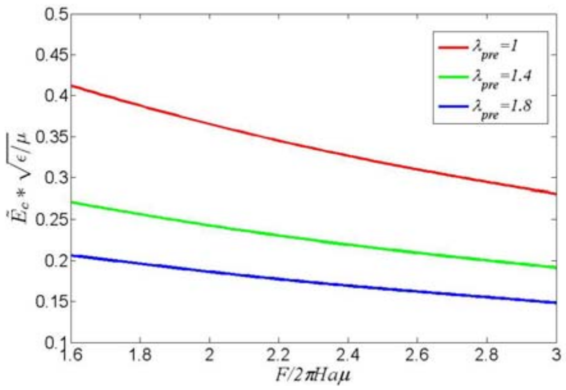

Figure 5. The effect of equal biaxial prestretch on the critical voltage

Figure 5 and Figure 6 show the effect of equal biaxial prestretch on the critical voltage and critical true electric field, respectively. As the prestretch increases, the critical voltage decreases while the critical true electric field increases. It is desirable for the cone actuator to work under a low voltage and a low true electric field, but generate large displacement output. Displacement output is an important performance of actuator and will also influence the force output. The displacement output vs preload is ploted in figure 7. The displacement output is decreases as the preload increases and

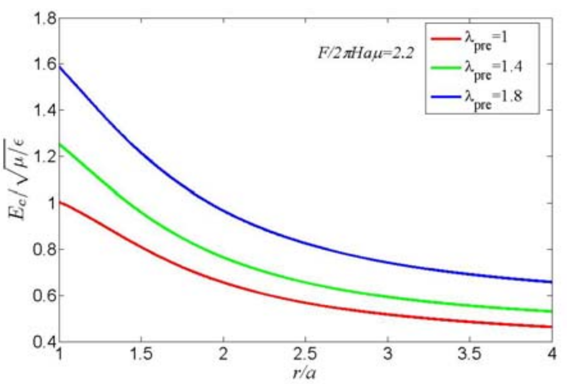

Figure 6. The effect of equal biaxial prestretch on the critical true electric field

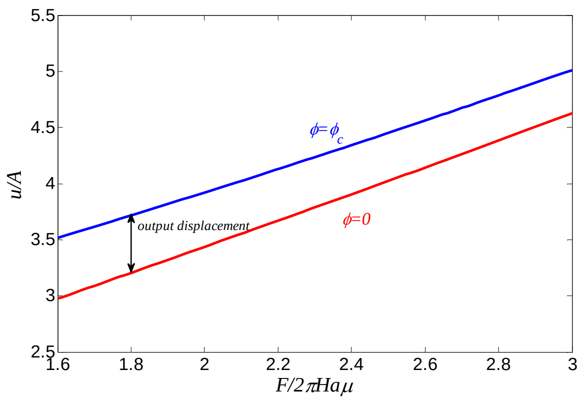

Figure 7. The dead preload vs the output displacement

critical voltage decreases. According to the presented theoretical model, when the actuator is applied by low preload and prestressed, the critical voltage is high, and the actuation stretch and displacement output is large. As a summary, The electromechanical instability occurs when the Hessian of free-energy function of anyone particle in the membrane ceases to be positive definite. The dead preload has significant influence on the electromechanical stability of cone actuator. As the preload increases, the critical voltage of the membrane decreases. Prestress can markedly increase the critical voltage and decrease the critical true electric field.

Acknowledgements

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References

[1] R. Pelrine, R. Kornbluh, Q. B. Pei, J. Joseph, High-speed electrically actuated elastomers with strain greater than 100%. Science, 287 (2000)836–839.
[2] A. O'Halloran, F. O'Malley, P. McHugh, A review on dielectric elastomer actuators, technology, applications, and challenges. J. Appl. Phys., 104 (2008)071101.
[3] X. H. Zhao, Z.G. Suo, Theory of dielectric elastomers capable of giant deformation of actuation. Phys. Rev. Lett., 104 (2010)178302.
[4] R. Heydt, R. Pelrine, J. Joseph, J. Eckerle, R. Kornbluh, Acoustical performance of an electrostrictive polymer film loudspeaker. J. Acoust. Soc. Am., 107 (2000)833–839.
[5] J.S. Plante, S. Dubowsky, Large-scale failure modes of dielectric elastomer actuators. Int. J. Solids Struct., 43 (2006)7727-7751.
[6] J. Zhu, H. Stoyanov, G. Kofod, Z.G. Suo, Large deformation and electromechanical instability of a dielectric elastomer tube actuator. J. Appl. Phys., 108 (2010)074113.
[7] X.H. Zhao, Z.G. Suo, Method to analyze electromechanical stability of dielectric elastomers. Appl. Phys. Lett., 91 (2007)061921.
[8] L.W. Liu, Y.J. Liu, B. Li, K. Yang, T.F, Li, J.S. Leng, Thermo-electro-mechanical instability of dielectric elastomers. Smart Mater. Struct., 20 (2011) 075004.
[9] B.X. Xu, R. Mueller, M. Klassen, D. Gross, On electromechanical stability of dielectric elastomer actuators. Appl. Phys. Lett., 97 (2010)162908.
[10] X.H. Zhao, W. Hong, Z.G. Suo, Electromechanical coexistent states and hysteresis in dielectric elastomers. Physical Review B, 76 (2007)134113.