Experimental Validation of the Relationship between Parameters of 3P-Weibull Distributions Based in J_C or K_{JC}

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Abstract The characterization of fracture resistance of ferritic steels in the ductile to brittle transition zone is problematic due to the great scatter of test results. The statistical treatment in the literature is mainly based on the Weibull distribution, but some authors based their analysis on such a distribution with two parameters (2P-W), while others use a three parameters Weibull distribution (3P-W). Besides this, it is not homogeneous the use of these distributions in terms of J or K, and in general is acceptable that the Weibull slope for a K based distribution is twice the corresponding slope for the distribution based on J values. In previous works, the authors have shown that this relationship between slopes is not valid, except for the case of a 2P-W distribution, and have proposed a new factor (ξ) different from two that is calculated once the three parameters of the 3P-W distribution are estimated. In this paper it is demonstrated, using datasets with 100% cleavage from the Euro Round Robin in order to analyze cases where the weakest link model is valid, that this new factor works quite well, and that even when 3P-W distributions based on J and K values are not equivalent, both could be used to describe the results of fracture mechanics toughness tests.

Keywords Fracture toughness, Ductile-to-brittle transition, Scatter, Weibull distribution

1. Introduction

The experimental determination of fracture toughness in ferritic steels in the brittle-to-ductile transition region is generally based on J_C tests because K_{IC} valid values require too large specimens to meet small scale yielding conditions. Additionally, this characterization is problematic because of the scatter in results that need to be adjusted with the aid of a statistical distribution, being the Weibull distribution the most employed in literature.

This distribution has been used with two (2P-W) or three parameter (3P-W), and in both cases, adjusting data from J_C tests, or data expressed in terms of K converted from J_C (K_{JC}). For instance, Landes and Shaffer [1], Iwadate *et al.* [2], Anderson *et al.* [3], Landes *et al.* [4], and Heerens *et al.* [5] made use of a 2P-W distribution based on J_C values, while Landes and McCabe [6], Neville and Knott [7], and Perez Ipiña *et al.* [8] based their analysis on the 3P-W distribution using J_C data. The use of such distributions based on K values was promoted by Wallin, with a 2P-W distribution [9], and later with a 3P-W distribution [10].

The parameters to be determined in the 2P-W distribution are the shape parameter (also known as Weibull slope), and the scale parameter. For a 3P-W distribution, the threshold parameter is added.
Besides the possibility of working with two or three parameters, and also with J or K data, some authors have proposed a fixed shape parameter with a given value: 2 when working with J_C [3, 4, 5, 11] and 4 when working with K_{JC} [10, 12, 13].

The advantage of using a 3P-W distribution with a fixed shape parameter is that an adjusted to experimental data distribution would be obtained with a smaller number of tests.

According to the theoretical deduction performed by Wallin [10], the value of the shape parameter is four when adjusting K values, and by the well-known relationship between K and J for small scale yielding, this value would be two when working with J values.

As was shown by Larrainzar *et al.* [14], this relationship is only valid for 2P-W distributions. For 3P-W distributions there is no exact equivalence between that expressed in terms of J and the one in terms of K.
Equations (1) and (2) present 3P-W distributions expressed in terms of J or K, respectively.

$$P = 1 - e^{-\left(\frac{J - J_{\min}}{J_0 - J_{\min}}\right)^{b_J}} \quad (1)$$

$$P = 1 - e^{-\left(\frac{K - K_{\min}}{K_0 - K_{\min}}\right)^{b_K}} \quad (2)$$

The scale parameter values are J_0 and K_0, while J_{min} and K_{min} are the threshold parameter values, and b_J and b_K are the shape parameter values of the distributions given by Eq. (1) and (2). All these parameters can be estimated by linear regression, using experimental data in J or these data converted to K.
As was already established, there is no exact equivalence between distributions expressed by Eqs. (1) and (2). The relationship between b_K and b_J given by Eq. (3) is not valid for a general case, instead that given by Eq. (4) has been proposed as a good approximation.

$$b_K = 2b_J \quad (3)$$

$$b_K = \xi . b_J \quad (4)$$

Where ξ is given by Eq. (5), and its value ranges between 1 and 2.

$$\xi = \frac{2K_0}{K_0 + K_{\min}} \quad (5)$$

There is an exact relationship between the threshold and scale parameters (Eq. 6 and Eq. 7).

$$K_0 = \sqrt{\frac{EJ_0}{(1-\nu^2)}} \quad (6)$$

$$K_{\min} = \sqrt{\frac{EJ_{\min}}{(1-\nu^2)}} \quad (7)$$

Where E is the material Young modulus, and < is the material Poisson coefficient.
In this work the relationship between shape parameters in 3P-W distributions, based in K, and J and given by Eq. (4), is validated with experimental data obtained from the Euro Fracture Toughness Dataset. For such a purpose, the parameters of both 3P-W distributions based in J and in K were estimated. Then it was performed a comparison between the parameters of the last one with those converted from 3P-W based in J (by means of Eqs. 4, 6 and 7).

2. Material and Method

Data taken from the Euro Fracture Toughness Dataset [15] were used in the present work. They

correspond to the Round Robin organized by the European Structural Integrity Society (ESIS) and all the information is available in ftp://ftp.gkss.de/pub/eurodataset.
The material tested in the project was a ferritic steel DIN 22NiMoCr37 forged, quenched and tempered.
Figure 1 shows the test matrix performed in the ESIS Round Robin. As the figure shows, tests were performed at different temperatures (-154°C, -110°C, -91°C, -60°C, -40°C, -20°C, 0°C and 20°C) and with different specimen thicknesses C(T) (½", 1", 2" and 4"), with a thickness to width ratio B/W=0.5. Specimens were fatigue pre-cracked to be inside the range 0.52 < a0/W < 0.6. Side grooving was performed after pre-cracking in a few specimens. Tests were carried out in order to obtain the fracture toughness at the point of fracture J_C.

From the tested sets, only those in which all the specimens presented cleavage were considered in the present work. They corresponded generally to the lowest temperatures and largest sizes and are color marked out in squares in Fig. 1. Figure 2 shows the results for all the analyzed sets. This selection was decided in order to avoid sets where two different failure modes coexist, implying that a 3P-W function could not adequately describe the scatter. It is important to note that some datasets included values greater than the allowed J_{max} for the corresponding thickness (½ T at -60°C, 1T at -40°C, 1T at -20°C and 2T at 0°C). Each J_C value was converted to its K_{JC} equivalent, by means of Eq. (8), considering E=210 GPa and <=0.3.

$$K_{J_C} = \sqrt{\frac{EJ_C}{\left(1-\nu^2\right)}} \qquad (8)$$

The parameters of both 3P-W distributions in terms of J and K were estimated for all the analyzed sets. In this way the slopes (b_J and b_K), the thresholds (J_{min} and K_{min}), and the scale (J_0 and K_0) parameters were obtained. Following, the parameters of another 3P-W distribution in terms of K ($K_0(J)$, $K_{min}(J)$ and $b_{K\xi}$), were calculated from the estimations of J_0, J_{min} and b_J already obtained, using Eqs. (6), (7) and (4) respectively.

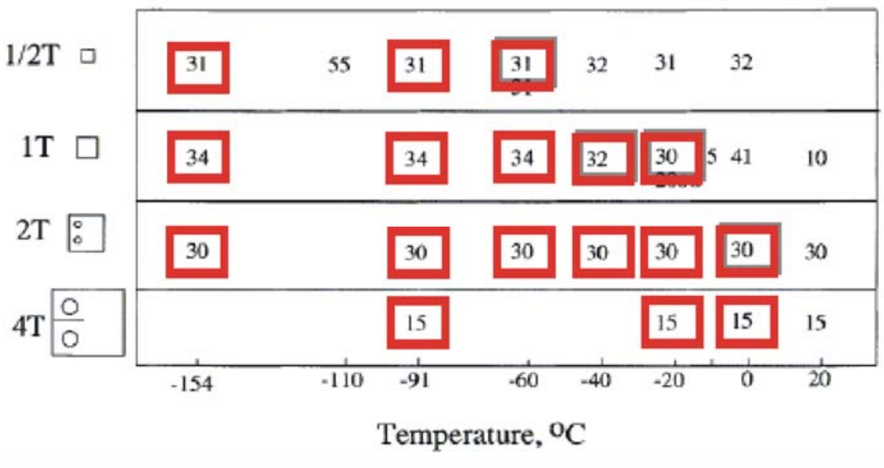

Figure1. Test matrix performed in the ESIS Round Robin. The sets marked out in squares presented only cleavage results and were analyzed in this work. Those shadowed correspond to data sets where some results exceeded J_{max}

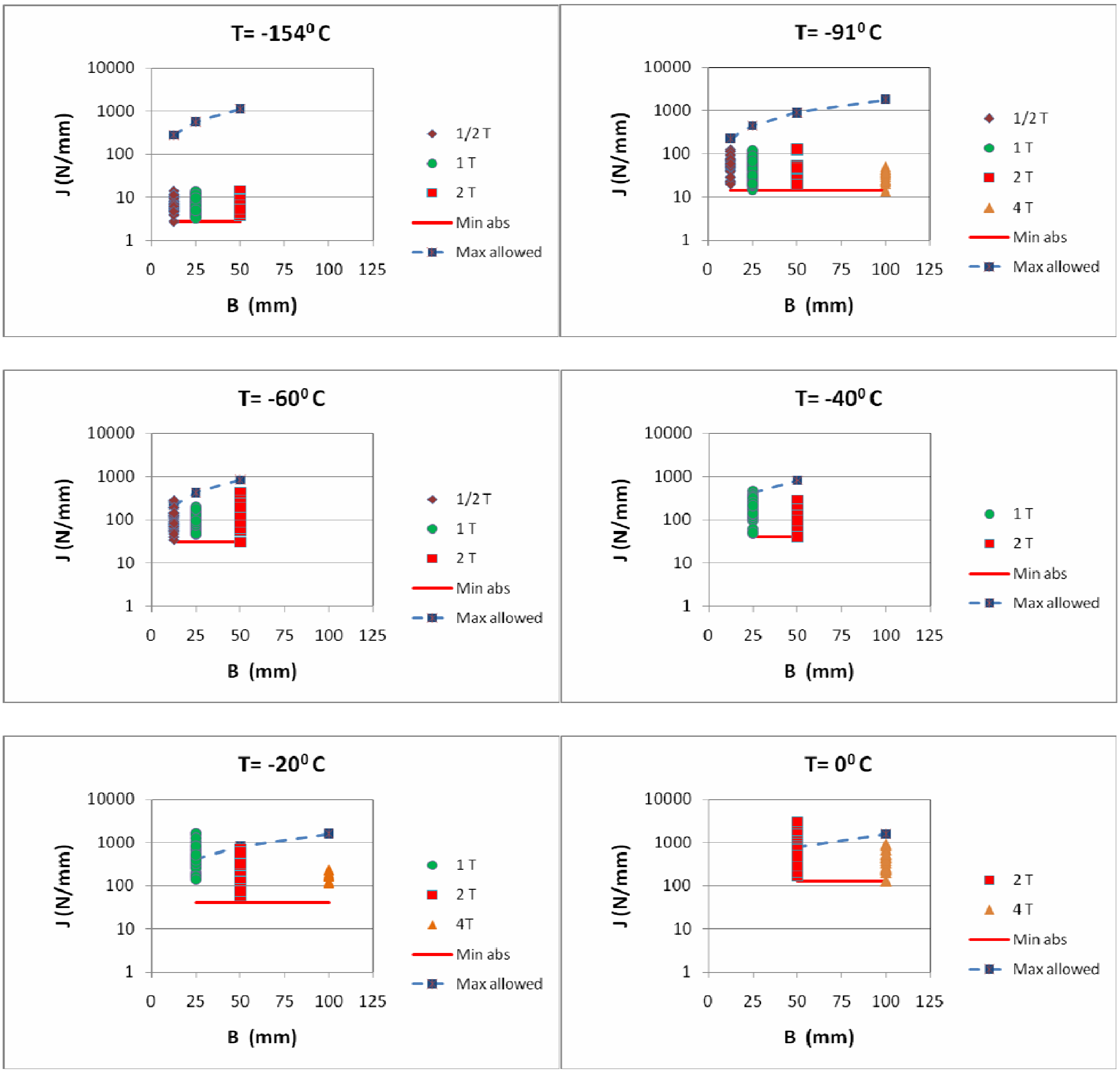

Figure 2. Experimental results for all the analyzed temperatures

3. Results, analysis and discussion

Table 1 shows the 3P-W parameters estimated for all the analyzed sets: the slopes (b_J and b_K), the thresholds (J_{min} and K_{min}), and the scale (J_0 y K_0) parameters. The parameters of the distribution in terms of K(J), calculated from the estimations of J_0, J_{min} and b_J already obtained, as well as the values of ξ , and the b_K/b_J ratio are also shown in the table.

Nearly all the sets produced "acceptable" values of Weibull parameters, especially K_{min}. The set corresponding to B=25mm and T=-40°C was the exception in which physically impossible values of J_{min} and K_{min} were obtained, so the threshold value was considered as zero. In this set, only one of the 32 results was larger than the J_{max} corresponding to this thickness, although there were more than one non-valid results in other sets and the Weibull parameters were physically "acceptable".

Table 1. 3P-W parameters in terms of **J** [kJ/m^2] and **K** [MPa/m$^{1/2}$], all temperatures and sizes

		Estimated from experimental data			Estimated from converted experimental data K_{JC}				Converted from estimated J parameters			
T (ºC)	Size	J_0	J_{min}	b_J	K_0	K_{min}	b_K	b_K/b_J	$K_0(J)$	$K_{min}(J)$	$b_{K\xi}$	ξ
-154	1/2T	7.92	1.80	2.22	42.67	18.54	3.23	1.45	42.76	20.41	3.01	1.35
	1T	7,28	3,06	1,57	40,86	26,05	1,95	1,24	41	26,57	1,90	1,21
	2T	6,31	3,67	1,31	38,02	28,94	1,50	1,15	38,16	29,10	1,49	1,13
-91	1/2T	65.66	9.95	1.85	123.08	34.38	3.21	1.73	123.09	47.93	2.67	1.44
	1T	55,68	11,84	1,44	112,91	47,93	2,08	1,44	113,35	52,27	1,97	1,37
	2T	41,21	17,37	1,44	97,15	61,39	1,87	1,3	97,52	63,31	1,75	1,21
	4T	35.52	4.07	2.68	90.49	22.43	4.54	1.69	90.54	30.65	4.00	1.49
-60	1/2T	115.91	28.72	1.39	162.52	77.28	1.92	1.38	163.55	81.41	1.86	1.36
	1T	107,13	41,19	1,41	156,74	94,08	1,84	1,30	157,23	97,50	1,74	1,23
	2T	157,71	16,12	1,75	190,06	50,64	2,83	1,62	190,77	60,99	2,65	1,52
-40	1T	236,76	0	2,11	233,75	0	4,22	2	233,75	0	4,22	2
	2T	144,49	22,55	2,07	182,12	62,58	3,20	1,55	182,60	72,14	2,97	1,43
-20	1T	564,71	125,55	1,08	357,50	162,19	1,50	1,39	361	170,21	1,47	1,36
	2T	277,41	37,70	1,52	252,25	77,60	2,43	1,60	253,02	93,27	2,22	1,46
	4T	190.96	58.45	2.97	209.89	101.59	4.44	1.49	209.92	116.14	3.83	1.29
0	2T	953.58	166.33	0.83	458.32	191.46	1.13	1.36	469.10	195.92	1.18	1.41
	4T	458.66	81.30	1.31	324.15	118.66	2.02	1.54	325.34	136.97	1.85	1.41

Table 1 shows that the parameters estimated by linear regression from K values converted from J (shown in column "Estimated from converted experimental data K_{JC}" in Table 1) and also using the parameters obtained by means of Eqs. (4), (6) and (7) (shown in column "Converted from J parameters" in Table 1) are very close. Figure 3 shows some Weibull cumulative distribution functions with parameters obtained by the two ways, together with the experimental points and the agreement is confirmed.

Table 2 and Fig. 4 show the comparison between ξ, obtained using Eq. (5), and the ratio b_K/b_J, where b_K and b_J are the shape parameters estimated for J experimental values and K_J. These values are quite similar, although b_K/b_J resulted always larger than ξ, except in one case. The horizontal line in the figure corresponds to the hypothetical case given by Eq. (3) (b_K=2b_J), which is not satisfied except when $J_{\min} = 0$, as already justified.

Figures 5 to 7 compare the three Weibull parameters in K: those obtained from the K_{JC} values and those obtained from J_C values (3P-W(J)) and transformed to K by using Eqs. (4), (6) and (7).

Figure 8 shows the predicted thresholds against the minimum experimental values. It can be observed from this figure and also from Fig. 5 and Table 1 that $K_{min}(J)$ values, those obtained using ξ, resulted less conservative than the K_{min} ones, although their values were always lower than the corresponding experimental minimum.

As it is seen from Fig. 3, it resulted clear that the methodology proposed (that uses Eqs. 4, 6 and 7) adjusts the experimental data very well.

Table 2. Difference $b_K / b_J - \xi$ relative to ξ

T (ºC)	W=25 mm	W=50 mm	W= 100 mm	W= 200 mm
-154	7.41%	2,48 %	1,77 %	-
-91	20.14%	5,11 %	7,44 %	13.10%
-60	1.47%	5,69 %	6,59 %	-
-40		0 %	8,39 %	-
-20		2,21 %	9,59 %	15.50%
0			-3.55%	9.22%

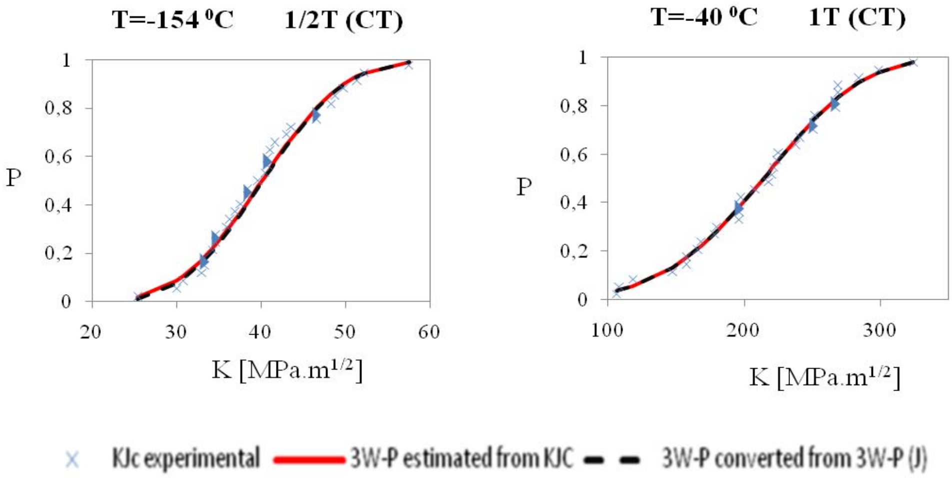

Figure 3. 3P-W distributions comparison with parameters obtained by linear regression and by J_0, J_{min} and b_J. conversion, for two datasets

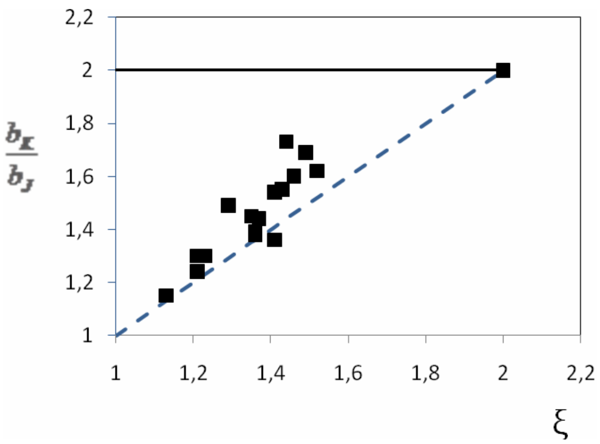

Figure 4. Comparison between b_K/b_J and ξ (R^2 for the linear regression equal to 0.83254927)

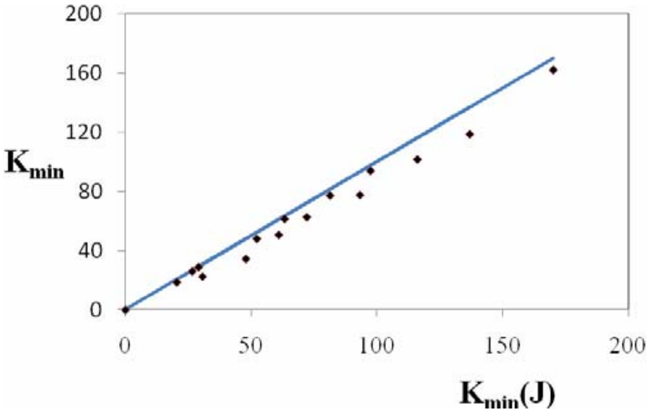

Figure 5. Comparison between $K_{min}(J)$ and K_{min} (R^2 for the linear regression equal to 0.9870869)

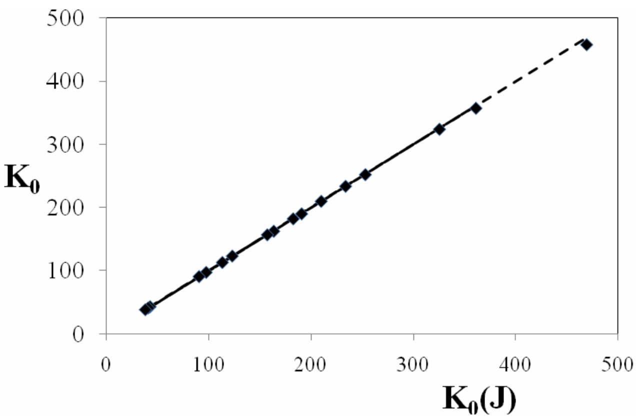

Figure 6. Comparison between $K_0(J)$ and K_0 (R^2 for the linear regression equal to 0.99979445)

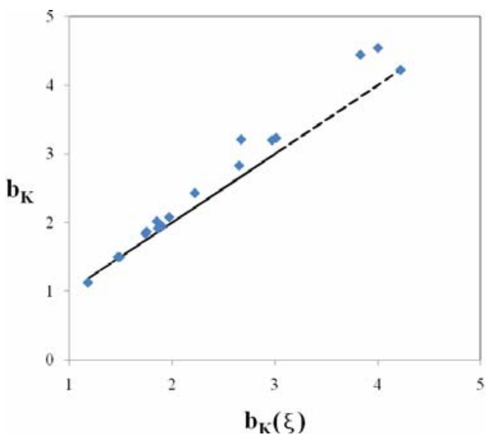

Figure 7. Comparison between $b_K(\xi)$ and b_K (R^2 for the linear regression equal to 0.97867302)

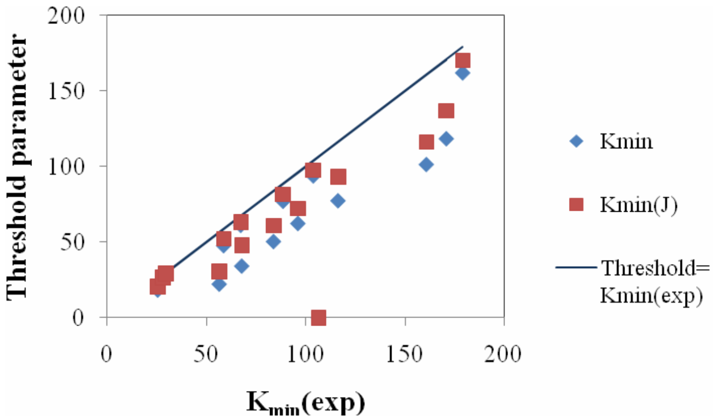

Figure 8. Comparison between $K_{min}(J)$, K_{min} and $K_{min}(exp)$

Table 3. Percentual differences between K_{min} and experimental K_{min}

	T (^{0}C)	$\frac{K_{min}-K_{min}(\exp)}{K_{min}(\exp)}(\%)$	$\frac{K_{min}(J)-K_{min}(\exp)}{K_{min}(\exp)}(\%)$
1/2T	-154	-27.06	-19.71
	-91	-49.27	-29.27
	-60	-12.76	-8.09
1T	-154	-7.00	-5.14
	-91	-18.26	-10.86
	-60	-9.37	-6.08
	-40 *	-100.00	-100.00
	-20	-9.44	-4.96
2T	-154	-2.27	-1.73
	-91	-8.72	-5.86
	-60	-39.54	-27.18

	-40	-34.86	-24.91
	-20	-33.27	-19.79
	0	-4.54	-2.31
4T	-91	-60.25	-45.69
	-20	-36.81	-27.76
	0	-30.55	-19.83

*: In this case, threshold parameter was forced to zero because a negative value was obtained from the estimation, resulting a difference of 100% between K_{min} and experimental K_{min}

There is no clear evidence that the 3P-W distribution based in J is better than the corresponding to that based in K values. Both fitted well the experimental results and predicted good threshold parameters.

4. Conclusions

- The theoretical relationship between b_J and b_K given by eq. (5) is consistent with the obtained from experimental data from the ESIS Round Robin.
- The Weibull slope in terms of K is not consistent with a fixed value equal to 4, instead it appears to calculate it from b_J by using Eq. (3), or estimating it from experimental data converted toK.
- The 3P-W parameters in terms of K, that can be estimated from experimental Jc converted to K_{Jc}, result more consistent with reality when they are calculated converting the corresponding estimated in terms of J.

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