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Analysis of fatigue crack initiation and propagation in ship structures

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Abstract

Ships structures are subjected to various types of cyclic loads from waves, wind and cargo operations that cause fatigue damages in the structures. There exist rules to regulate the structural design with sufficient fatigue strength to survive their service period. However, fatigue cracks do occur earlier than expected in numerous locations of e.g. ocean-crossing container vessels. The presence of fatigue cracks greatly affects a ship's safety and serviceability. Managing how initiated cracks grow is an important task to ensure a safety and cost-effectiveness ship transportation. The objective of this study is to develop a robust method, which can be used to predict crack growth and crack maintenance in ship structures. A longitudinal stiffener of a 2800TEU container vessel with full-scale measurements of e.g. strain signals and operation conditions are used in the study. Firstly, a spectral *S*-*N* fatigue analysis is adopted to predict when the first crack occurs in the ship's most fatigue-critical region. Then the crack's growth characteristics are modeled by the code FRANC2D. Finally, the time needed for the crack initiation and growth are studied in terms of structural maintenance plan, i.e. crack growth management.

Keywords Fatigue crack, hydrodynamic loads, ship structure, spectral method and wave height.

1. Introduction

The size of ships has increased rapidly during the last decade as a result of the fast growth of the global shipping market and the developments of construction technologies that also makes the high-tensile steels widely used in ships. The increase of dimensions and high-tensile steel usage in ships made the ship structures more flexible when they are operated in waves. The wave (hydrodynamic) loads can cause continuously changing stresses and result in fatigue damages in ship structures, which challenges the ship's structural integrity and thereby its safety. Therefore, ship structures should be designed with sufficient fatigue strength based on the rules [1], where fatigue strength is assessed using stress-based approaches (the high-cycle fatigue analysis). In ship class rules, the stress range distribution is normally provided for the fatigue assessment. But for a novel ship design, the stress range distribution is not available and should be computed by a direct calculation method that demands huge computation powers to consider the ship's specific operation conditions and encountered wave environments. However, large uncertainties cannot be avoided in the above two fatigue design processes [2]. For example, a ship's real encountered wave environments may be different from that used and proposed in the fatigue design rules [3, 4]. The results from the hydrodynamic loads analysis can be large [5]. There are additional factors which are also often disregarded in the fatigue analysis of ship structures, such as analysis of the effects of redistribution of residual stresses from manufacturing, corrosion, weld defects, etc., and these factors contribute to even larger uncertainty in the fatigue assessment. The sum of all these factors may explain why fatigue cracks are found earlier than expected in many vessels. Repairing the entire minor to moderate cracks is expensive and it is seldom an alternative, due to the economic issues related to the maintenance process. Hence, the understanding if "initiated" and visible cracks are critical to the structure's integrity becomes an important issue. Tools and methods for maintenance plan based on crack growth management should therefore be developed. In the literature, crack propagation under various sailing conditions has been investigated in e.g. [6]. A summary of the progresses of using fracture mechanics in the maritime industry is summarized in [7].

The objective with the current study is to present the derivation of a useful method that can be used to predict the fatigue crack growth in a ship structure. In the development, the special loading characteristics of ship structures are dealt with. It has been formulated to assist maintenance planning with respect to e.g. repair of fatigue cracks. A structural detail with an initiated crack in a 2800TEU container ship is used to demonstrate the approach using fatigue loads from a direct calculation. The results are compared with full-scale measurements made on the vessel. Finally, based on the operation conditions for the current case study vessel and the existing crack, the number of voyages the ship structure can undertake before final rupture is investigated.

2. A spectral method for ship fatigue assessment

For the ship fatigue assessment, the long-term structural stresses are divided into a series of short stationary signals [8]. A stationary process can last for from 20 minutes to several hours. The structural stress under a stationary sea state is caused by the wave loads applied on ships. The waves in a sea state is often described by the significant wave height, H_s, the wave period T_p, and a specific wave spectrum $S(H_s, T_p)$. In order to estimate a ship's fatigue damage under a sea state, its structural stress is often assumed to be Gaussian distributed [9]. Hence, both the hydrodynamic loads and the structural stresses can be computed by linear theories.

2.1. High-cycle fatigue analysis

Ship structures are designed to behave elastically during its design life of around 20 years. The fatigue strength is assessed by stress-based approaches, i.e. high-cycle fatigue design principles. In the analysis, the material behavior is characterized by a *S-N* curve, with a log-linear dependence between the number of cycles to failure N, and the stress cycle range S, $\log(N) = \alpha - m\log(S)$. Different *S-N* curves exist for different materials, geometries, welds, etc., the parameters a and m are usually categorized based on the properties of structural details in the class rules. The stress ranges, here denoted by S_i, ($i = 1,\ldots,n$), can be obtained by the rainflow counting method for each sea state. Finally, the accumulated damage is calculated using the linear Palmgren-Miner law as:

$$D = \sum_{i=1}^{n} \frac{S_i^m}{\alpha} \tag{1}$$

In order to estimate the fatigue damages accumulated in a sea state, it is necessary to get the total number and distribution of the stress ranges S_i. Since the stress is assumed to be Gaussian, it is sufficient to get the spectrum of structural stresses for the ship fatigue assessment.

2.2. Ship structural stresses in a sea state

The variability of structural stresses, denoted by $X(t)$ here, is mainly caused by the change of the wave loadings applied on ships. Hence, it is essential to get the correct wave (hydrodynamic) loads.

The structural stresses due to the wave loads can be computed by beam theory. To simulate and evaluate realistic ship operation conditions, the computation has to be done for many sea states, at various ship speeds, U, and different heading angles, θ. In general, a frequency domain analysis is used to first compute the Response Amplitude Operators (RAOs) of the hydrodynamic loads using linear potential strip theory [10]. Then, by means of beam theory, the transfer function of structural stresses $H_\sigma(\omega|U, \theta)$ is calculated using the section modulus of the structural detail of interest. Often, a stress concentration factor is added in the transfer function to get the local stresses. Thereafter, the stress spectrum under arbitrary sea states, $S_\sigma(\omega|H_s, T_p)$, can be computed as:

$$S_\sigma(\omega\,|\,U,\theta,H_s,T_p) = |\,H_\sigma(\omega\,|\,U,\theta)\,|^2 \cdot S_e(\omega\,|\,H_s,T_p), \tag{2}$$

where $S_e(\omega|H_s, T_p)$ is the encountered wave spectrum. It is not always explicitly derived for all wave frequencies, but the spectral moments are rather easy to obtain and of great interest in fatigue analysis. The n-th order spectral moments is calculated by:

$$\lambda_n = \int_0^\infty \left|\omega + \omega^2 U \cos\theta / g\right|^n H_\sigma^2(\omega\,|\,U,\theta) S(\omega\,|\,H_s,T_p)\,d\omega. \tag{3}$$

Let R denote the local maxima of the Gaussian stress signal X in a sea state. The distribution of R can be described by Rice's distribution function:

$$F_R(r) = \Phi\left(\frac{r}{\varepsilon\sigma_x}\right) - \sqrt{1-\varepsilon^2}\,\Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\frac{r}{\sigma_x}\right) e^{-\frac{1}{2}\left(\frac{r}{\sigma_x}\right)^2} \tag{4}$$

where Φ is the standard normal cumulative distribution function, σ_x is the standard deviation of X and $\sigma_x = \sqrt{\lambda_0}$, ε is the spectral width parameter. If $\varepsilon = 0$, Eq. (4) becomes Rayleigh distribution:

$$F_R(r) = 1 - e^{-\frac{r^2}{2\sigma_x^2}}, \text{ where } R \geq 0 \tag{5}$$

For the narrow band Gaussian process, the number of local maxima can be computed through the zero-upcrossing frequency of the signal $X(t)$ as $f_z = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$.

2.3. A spectral fatigue method

Since the waves in a stationary sea state are actually random processes, the stress cycle range S is also a random variable with the probability density function (pdf) denoted by $f_S(s)$. Then, the expected value of S^m is computed by $E[S^m] = \int_0^\infty s^m f_S(s)\,ds$. For a zero mean narrow band Gaussian stress $X(t)$, the stress cycle range S is approximated by two times the stress amplitude R, i.e. $S \approx 2R$. Subsequently, by means of Eq. (5), $E[S^m]$ can be computed by:

$$E[S^m] \approx \int_0^\infty (2r)^m f_R(r)\,dr = \left(2\sqrt{2}\sigma_x\right)^m \Gamma\left(\frac{m}{2}+1\right) \tag{6}$$

where $\Gamma(x)$ is the gamma function. The expected fatigue damage computed by Eq. (1) becomes:

$$E[D] = \frac{N_0}{\alpha} E\left[S^m\right] \approx \frac{N_0}{\alpha}\left(2\sqrt{2}\sigma_x\right)^m \Gamma\left(\frac{m}{2}+1\right) \tag{7}$$

where N_0 is the expected number of stress cycles and computed by $N_0 = T \cdot f_z$ for $X(t)$, $t \in [0, T]$. Finally, the expected fatigue damage caused by the narrow band stress $X(t)$ denoted by D_T, is:

$$D_T = E[D] \approx \frac{T}{2\pi\alpha}\sqrt{\frac{\lambda_2}{\lambda_0}}\left(2\sqrt{2\lambda_0}\right)^m \Gamma\left(\frac{m}{2}+1\right) \tag{8}$$

Equation (8) is also known as the narrow band approximation and works quite well even for stress signal with spectral width parameter ε up to 0.5 [10].

3. Crack propagation analysis for ship structures

Ships follow specific inspection and maintenance plans based on rules and regulations. Because of large costs during the repair process [11], ship owners would like to repair ship defects and cracks during the ship's regular hull survey, which should be carried out from every 2 to 5 years depending on the type and age of the vessel. However, it is not practical or possible to repair all the cracks at once. Consequently, it is of great interest to study when fatigue cracks propagate and reach a critical length which requires immediate repairing. In the following, an efficient way for such an analysis is derived and it has been limited to Model I crack propagation according to linear elastic fracture mechanics principles.

3.1. Fatigue crack propagation analysis

The rate of fatigue crack propagation under cyclic loads can be described by the Paris' law [12] as:

$$\frac{da}{dN} = C \cdot \Delta K^k \tag{9}$$

where a is the crack length, N is the number of cycles, da/dN is the crack growth per load cycle, and C and k are material parameters from experiments. The ΔK is the range of the stress intensity factor during a load cycle, i.e. $\Delta K = K_{max} - K_{min}$, where the stress intensify factor K is defined as $K = \sigma f(a/w)\sqrt{\pi a}$, in which σ is the tensile stress perpendicular to the crack plane, $f(a/w)$ is the dimensionless parameter in terms of the crack geometry. The value of K can be difficult to describe analytically for ship structures due to their geometrical complexity. Alternatively, codes such as FRANC2D [13] can be used to compute K as a function of the crack length. Let $Y = f(a/w)\sqrt{\pi a}$. The stress intensity factor can be written as $K = \sigma Y$. For the computation of Y by Franc2D, the crack growth is treated as a series of stages, i.e. the crack grows from a_j, $j = 0,1,\ldots, M$, corresponding to the value of Y_j, $j = 0,1,\ldots, M$. Here, a_0 is the initial crack length and a_M represents the critical crack length defined for crack repair. Finally, the stress intensity factor range becomes $\Delta K = \Delta\sigma Y$.

3.2. Fatigue crack growth under a sea state

Following the reasons in Section 2.3, here the stress signal $X(t)$ is also assumed to be a zero mean narrow band Gaussian process. Further, it is assumed that the compressive stress does not contribute to the crack propagation. Hence, the stress range $\Delta\sigma$ equals to the local maximum of $X(t)$, $\Delta\sigma = R$. Let T denote a sea state lasting period, say 30 minutes, and assume that Y is constant for a crack from a_j to a_{j+1}, i.e., $Y = Y_j$, which is computed by fracture mechanics codes such as the FRANC2D. The expected number of stress cycle N_0 is known from Section 2.3. Then the expected crack growth in the sea state can be computed by:

$$\Delta a_T = E\left[\sum_{i=1}^{n} C\Delta K_i^n\right] = CN_0 Y_j^k E[\Delta\sigma^k] = \frac{TC}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}\Gamma\left(\frac{k}{2}\right)\left(\sqrt{2\lambda_0}Y_j\right)^k \tag{10}$$

where λ_0 and λ_2 are the spectral moments of $X(t)$ in the sea state as Eq. (3). By means of Eq. (10), it is straightforward to compute how many sea states a ship can sail until the crack of interest reaches to a_M, which is the critical crack length that requires repair, cf. crack growth management.

3.3. Sailing wave environments and ship response

When a ship sails across the ocean, e.g. on the North Atlantic trade, the ship typically encounters one or two storms with high significant wave height H_s, see Fig. 1 (left) for an example. Depending on the weather forecast information, captains can choose different routes for safety and economics reasons. For the prediction of fatigue crack growth, it is of great interest to know the distribution of the wave environment a ship will encounter in a few years. A large amount of data is available, such as wave measurements from satellites, hindcast or buoys, which can be used for this purpose. There also exist statistical models built up based on the data, e.g. [14], which can be used to simulate the mean and covariance of wave environments along various ship routes.

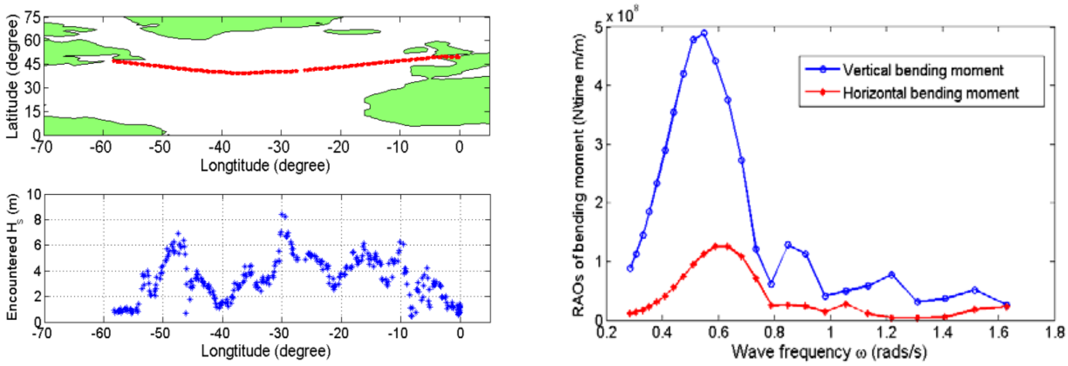

Figure 1. (Left) One voyage from Europe to North America with the significant wave height H_s for all the encountered sea states during the voyage; (Right) response amplitude operators of two bending moments for a ship operating with 10 m/s forward speed and heading angle 20 degrees.

For ship fatigue assessment, the structural response should be computed for all encountered sea states. In the maritime industry, the response is often described by the transfer function, $H_\sigma(\omega|U, \theta)$, which depend on the ship speed and the heading angle. When sailing in the North Atlantic Ocean from Europe to North America, ships usually have to go against waves with a heading angle to the wave encounter direction that varies between 20 to 50 degrees. As a result, the normal stress signal $X(t)$ is composed of stresses from the ship's vertical bending, horizontal bending and Vlasov Torsion. In the right plot of Fig. 2, an example of these two bending moments is depicted.

4. A case study: fatigue analysis of ship structures

A structural detail amidships (see Fig. 2) of a 2800TEU container ship is used to demonstrate the approaches for ship fatigue analysis presented in the previous sections, i.e. fatigue initiation in Section 2 and crack propagation in Section 3. The purpose of the analysis is to explain how the two analysis methods can be implemented to plan the maintenance of ship structures. The structural detail of interest was identified from reports, which stated that fatigue cracks were initiated after less than half of the vessel's fatigue design life. Hence, instruments were installed onboard to measure the time series of strains (stresses), significant wave height, heading angle and ship speed when crossing the North Atlantic. The measurement campaign includes 14 voyages where 7 go from Europe (EU) to North America (NA), while the other 7 go to EU from NA. For the following fatigue analysis, the parameters in the *S-N* curve [9] and the Paris' law [15] are listed in Table 1 .

Table 1. Material parameters in the *S-N* curve and in Paris law for the AH36 steel.

	S-N curve		Paris law	
Parameters	α	m	C	K
Values	$10^{12.76}$	3	1.45×10^{-11}	2.75

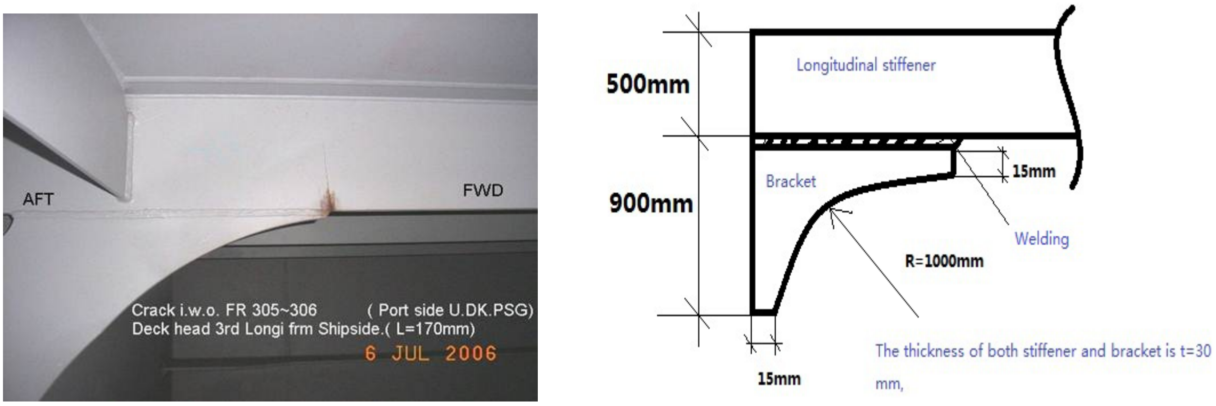

Figure 2. A fatigue crack in the case study vessel and the structural detail in the fracture analysis.

4.1. Fatigue crack initiation analysis

Firstly, by means of the measured stress signals, the fatigue damages accumulated during each individual voyage are computed by the rainflow counting method – they are denoted as the "observed" damages. Secondly, the transfer function of structural stress is computed by means of a 2D strip theory hydrodynamic analysis and the simple beam theory for structure analysis. Then, combining the transfer function with the wave measurements (H_s, T_p) for all encountered sea states, the fatigue damages are computed by the spectral method as Eq. (8). The results from both methods are presented in Fig. 3. It is of interest to focus the study of the fatigue damages during winter season voyages from Europe to North America where the ship were operated in the harshest conditions. For these voyages, the spectral method gives maximum 30% discrepancy from the observed damages, but in the long-term analysis, the spectral method works well in comparison with the rainflow method. The results also confirm the initiation of a fatigue crack observed

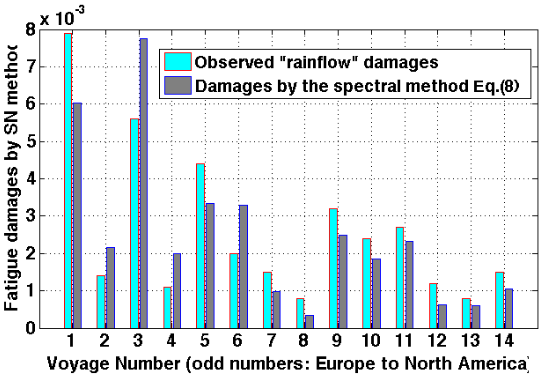

Figure 3. Observed fatigue damages and damages computed by the spectral method introduced in Section 2.3 for all measured voyages. Voyages 1-7 represent winter seasons, 8-14 represent summer seasons.

4.2. Crack propagation analysis and maintenance plan

A ship's structure integrity can be strongly affected by fatigue cracks depending on their locations. Sometimes, the cracks can lead to undesirable consequences, such as oil leakage and compartment flooding, or even structural failure of the entire ship. In principle, the deck plates and longitudinal stiffeners/girders have to undertake the ship's global strength. The cracks around these areas should be repaired well in time before they reach to a critical value. For this purpose, the crack propagation analysis is needed to plan the cracks maintenance. For the example container ship, the critical crack length is assumed to be 200 mm. The crack propagation analysis is performed using the approach presented in Section 3.2. It enables us to estimate how many voyages or sea states the ship can sail before the crack reaches to 200 mm. The initial and interval crack growth length is set to be 5 mm for the linear elastic fracture mechanics analysis. A strong beam is attached on the top of the stiffener to model the deck effect as shown in Fig. 2. A crack is initiated at the connection between the longitudinal stiffener and the associated bracket with high stress concentration. The crack growth path and corresponding stress intensity factors K_{I} are computed using FRANC2D [13]. For the current boundary and loading conditions, an analysis showed that crack propagation was governed by Mode I. The final stage of the crack propagation and K_{I} are presented in Fig. 4.

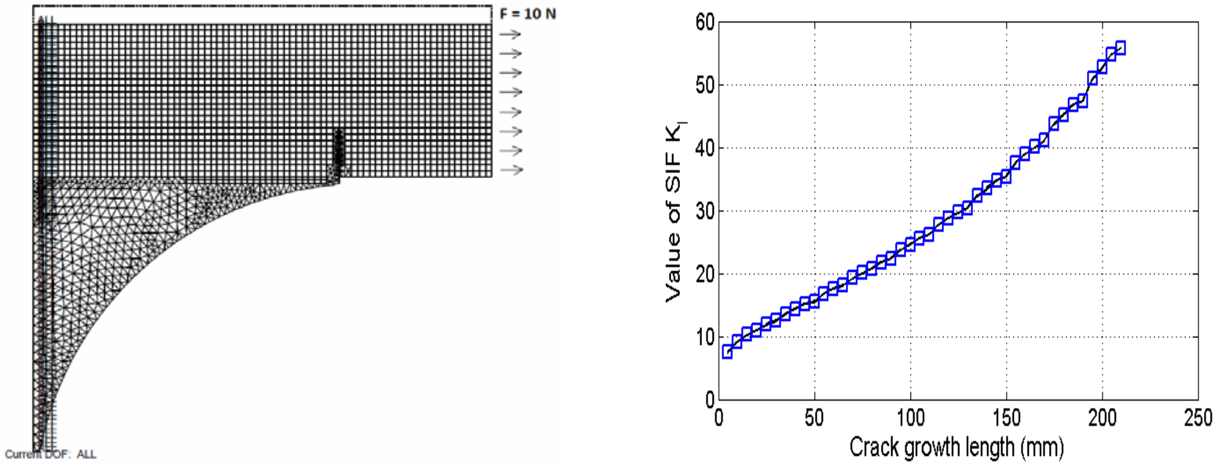

Figure 4. Fatigue crack growth path and stress intensity factors computed by FRANC2D, where the initial crack length is 5mm, the crack is propagated straight upwards, and the element is set to 20 mm.

In order to predict the fatigue crack growth, it is necessary to get the wave environments, which the ship will encounter in the future years' operation. In this study, the ship is assumed to sail along the same routes as the measured 14 voyages. Further, it is assumed that the ship is operated with the same speed and heading angle as the measured ones. The significant wave heights H_s along the measured routes are simulated using a spatio-temporal wave model presented in [14]. It contains the covariance structure of the ocean field in both time and space and enables us to simulate the value of H_s and their correlations for all locations (sea states) along the measured routes. Because of the natural variability of the wave environments, the value of H_s could change as in Fig. 5, together with the observed in one winter voyage. In the study, a stationary sea state is assumed to be 30 minutes and described by the Pierson-Moskowitz (P-M) power spectrum density which is expressed in terms of the significant wave height, H_s, and the wave period, T_p.

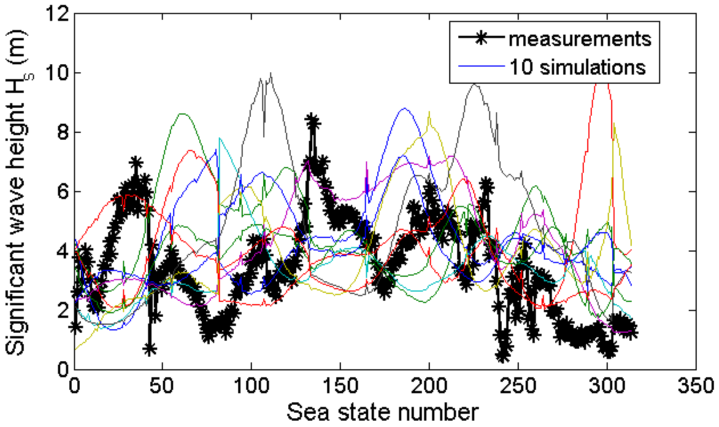

Figure 5. Significant wave heights along the route in Fig. 1 using the spatio-temporal model [14].

The crack propagation analysis is divided into various crack growth stages/intervals. For each stage, the crack increment is computed by Eq. (10) for the ship in a specific condition (H_s, T_p, U and θ). Subsequently, how many sea states the ship can sail for the crack growing from a_i to a_{i+1} can be also calculated. Finally, the repair time, i.e. when the crack reaches to the critical length 200 mm, can be easily predicted. Using the repeated simulations of encountered H_s as in Fig. 5, the mean and variance of the fatigue crack growth under a certain period can be estimated. Therefore, the variation of the total sailing time until the crack propagates to a specific length can be predicted.

Using the wave environments from the wave model simulation [14], the ship can sail for 2.53 years before the crack reaches 200 mm. If the waves are simulated many times from the other approach, the median value of such a period is 2.74 years, while the standard deviation of the period is 0.34 year. Within each interval, the expected number of sea states needed for the crack to grow 5 mm is shown in Fig. 6, as well as the standard deviation of the number. When the crack length is short, more sea states are needed for the crack to propagate 5 mm. It can grow very fast when the crack is close to its critical value. In this case, only a few sea states can cause the crack to propagate 5 mm. It should be noted that the number of sea states for the crack to grow 5 mm does not always decrease as the crack length increase, such as the crack grows to 25 mm, 50 mm etc. This is because the crack can grow slower when the ship meets more calm sea state, e.g. during summer, or/and sails with a better operation condition (speed and heading angles with respect to waves, and less loadings etc.). These findings can be used in a ship fatigue routing plan, i.e. optimization of ship courses, ship speeds, heading angles and loading conditions. It has to be combined with the shipping schedule, structural ultimate safety, onboard weather forecast information and regular inspection to maximize the ship's serviceability before the regular inspection and maintenance. Further, the large standard deviation in Fig. 6 indicates that the ship has a potential to extend its repair time dramatically if the ship's operation is well planned.

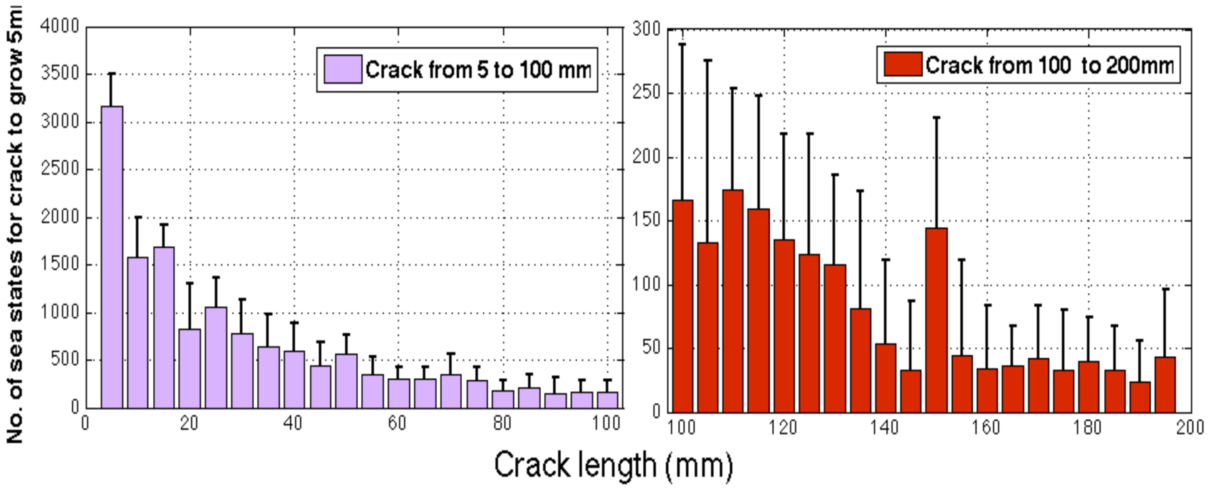

Figure 6. Median and standard deviation of the sailing periods (number of sea states) when the fatigue crack reaches 200 mm. Bars: median values of sea state number; lines: the standard deviation of sea state number needed for the crack to propagate 5 mm.

5. Discussion and conclusions

Conventional fatigue design of ship structures is carried out using high-cycle fatigue approaches, with stress ranges from either empirical data or direct calculations. Because of large uncertainties involved in the ship fatigue design process, such as the variation of encountered wave environments, computation of hydrodynamic fatigue loads and stress concentration factor, etc., fatigue cracks occur much earlier than expected. It is of great interest to study the crack propagation conditions in order to design and plan a maintenance strategy based on crack growth management, which ensures e.g. safety of the vessel. In this study, an efficient method for fatigue crack propagation analysis in ships was derived based on the narrow-band spectral fatigue method. The spectral method was validated by full-scale measurements on a 2800TEU container ship, to give accurate prediction of stress range distributions. In a crack propagation analysis, a case study using an example from reality of a structural detail prone to fatigue was used to demonstrate the application of the proposed method. The results show that depending on the encountered wave environments, the crack is critical and needs to be repaired within 2 to 3 years. During the crack propagation process, many sea states were required for the crack to grow 5 mm, but when the crack reached close to 200 mm, only a few sea states grew the crack 5 mm. It indicates the necessity to repair the crack in time since it can grow so fast that structural integrity and safety may be jeopardized.

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