Combined Self-Consistent and Mori-Tanaka Approach for Evaluation of Elastoplastic Property of Particulate Composites

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Abstract Suppose the particles in with volume fraction c in an RVE of a particulate composite are separated into two groups, with volume fractions $(1-\lambda^{-1})c$ and c/λ over the RVE, respectively, a combined self-consistent and Mori-Tanaka approach is proposed for the evaluation of the effective elastoplastic property of particulate composites. The particles in Group I and the original matrix form a fictitious matrix, and its mechanical property is determined with the self-consistent scheme. The RVE of the composite consists of the fictitious matrix and Group of particles, and its mechanical property is determined with the Mori-Tanaka scheme. The conventional Mori-Tanaka scheme and self-consistent scheme can be obtained as the two limit cases as λ =1 and λ =∞, respectively. The constitutive behavior of the particles in Group I is identical with that in Group II. The effective elastoplastic behavior of some typical particulate composites is evaluated, and the comparison with the experimental results demonstrates the validity of the proposed approach. The induced λ can serve as a parameter related to the actual property of composites and identified by experiment, for a more accurate evaluation of the effective elastoplastic property of particulate composites.

Keywords Particulate composite, Elastoplasticity, Effective property, Mean-field approach

1. Introduction

The overall property of a composite is governed by the properties of their constituents and the microstructures. In order to achieve the desired property of a composite, an appropriate approach for the evaluation of the overall property of the composites is essential. Many micromechanics approaches have been developed [1-8] and extensively used to evaluate the effective properties of composites. The homogenization approaches have also been developed for the evaluation of the elastoplastic or elastoviscoplastic properties of composites [9-11].

It has been shown that there might be a big gap between the effective properties of a composite obtained respectively with the conventional Mori-Tanaka scheme and the conventional self-consistent scheme [12]. Since both schemes are extensively used in the evaluation of the effective property of composites, a question one may ask is whether one can evaluate more accurately the effective property of a composite with the combination of these two schemes. A combined self-consistent and Mori-Tanaka approach was proposed by Peng et al [13] for the evaluation of the effective elastic property of particulate composites. It was shown that the conventional Mori-Tanaka scheme and self-consistent scheme can be obtained as the two limit cases of the approach. The effective elastic properties of some typical particulate composites were evaluated and compared with experimental results, and the satisfactory agreement between the computed and the experimental results demonstrates the validity of the proposed approach.

In this article, the combined self-consistent and Mori-Tanaka approach [13] is extended to the evaluation of elastoplastic properties of particulate composites. The effective elastoplastic responses of some typical particulate composites are analyzed and compared with experimental results.

2. Extension of Combined Self-consistent and Mori-Tanaka Approach

An RVE of a particulate composite consists of the matrix of mechanical property $\mathbf{L}^m$ and particles of mechanical property $\mathbf{L}^c$ with total volume fraction c. The particles are separated into two groups by introducing a parameter λ ($\lambda \geq 1$): Group I contains particles with total volume fraction $(1-\lambda^{-1})c$ over the RVE, and Group II contains particles with total volume fraction c/λ over the RVE. The effective elastoplastic property of the composite can be determined by two steps: in the first step, the particles in Group I are embedded in the original matrix to form the fictitious matrix (with equivalent particle volume fraction $\hat{c}$), and the elastoplastic property of the fictitious matrix, $\hat{\mathbf{L}}^m$, is determined with the self-consistent scheme; and in the second step, the particles in Group II are further distributed randomly in the fictitious matrix to form the RVE, and the effective elastoplastic property of the composite, $\bar{\mathbf{L}}$, is determined with the Mori-Tanaka scheme, the particle inclusions in Group II should be necessarily sufficient to meet the requirement of the application of the Mori-Tanaka scheme.

Suppose the volume of the RVE is V, the volume of the original matrix is $V_m = (1-c)V$ and the volume of the particle inclusions in Group I is $(1-\lambda^{-1})cV$, the volume of the fictitious matrix is $\hat{V}_m = V_m + (1-\frac{1}{\lambda})cV = (1-\frac{c}{\lambda})V$, and the equivalent volume fraction of the particles in the fictitious matrix is

$$\hat{c} = (1-\frac{1}{\lambda})cV \bigg/ \left[(1-\frac{1}{\lambda})cV + (1-c)V\right] = \frac{\lambda-1}{\lambda-c}c\,. \tag{1}$$

In the first step, making use of the Hill's self-consistent scheme, the elastoplastic tensor of the fictitious matrix can be determined as follows by averaging the stress rate field over $\hat{V}_m$, i.e.,

$$\hat{\mathbf{L}}^m = (1-\hat{c})\mathbf{L}^m \cdot \mathbf{A}^m + \hat{c}\mathbf{L}^c \cdot \mathbf{A}^c\,, \tag{2}$$

where
$$\mathbf{A}^c = [\mathbf{L}^* + \mathbf{L}^c]^{-1} : [\mathbf{L}^* + \hat{\mathbf{L}}^m], \qquad \mathbf{A}^m = [\mathbf{L}^* + \mathbf{L}^m]^{-1} : [\mathbf{L}^* + \hat{\mathbf{L}}^m]\,, \tag{3}$$

and
$$\mathbf{L}^* = \hat{\mathbf{L}}^m : \left(\mathbf{S}^{-1} - \mathbf{I}_4\right). \tag{4}$$

$\hat{\mathbf{L}}^m$ relates the stress rate $\hat{\dot{\boldsymbol{\sigma}}}^m$ and the strain rate $\hat{\dot{\boldsymbol{\varepsilon}}}^m$ of the fictitious matrix by

$$\hat{\dot{\boldsymbol{\sigma}}}^m = \hat{\mathbf{L}}^m : \hat{\dot{\boldsymbol{\varepsilon}}}^m\,, \tag{5}$$

and the stress rate and the strain rate in the original matrix and the particles are determined with

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}}^c &= \mathbf{A}^c : \hat{\dot{\boldsymbol{\varepsilon}}}^m, & \dot{\boldsymbol{\sigma}}^c &= \mathbf{L}^c : \dot{\boldsymbol{\varepsilon}}^c, \\ \dot{\boldsymbol{\varepsilon}}^m &= \mathbf{A}^m : \hat{\dot{\boldsymbol{\varepsilon}}}^m, & \dot{\boldsymbol{\sigma}}^m &= \mathbf{L}^m : \dot{\boldsymbol{\varepsilon}}^m., \end{aligned} \tag{6}$$

In the second step, making use of the Mori-Tanaka scheme and keeping in mind that the RVE of the material is composed of the fictitious matrix and the particle inclusions in Group II (with total volume fraction c/λ) and the domain for averaging strain (or stress) is $\hat{V}^m + \frac{c}{\lambda}V = V$, the corresponding expression can easily be obtained as

$$\dot{\boldsymbol{\varepsilon}}^c = \tilde{\mathbf{A}}^c : \hat{\dot{\boldsymbol{\varepsilon}}}^m, \qquad \dot{\boldsymbol{\sigma}}^c = \mathbf{L}^c : \dot{\boldsymbol{\varepsilon}}^c, \tag{7}$$

where $$\tilde{\mathbf{A}}^c = [\tilde{\mathbf{L}}^* + \mathbf{L}^c]^{-1} : [\tilde{\mathbf{L}}^* + \hat{\mathbf{L}}^m], \quad \text{with} \quad \tilde{\mathbf{L}}^* = \hat{\mathbf{L}}^m : (\mathbf{S}^{-1} - \mathbf{I}_4), \tag{8}$$

$$\bar{\mathbf{L}} = \hat{\mathbf{L}}^m + \frac{c}{\lambda}\left(\mathbf{L}^c - \hat{\mathbf{L}}^m\right) : \tilde{\mathbf{A}}^c : \left[\left(1 - \frac{c}{\lambda}\right)\mathbf{I} + \frac{c}{\lambda}\tilde{\mathbf{A}}^c\right]^{-1}, \tag{9}$$

$$\dot{\bar{\boldsymbol{\sigma}}} = \bar{\mathbf{L}} : \dot{\bar{\boldsymbol{\varepsilon}}}, \tag{10}$$

$$\hat{\dot{\boldsymbol{\varepsilon}}}^m = \left[\left(1 - \frac{c}{\lambda}\right)\mathbf{I} + \frac{c}{\lambda}\tilde{\mathbf{A}}^c\right]^{-1} : \dot{\bar{\boldsymbol{\varepsilon}}}. \tag{11}$$

It can be seen easily by comparing Eq. (8) with Eqs. (3) and (4) that $\tilde{\mathbf{L}}^* = \mathbf{L}^*$ so that $\tilde{\mathbf{A}}^c = \mathbf{A}^c$, indicating that the response of the particles in Group I given by Eq. (6) is identical with that of the particles in Group II given by Eq. (7). This fact, on one hand, implies the consistency of the constitutive behavior of the two parts of the particles in the extended approach, and on the other hand, brings convenience to the corresponding analysis.

Eqs. (9) and (11) can be reduced to the results of the conventional Mori-Tanaka scheme if λ =1 (noticing that if λ=1, there would be no particles in the fictitious matrix), $\hat{\mathbf{L}}^m = \mathbf{L}^m$. If λ is sufficiently large so that $c/\lambda \to 0$ (implying that there would be no particles in Group II), one obtains $\bar{\mathbf{L}} = \hat{\mathbf{L}}^m$ (Eq. (9)), $\hat{\dot{\boldsymbol{\varepsilon}}}^m = \dot{\bar{\boldsymbol{\varepsilon}}}$ (Eq. (11)), so that $\hat{\dot{\boldsymbol{\sigma}}}^m = \dot{\bar{\boldsymbol{\sigma}}}$ (Eqs. (10) and (5), noticing that $\dot{\bar{\boldsymbol{\sigma}}} = \dot{\boldsymbol{\sigma}}^0$ in the Mori-Tanaka scheme), the extended approach is reduced to the conventional self-consistent scheme.

Thus, it is known that the conventional Mori-Tanaka scheme and the conventional self-consistent scheme can be obtained as the two limit cases of the proposed approach as λ=1 and $\lambda=\infty$, and the variation of λ may provide the evaluation of the elastoplastic property varying between the result given by the Mori-Tanaka scheme and that given by the self-consistent scheme. In practical application, the introduced λ may serve reasonably as an adjustable parameter that can be associated with the actual properties of composites and determined by experiment. This is important because the mechanical properties of composites are affected by many factors such as compositions, microstructures, internal constraints, etc., and the fabrication of composites may inevitably involve various unexpected factors that also strongly affect the overall property of the composites. The effects of these factors could be comprehensively considered with λ.

3. Constitutive Model and Numerical Algorithm

3.1. Elastoplastic constitutive equation

Assuming both constituents of a composite are initially isotropic and plastically incompressible, in the case of isothermal and small deformation, the following incremental form of the elastoplastic relationship can be obtained [14]

$$\Delta\mathbf{s}(z) = A\Delta\mathbf{e}^p + \mathbf{B}(z_n)\Delta z \tag{12}$$

where $\mathbf{s}$ and $\mathbf{e}^p$ are deviatoric stress and plastic strain, respectively,

$$A = \sum_{r=1}^{3} k_r C_r \qquad \mathbf{B}(z_n) = -\sum_{r=1}^{3} k_r \alpha_r \mathbf{s}^{(r)}(z_n), \tag{13}$$

$$\Delta\mathbf{s}^{(r)}(z) = k_r (C_r \Delta\mathbf{e}^p - \alpha_r \mathbf{s}^{(r)}(z_n))\Delta z, \quad k_r = \frac{1 - e^{-\alpha_r \Delta z}}{\alpha_r \Delta z}, \qquad z = z_n + \Delta z, \tag{14}$$

z is generalized time that is non-negative and increases monotonically during any plastic deformation in terms of the following definition

$$\Delta z = \frac{\Delta\zeta}{f(z)}, \qquad \Delta\zeta^2 = \Delta\mathbf{e}^p : \Delta\mathbf{e}^p, \tag{15}$$

$f(z)$ is a hardening function, C_r and α_r (r =1,2,3) are material constants. The deviatoric elastic relation is

$$\Delta\mathbf{e} - \Delta\mathbf{e}^p = \frac{\Delta\mathbf{s}}{2G} \tag{16}$$

where $\mathbf{e}$ is deviatoric strain tensor and G is shear modulus. The volumetric constitutive relation can be expressed as

$$tr(\boldsymbol{\sigma}) = 3K tr(\boldsymbol{\varepsilon}) \qquad \text{or} \qquad tr(\Delta\boldsymbol{\sigma}) = 3K tr(\Delta\boldsymbol{\varepsilon}), \tag{17}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are stress and strain tensors, respectively, and K is bulk modulus. Rewriting Eq. (15) as

$$\Delta z = \frac{\Delta\mathbf{e}^p}{f^2 \Delta z} : \Delta\mathbf{e}^p \tag{18}$$

and making use of the following relationships

$$\Delta\mathbf{s} = \Delta\boldsymbol{\sigma} - \frac{1}{3} tr(\Delta\boldsymbol{\sigma})\mathbf{I}_2 \qquad \Delta\mathbf{e} = \Delta\boldsymbol{\varepsilon} - \frac{1}{3} tr(\Delta\boldsymbol{\varepsilon})\mathbf{I}_2, \tag{19}$$

where $\mathbf{I}_2$ is the identity tensor of rank two, the incremental constitutive equation can be derived from Eqs. (12), (16) and (17) as [14]

$$\Delta\boldsymbol{\sigma} = \mathbf{L} : \Delta\boldsymbol{\varepsilon}, \tag{20}$$

where

$$\mathbf{L} = 2G_p \mathbf{I}_4 + \left\{ K - \frac{2}{3} G_p \right\} \mathbf{I}_2 \otimes \mathbf{I}_2 + \frac{(2G)^2}{(2G + A)^2 f^2(z)} \frac{1}{a} \mathbf{B} \otimes \frac{\Delta\mathbf{e}^p}{\Delta z}, \tag{21}$$

with
$$G_p = \frac{GA}{2G+A}, \qquad a = 1 + \frac{\mathbf{B} : \Delta \mathbf{e}^p}{(A+2G) f^2 \Delta z}. \tag{22}$$

If the particles are assumed linearly elastic, the constitutive relationship of the particles can easily be obtained as follows by ignoring the terms related to plastic deformation and noticing that for elastic material, A is sufficiently large and G_p tends to G (Eqs. (22) and (21))

$$\mathbf{L} = 2G\mathbf{I}_4 + \left\{ K - \frac{2}{3} G \right\} \mathbf{I}_2 \otimes \mathbf{I}_2 . \tag{23}$$

Eq. (20) can be specified as follows for the matrix and the particles in a particulate composite

$$\Delta \boldsymbol{\sigma}^m = \mathbf{L}^m : \Delta \boldsymbol{\varepsilon}^m \qquad \text{and} \qquad \Delta \boldsymbol{\sigma}^c = \mathbf{L}^c : \Delta \boldsymbol{\varepsilon}^c \tag{24}$$

3.2. Numerical Algorithm

For simplicity, we assume in the analysis that the particles are of spheres with identical size, if we further ignore the minor difference between the result obtained with anisotropic Eshelby tensor and that obtained with isotropic Eshelby tensor provided the plastic deformation is moderate [15], both the original matrix and the fictitious matrix can be approximated as overall isotropic, then the following Eshelby tensor for isotropic media is adopted in the analysis [10],

$$\mathbf{S} = a\, \mathbf{I}_2 \otimes \mathbf{I}_2 + b[\mathbf{I}_4 - \mathbf{I}_2 \otimes \mathbf{I}_2], \tag{25}$$

with
$$b = \frac{2(4-5\nu)}{15(1-\nu)}, \qquad a = 1 - b . \tag{26}$$

The analysis of the response of a composite material involves the constitutive behavior of the original matrix and that of the particles, which are determined with Eq. (6). For a stress-controlled process, the numerical algorithm for the analysis of the elastoplastic response of a two-constituent composite is stated as follows: For the prescribed particle volume fraction c and the parameter λ, $\hat{c}$ can be calculated with Eq. (1), and with the results obtained in the k-th iteration of the l-th increment of loading, such as $(\Delta\bar{\boldsymbol{\varepsilon}})_{(l)}^{(k)}$ and of the composite, $(\hat{\mathbf{L}}^m)_{(l)}^{(k)}$ of the fictitious matrix, $(\Delta\boldsymbol{\varepsilon}^m)_{(l)}^{(k)}$, $(\Delta\boldsymbol{\sigma}^m)_{(l)}^{(k)}$ and $(\mathbf{L}^m)_{(l)}^{(k)}$ of the matrix, and $(\Delta\boldsymbol{\varepsilon}^c)_{(l)}^{(k)}$, $(\Delta\boldsymbol{\sigma}^c)_{(l)}^{(k)}$ and $(\mathbf{L}^c)_{(l)}^{(k)}$ of the particles, in the following (k+1)th iteration, $(\mathbf{L}^*)_{(l)}^{(k+1)}$, $(\mathbf{A}^c)_{(l)}^{(k+1)}$ and $(\mathbf{A}^m)_{(l)}^{(k+1)}$ can be calculated with Eqs. (4) and (3) respectively, and then, keeping in mind $\tilde{\mathbf{A}}^c = \mathbf{A}^c$, $\bar{\mathbf{L}}_{(l)}^{(k+1)}$ of the composite can be immediately obtained with Eq. (9). Given the l-th increment of stress $(\Delta\boldsymbol{\sigma}^0)_{(l)}$, $(\Delta\bar{\boldsymbol{\varepsilon}})_{(l)}^{(k+1)}$ can be solved from Eq. (10), and $(\Delta\hat{\boldsymbol{\varepsilon}}^m)_{(l)}^{(k+1)}$ with Eq.(11), then $(\Delta\boldsymbol{\varepsilon}^m)_{(l)}^{(k+1)}$, $(\Delta\boldsymbol{\sigma}^m)_{(l)}^{(k+1)}$, $(\Delta\boldsymbol{\varepsilon}^c)_{(l)}^{(k+1)}$ and $(\Delta\boldsymbol{\sigma}^c)_{(l)}^{(k+1)}$ can be obtained with Eq. (6), the mechanical property of the matrix $(\mathbf{L}^m)_{(l)}^{(k+1)}$ and that of particles $(\mathbf{L}^c)_{(l)}^{(k+1)}$ can be updated with Eqs. (21) and (23), respectively, than the mechanical property of the fictitious matrix, $(\hat{\mathbf{L}}^m)_{(l)}^{(k+1)}$, can be calculated with Eq. (2). The iterative process continues until the following inequality is satisfied,

$$\delta = \frac{\left\| \Delta \overline{\boldsymbol{\varepsilon}}_{(l)}^{(k+1)} - \Delta \overline{\boldsymbol{\varepsilon}}_{(l)}^{(k)} \right\|}{\left\| \Delta \overline{\boldsymbol{\varepsilon}}_{(l)}^{(k+1)} \right\|} \le \delta_0 , \tag{27}$$

where δ_0 is the tolerant error and δ_0=0.0001 is used in computation. After superimposing the derived increments on the corresponding quantities up to that after the (l-1)th increment of loading, one obtains $(\boldsymbol{\sigma}^0)_{(l)}$ and $(\overline{\boldsymbol{\varepsilon}})_{(l)}$ of the composite, and then starts the computation for the next increment of loading.

3. Numerical Examples and Verification

The elastoplastic behavior of some typical particulate composite materials subjected to tensile/compressive loading is computed and compared with experimental results.

The material constants involved in the proposed approach includes elastic/elastoplastic material constants of each constituent and λ. For the composites consisting of purely elastic particles and elastoplastic matrix, the elastic property of each constituent is usually provided by material suppliers or can be determined with conventional test; and the plastic constants of the matrix can be identified with simple test (e.g., tensile test) of pure matrix; λ can then be identified by fitting simple testing result (e.g., σ-ε curve) of the composite.

The variation of the tensile response of an Al/Al_2O_3 composite with respect to the volume fraction of Al_2O_3 particles is computed and shown in Fig. 1. The material constants adopted are given in Table 1, in which the Al matrix is assumed elastoplastic and the Al_2O_3 particles are purely elastic. The elastic constants are adopted from [16]. The plastic constants of the matrix are identified with the empirical relation $\sigma = 130 \times \left[1 + \left(\varepsilon^p \right)^{0.2} \right]$ [16], and the hardening function $f(z)$ (in Eq.(15)) takes the following form

$$f(z) = d - (d-1)e^{-\beta z} \tag{28}$$

where d and β are material constants. It can be seen in Fig. 1 that the σ - ε curve of the Al matrix (c=0) computed with the obtained material constants can reasonably replicate the empirical σ-ε curve. The ultimate strength of the composite, σ_u, increases with the increase of c.

Table 1 Material constants of Al/Al_2O_3

Constituent	E (GPa)	v	C_1/α_1, C_2/α_2, C_3/α_3 (MPa)	α_1, α_2, α_3	d, β
Al	69	0.345	112, 25, 17.5	2500, 800, 200	1.25, 16
Al_2O_3	400	0.25	--------------------	------------------	---------

The effective σ - ε curves of c=0.34 corresponding to different λ is shown in Fig.2(a), where the σ-ε curve is hoisted with the increase of λ, but all the curves lie between the two bounds determined respectively by λ=1 and $\lambda \rightarrow \infty$. There is a remarkable gap between the σ-ε curves evaluated respectively with λ=1 and $\lambda \rightarrow \infty$. The variation of σ_u against λ for c=0.34 is shown in Fig. 2(b), where the results given by the conventional Mori-Tanaka scheme and the conventional self-consistent scheme are independent of λ and serve as the lower and the upper bounds, respectively. The results given by the combined self-consistent and Mori-Tanaka approach starts from that obtained with Mori-Tanaka scheme, increases with λ, and approaches the asymptotic value determined with self-consistent scheme. It can be seen that the experimental result also lies in the region bounded by the curve with λ=1 and that with λ=∞. It can be found that the σ-ε curve with λ=8 can well fit the experimental result. All the curves in Fig. 1 are computed with λ=8, the obtained results fit well the empirical results at c=0 and experimental results at c=0.34, and the curves obtained at different c seems reasonable.

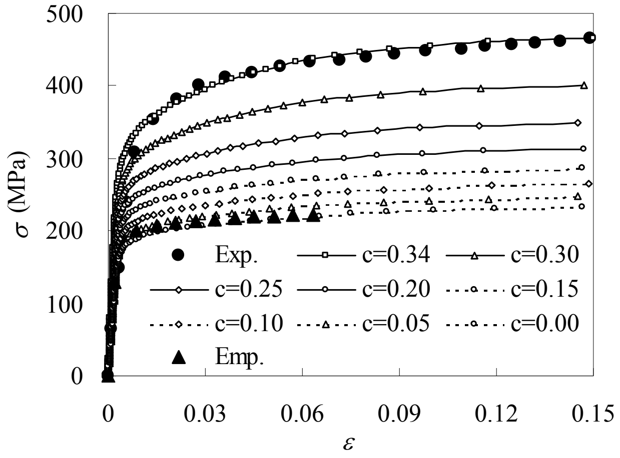
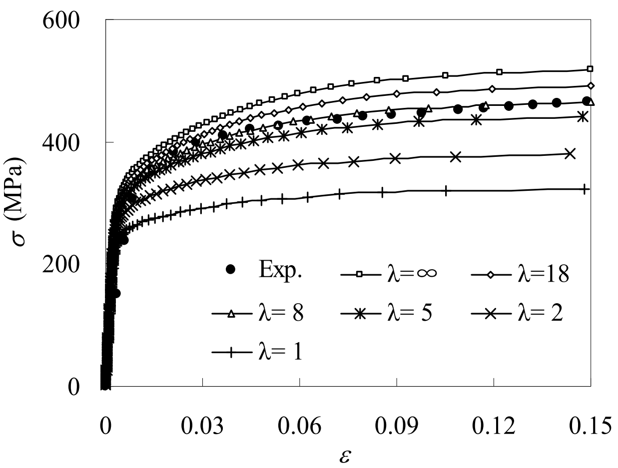
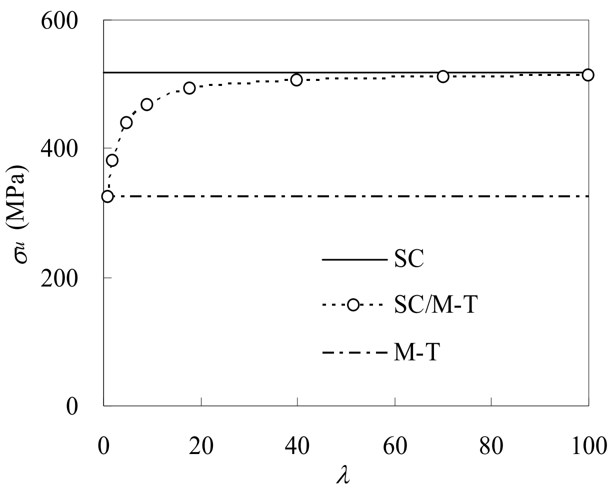


Fig. 1 Experimental and computational σ-ε curves of Al/Al_2O_3 composite



(a) Effective σ-ε curves of Al/Al_2O_3 composite using different λ (c=0.34)



(b) Variation of ultimate strength against λ (c=0.34)

Fig. 2 Effect of λ on σ-ε curves of Al/Al_2O_3 composite

It can be found in Fig. 2 that λ strongly affects the evaluated effective property of the composite. If λ changes between 1 and infinite, the evaluated effective property of the composite will change between the result determined by the Mori-Tanaka scheme and that by the self-consistent scheme, with the tendency definitely identical to that caused by the decrease of particle size, which was reported frequently and investigated extensively by many researchers [17-20]. It implies the possibility for the extended approach to describe implicitly and phenomenologically the size-effect of particulate composites. Suppose the volume of a RVE is fixed, given particle volume fraction c, the number of particle inclusions in the RVE, N, should reflect the size information of the particles. For instance, N should be a smaller value for larger particles, and vice versa. Suppose the particle volume fraction c is fixed in both Composite 1 and Composite 2, and the particles are of identical spheres of diameter φ_1 in Composite 1 and diameter φ_2 in Composite 2, letting $N_1=\lambda_1 m$ for Composite 1 and $N_2=\lambda_2 m$ for Composite 2, where m is a constant related to the volume of RVE and the minimal number of particles required by the application of the Mori-Tanaka scheme, one can easily obtain the relation between λ_1 and λ_2 of the two composites as $\lambda_2=\left(\varphi_1/\varphi_2\right)^3\lambda_1$. It indicates that if λ_1 for the evaluation of the effective property of Composite 1 is fixed or fitted from experimental data, λ_2 for that of Composite 2 can be approximately estimated with the above relationship without necessity of being identified individually.

Fig. 3 shows the capability of the present approach in the description of the effective σ-ε relation of the composite consisting of Al356 (T4) matrix and 15% SiC particles. The material constants adopted are listed in Table 2, where it is assumed that the Al356 (T4) matrix is elastoplastic and the SiC particles are purely elastic. The elastic constants of the two constituents are the same as those used by Lloyd [17]. The plastic constants of the matrix are identified by fitting the empirical relation $\sigma=86+141.7\left(\varepsilon^p\right)^{0.365}$ given in [17]. It can be seen in Fig. 3 that the σ-ε curve of the matrix obtained by the extended approach (with c=0) agrees reasonably with that given by the empirical relation [17]. For c=0.15 and φ=16 μm, the effective σ-ε curve obtained with the extended approach using λ_{16} =1, corresponding to the result using the Mori-Tanaka scheme, can reasonably fit the experimental result. Experimental results show that, even if the volume fraction of particles keeps unchanged, the composite with smaller SiC particle inclusions (e.g., φ=7.5 μm) exhibits larger resistance against irreversible deformation. The effective σ-ε curve of the composites using $\lambda=\left(16/7.5\right)^3\lambda_{16}\approx 9.71$ is shown in Fig. 3 with solid line, which agrees reasonably with the experimental effective σ-ε curve. The effective σ-ε curve computed with the self-consistent scheme is also shown in Fig. 3 for comparison.

Table 2 Material constants of Alcoa X2080/SiC composite

Constituent	E (GPa)	v	C_1/α_1, C_2/α_2, C_3/α_3 (MPa)	α_1, α_2, α_3	d, β
Al356 (T4)	70	0.33	72.14, 31.25, 18.0	7000, 800, 200	1.75, 30
SiC	490	0.17	--------------------	----------------	--------

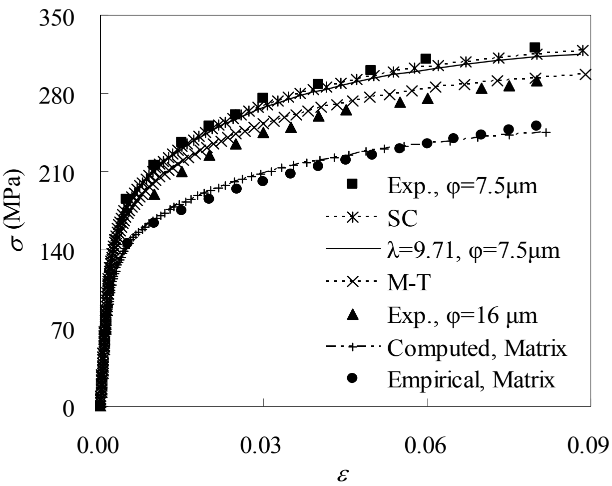


Fig. 3 Experimental and computational σ-ε curves of Al356/SiC composite

5. Conclusion and discussion

The combined self-consistent and Mori-Tanaka approach for the evaluation of the elastic property of particulate composites [13] is extended to the evaluation of the elastoplastic property of particulate composites. Several examples are exhibited and compared with the experimental results. The following conclusion can be drown from the analysis:

(1) The comparison between the computed and the experimental results shows that the effective elastoplastic property of the composites can be satisfactorily evaluated with the extended approach by properly choosing the parameter λ, demonstrating the validity of the extended approach.

(2) The results given by the Mori-Tanaka scheme and the self-consistent scheme can be obtained as two bounds of that by the extended approach as the $\lambda=1$ and $\lambda=\infty$, respectively, and the variation of λ between $\lambda=1$ and $\lambda=\infty$ yields the results lying between the two bounds.

(3) The constitutive behavior of the particles outside of the introduced fictitious matrix is identical with that of the particles inside the fictitious matrix, indicating the consistency of the particle behavior in the combined approach.

(4) The introduced parameter λ can take into account comprehensively the effects of the factors such as microstructures (e.g., the size and shape of the inclusions), internal constraints (e.g., interfacial condition between different constituents), etc., as well as various unexpected factors during the fabrication of composites, which may strongly affect the overall property of the composites. In other words, λ can be associated with the actual property of composites and identified with experiment for a more accurate evaluation of the effective property of composites.

(5) The approach can describe implicitly the effect of particle size on the elastoplastic property of composites. With the help of the baseline experimental data of a composite with the particles of a specified size, the mechanical properties of composites with the same constituents but different size of particles could be estimated approximately with the proposed approach.

In this article we like to suggest a method parallel with the conventional mean-field schemes. In order to show more clearly the capability of the approach, we do not introduce other influencing factors, for example, higher-order gradients of deformation or isotropisation of tangent stiffness tensor, etc., to avoid blurring the advantages of the proposed approach.

It should be mentioned that, in the proposed approach with the increase of λ, the obtained effective elastoplastic response exhibits the tendency definitely identical with the size-effect phenomenon. Although it is known that “such effects cannot be included in the conventional inclusion-infinite matrix based Eshelby problem” [21], we are still interested in this capability. It can be accounted for with the difference between the assumptions used in the Mori-Tanaka scheme and in the self-consistent scheme. It is known that, although the Mori-Tanaka scheme involves the interaction between mediums, when any particle is added, the matrix is always considered as the original one without taking into account the existing reinforcement by other particles. Thus, in mathematics, the particles in an RVE can be assumed necessarily sufficient and randomly distributed, which implies relatively large size of the particle inclusions. However, in the conventional self-consistent scheme, the property of the matrix is assumed to be that after all the particle inclusions are added, which implies sufficiently large number of sufficiently small inclusions in the RVE. Since the proposed approach could adjust the property of the fictitious matrix (from that of the original matrix to that of the matrix with all particle inclusions being added) by adjusting λ, it should be able to describe the particle size effect to some extend. The introduced parameter λ can, on one hand, take into account averagely the enforcement in the fictitious matrix, and on the other hand, related to the ratio of the particle size to the reference particle size, which account for why it possesses the capability of describing the relative “size-effect” of particulate composites.

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References

[1] A. V. Hershey, The elasticity of an isotropic aggregate of anisotropic cubic crystals. J Appl Mech, 21(1954) 236–240.
[2] E. Kröner, Berechung der elastischen konstanten des vielkristalls aus den konstanten des einkristalls. Zeitschrift für Physik, 151(19584) 504–518.
[3] B. Budiansky, On the elastic moduli of some heterogeneous materials. J Mech Phys Solids, 13(1965) 223–227.
[4] R. Hill, A self-consistent mechanics of composite materials. J Mech Phys Solids, 13(1965) 213–222.
[5] T. Mori, K. Tanaka, Average stress in matrix and average elastic energy of materials with misfitting inclusions. Acta Metall, 21(1973) 571–574.
[6] S. Nemat-Nasser, M. Hori, Micromechanics: Overall properties of heterogeneous materials. Elsevier, North-Holland, 1993.
[7] R. M. Christensen, K. H. Lo, Solutions for effective shear properties in three phase space and cylinder model. J Mech Phys Solids, 27(1979) 315–330.
[8] Y. Huang, K. X. Hu, X. A. Wei, Chandra, A generalized self-consistent mechanics method for a composite with multi-phase inclusions. J Mech Phys Solids, 42(1994) 491–504.
[9] J. W. Hutchinson, Bounds and self-consistent estimates for creep of polycrystalline materials. Proc Royal Soc London. Series A, Math Phys Sci, 1976, 348(1652) 101–127.
[10] M. Berveiller, A. Zaoui, An extension of the self-consistent scheme to plastically flowing polycrystals. J Mech Phys Solids, 26(1979) 325–344.
[11] A. Molinari, G. R. Canova, S. Ahzi, A self consistent approach of the large deformation polycrystal viscoplasticity. Acta Metall, 35(1987) 2983–2994.
[12] R. M. Christensen, A critical evaluation for a class of micro-mechanical models. J Mech Phys Solids, 38(1990) 379–404.
[13] X. Peng, N. Hu, H. Zheng, H. Fukunaga, Evaluation of mechanical properties of particulate composites with a self-consistent and Mori-Tanaka approach. Mech Mat, 41(2009) 1288–1297.
[14] J. Fan, X. Peng, A physically based constitutive description for nonproportional cyclic plasticity. J Engng Mat Tech, 113(1991) 254-262.
[15] X. Peng X. Long, J. Fan, Micro-Macro Description for elastoplasticity of materials with lamellar microstructure using anisotropic Eshelby tensor, Mech Adv Mat Structs, 16(2009) 542-551.
[16] M. Kouzeli, D. C. Dunand, Effect of reinforcement connectivity on the elasto-plastic behavior of aluminum composites containing sub-micron alumina particles. Acta Mat, 51(2003) 6105–6121.
[17] D. J. Lloyd, Particle-reinforced aluminumand magnesiummatrix composite. Int Mat Rev, 39(1994) 1–23.
[18] K. Derrien, D. Baptiste, D. Guedra-Degeorges, J. Foulquier, Multiscale modeling of the damaged plastic behavior and failure of Al/SiCp composites, Int J Plasticity, 15(1999) 667-685.
[19] Y. Nakamura, M. Yamaguchi, M. Okubo, T. Matsumoto, Effects ofparticle size on mechanical and impact properties of epoxy resin filled with spherical silica. J Appl Polymer Sci, 45(1992) 1281–1289.
[20] P. Barai, G. J. Weng, The competition of grain size and porosity in the viscoplastic response of nanocrystalline solids. Int J Plasticity, 24(2008) 1380–1410.

[21] V. Marcadon, E. Herve, A. Zaoui, Micromechanical modeling of packing and size effects in particulate composites. Int J Solids Structs, 44(2007) 8213–8228.