J-integral Calculation of Nonlinear Fracture for Colloidal Soft Material

Hongyan Wang, Jun Lei, Qingsheng Yang *

Department of Solid Mechanics, Beijing University of Technology, 100124, China
* Corresponding author: qsyang@bjut.edu.cn; Tel & Fax: 86-10-67396333

Abstract J-integral of nonlinear fracture for the colloid soft material is calculated by the finite element method in this paper. The J-integral is based on the concept of conservation of energy, hardly depending on the crack-tip stress singularity. Therefore, the fracture of colloidal soft material could be characterized by the J-integral. In this paper, a limited area near the crack tip replacing the integral loop is used to calculate J-integral value. J-integral within one unit is approximated by summation of the formulas related with gauss points. Such repeatedly, J-integral corresponding to integral path is calculated. Effect of different loading conditions and crack lengths on J-integral of PVA-H nonlinear fracture mainly is emphasized. This study shows that the J-integral can effectively characterize the nonlinear fracture of colloidal soft material.

Keywords PVA-H, Fracture, J-integral, Finite element method

1. Introduction

The wide use of hydrogels in the biomedical field is getting more and more frequent in recent years, and enormous potential in the tissue engineering is showed for the swelling and indissolvable features in the water. Especially, polyvinyl alcohol hydrogel (PVA-H) is a very promising material. A few studies have confirmed that the biomechanical properties of the polyvinyl alcohol hydrogel are similar to that of the articular cartilage, which is based on the good biocompatibility and is able to be a partial substitute for the articular cartilage, delay or prevent the occurrence of traumatic osteoarthritis. So PVA-H has become an alternative material with great promise in a clinical context [1-2]. However, after implantation into the human body PVA-H will appear wearing and cracks which can impact its function as well as what articular cartilage damage seriously affects. Therefore, the study of the PVA-H fracture characteristics is obviously important, but few international scholars research on the aspect at present.

Although the mechanical constitutive relation of the PVA-H is complicated, a relatively simple elastic-plastic theory model is adopted to research PVA-H material by simplifying the relation in this paper. On linear elasticity or small range yield condition, PVA-H crack tip energy release rate is evaluated by J-integral fracture criterion which is commonly used to deal with nonlinear fracture problem. The J-integral is based on the concept of conservation of energy, hardly depending on the crack-tip stress singularity and treating specially for the unit at the crack tip. Hence, J-integral can effectively characterize the nonlinear fracture of PVA-H material.

In this paper, aiming at PVA-H as articular cartilage repair material, the energy release rate of PVA-H crack tip is calculated by finite element method. J values of the crack tip under different loading conditions and different integration paths are obtained, also in consideration of the impact of the crack length on the J-integral.

2. Theoretical model

Polyvinyl alcohol aqueous solution gel can be made into polyvinyl alcohol hydrogel elastomer by repeated freezing-melting method. In 2007, Pan et al. [3] obtained the PVA-H curves of the stress and instant tensile modulus with changing strain through five freezing-melting cycles (see Fig.1). PVA gel is a kind of viscoelastic material, whose stress-strain relationship has typical viscoelastic properties. Taking the modulus-strain relationship into consideration, it is known that the tensile modulus of PVA gel is significantly various in different strain range. When strain range is 0 ~ 100%, PVA gel whose tensile modulus is essentially unchanged has elasticity characteristic, so its stress-strain shows a good linear relationship. In 100 ~ 250% strain, PVA tensile modulus increases up to 4 times with the increment of strain. In this range, PVA gel demonstrates typical viscoelastic character, whose stress-strain relationship is distinctly nonlinear.

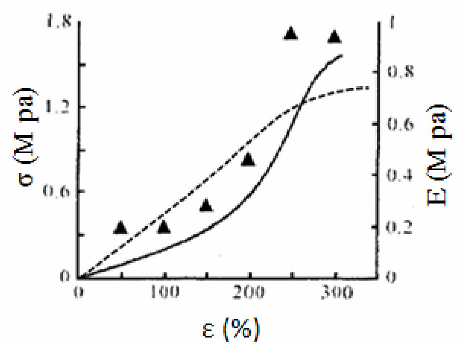

Fig.1 Stress-strain, modulus-strain and simplified stress-strain characteristic of PVA hydrogel

—: tension strength ▲: tension modulus ----: simplified stress-strain

Rice [4] introduced the J integral theory to analyze the crack behavior in an elastic body. This theory can quantitatively describe stress-strain field intensity of elastic materials with a crack, which has clear definition and strict theoretical basis. Consider a homogeneous plate containing a penetrating crack. There is no external force near the crack tip, but tension stress acting on the homogeneous plate produces 2D stress-strain field around the crack. J integral is defined as

$$J=\int_{\Gamma}\left(\mathrm{w}\mathrm{d}y-\mathrm{T}\frac{\partial U}{\partial x}ds\right) \tag{1}$$

Where Γ loop shown in Fig.2 is around the crack tip, which starts from the crack lower surface and ends at the upper surface (counterclockwise direction), w is the strain energy density of the plate, T is the force vector acting on micro arc, and U is the displacement on Γ loop.

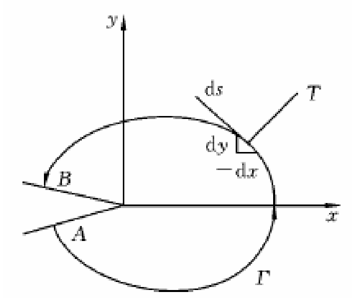

Fig.2 Schematic of the J-integral path

Rice [4] proved the path-independence of the J integral in the case of linear and nonlinear elastic material. Eq. (1) is not suitable for numerical calculation, because of not only the infeasibility of calculating stress and strain on Γ loop but the results are not always compatible when integral circuit is very close to the crack tip. Therefore, Shih [5] and Raju et al. [6] proposed the equivalent integral area method to compute J-integral by numerical calculation. A limited area near the crack tip replacing the integral loop is used to calculate J-integral value. According to divergence theorem, Eq. (1) translates into

$$J=\int_A \left(\sigma_{ij}\frac{\partial u_j}{\partial x_1}-\omega\delta_{1i}\right)\frac{\partial q}{\partial x_i}dA \tag{2}$$

For 2D crack analysis, Eq. (2) can be divided into

$$J=\int_A \left[\left(\sigma_{xx}\frac{\partial u}{\partial x}+\tau_{xy}\frac{\partial v}{\partial x}-\omega\right)\frac{\partial q}{\partial x}+\left(\tau_{xy}\frac{\partial u}{\partial x}+\sigma_{yy}\frac{\partial v}{\partial x}\right)\frac{\partial q}{\partial y}\right]dA \tag{3}$$

In order to easily calculate the integral numerically, function q(x, y) must have a certain value on each node of the integral area. Shih etc. proofed that the J integral is not sensitive to the assumption form of q(x, y), i.e., q(x, y) can be arbitrarily selected [5]. But the boundary values of the integral area are strictly prescribed. For example, q = 1on the internal boundary, then q=0 on the external boundary for a plane problem. Equivalent integral area method is very convenient in finite element analysis, surely becoming the common tools calculating J integral.

In this paper, the plane stress four node unit calculating the J integral of the PVA-H crack tip is used, which is corresponding to standard CPS4 in ABAQUS. Using the gauss integral method, J-integral within one unit is approximated by summation of the formulas related with gauss points, i.e.

$$\overline{J} \approx I(r_1,s_1)+I(r_2,s_2)+I(r_3,s_3)+I(r_4,s_4) \tag{4}$$

In which

$$I(r,s)=\left[\left(\sigma_{xx}\frac{\partial u}{\partial x}+\tau_{xy}\frac{\partial v}{\partial x}-\omega\right)\frac{\partial q}{\partial x}+\left(\tau_{xy}\frac{\partial u}{\partial x}+\sigma_{yy}\frac{\partial v}{\partial x}\right)\frac{\partial q}{\partial y}\right]\det(J^e) \tag{5}$$

$$r=\left\{-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right\},\quad s=\left\{-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right\}$$

Thus, J integral of PVA-H corresponding to one unit is obtained. Such repeatedly, J values of all units covering integral area are calculated. By determining J integral of different integral paths, whether the J integral of nonlinear fracture of PVA-H depends on the integral loop near the crack tip or not is given. Moreover, the influence of loadings and crack lengths on the J integral of nonlinear fracture of PVA-H is obtained by changing different loadings and crack lengths.

2. Determination of J integral

Figure 3 shows a center crack in a limited plate subjected to tension stress. It has a span 2H=400mm, a width 2W=200mm, a thickness B=1mm and the length of the crack a=20mm. Tensile modulus of PVA-H material is various as different manufacture method. According to the results given by Gao etc. [7], PVA-H with the following material properties is considered

$$E=10\text{Mpa},\ \nu=0.49,\ \sigma_s=4.47\text{Mpa}$$

Where E is the elastic modulus, ν is the Poisson ratio, σ_s is the tensile strength.

According to the asymmetry, the finite element model of a quarter PHV-H plate is shown in Fig.4 by ABAQUS pretreatment with transverse 100 division and vertical 200 division. Then all the information including geometric and material properties of the model extracted from ABAQUS is input into the FORTRAN input data file. The units from 1 to 20 are free, and the twenty-first node is the crack tip. Hence, y direction displacement of the right nodes of the crack tip and x direction displacement of the left boundary nodes is restrained, then the upper boundary nodes are applied by tensile load σ =0.1Mpa.

Equivalent integral area (gray section) including 8*2 units near the crack tip is selected shown in

Fig.5. J value of nonlinear fracture of PVA-H by FORTRAN program is given as J= 0.065082349N/mm.

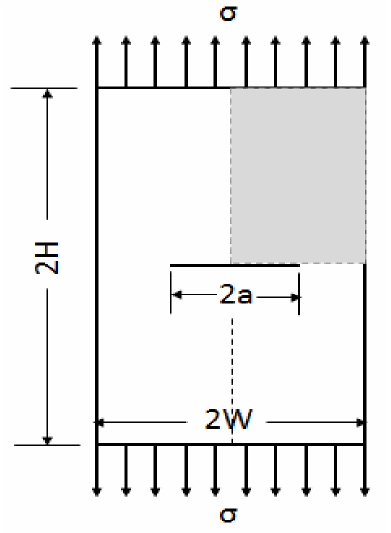

Fig.3 A center crack in a limited plate subjected to tension stress.

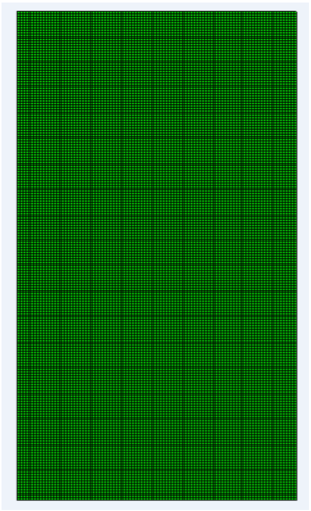

Fig.4 PHV-H plate finite element model

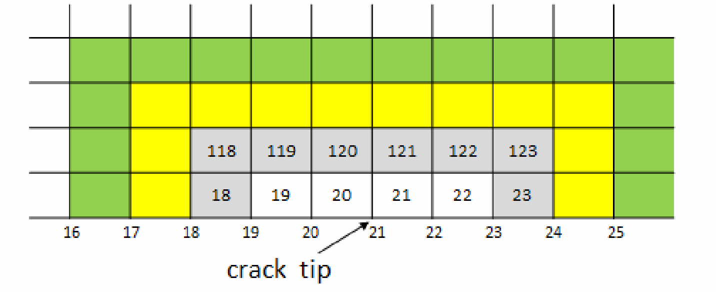

Fig.5 Equivalent integral area

Change tensile load σ , respectively, σ = 0.1、0.4、0.7、1.0、1.5 and 2.0Mpa. The calculated J integral values by FORTRAN in equivalent integral area are plotted in Fig.6. It shows that the J integral of nonlinear fracture of PVA-H increases as the increment of the applied loading, approximately, for square relation.

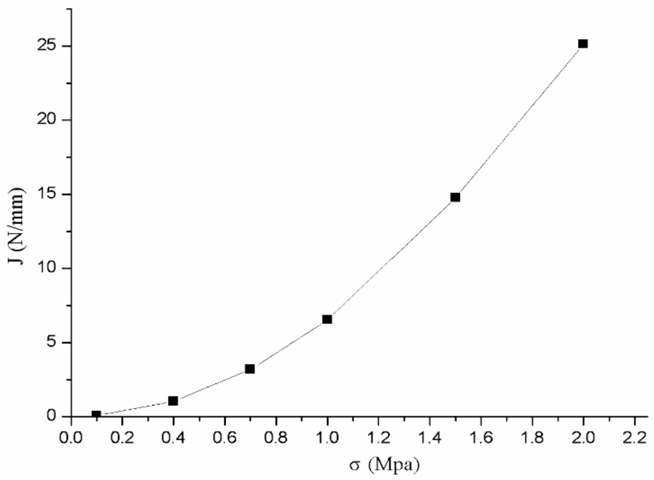

Fig.6 J-σ curve under different loadings

Each layer starting from the third layer outside of the crack tip is taken as integral path. The largest number of units of the layer is 24*2. Fig.7 plots the J-integral by FORTRAN in equivalent integral area against different integral paths under different loadings. It shows J integral in a certain loading remains unchanged as the increment of the chosen integral units, namely the J integral of nonlinear fracture of PVA-H is independent on the integral path. The result is found in very nice agreement with path-independent of the J integral in the case of linear and nonlinear elastic materials proved by Rice. Moreover, the bigger the applied loading, the greater the J integral value in the same integral path.

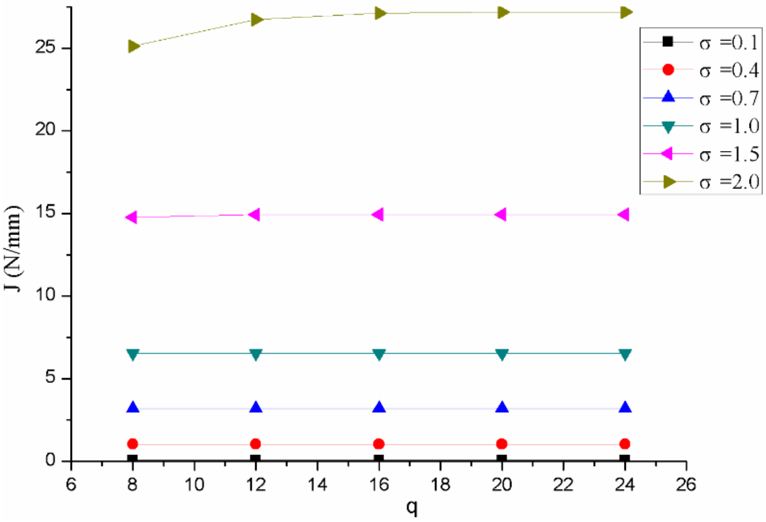

Fig.7 J curves for different integral paths under different loadings

In order to get the influence of the crack length on the J integral of nonlinear fracture of PVA-H, the location of the crack tip is changed from the eighteenth to the twenty-four node and the crack length ranges from 34 mm to 46 mm. The equivalent integral area is similar to that of Fig.5 (gray section). When the crack tip moves one unit length to the left or to the right, correspondingly, the equivalent integral area also moves a unit length to the left or to the right for guaranteeing the area covering 8*2 units. Change the tensile load, respectively, σ = 0.1、0.4、0.7、1.0、1.5 and 2.0Mpa. The calculated J values by FORTRAN in equivalent integral area are shown in Fig.8. It shows that J integral value at the crack tip increases linearly along with the PVA-H crack length increasing. It is clear that J integral or energy release rate of nonlinear fracture of PVA-H is severely affected by the crack length.

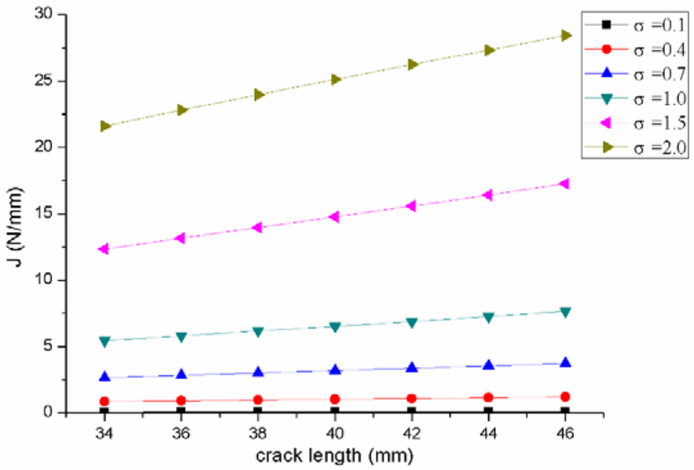

Fig.8 Effect of crack length on J-integral under different loadings

3. Conclusin

In this work, J-integral of nonlinear fracture for the colloid soft material with a center crack is calculated by the finite element method. When the integral circuit is very close to the crack tip, using the equivalent integral area method simulates J integral numerically. Through the divergence theorem, a limited area near the crack tip replacing the integral loop is used to calculate J-integral value. The PVA-H colloid soft material is simplified to elastic plastic model. J-integral within one unit is approximated by the gauss integral method. By summing for the J values of all units, J integral of the equivalent integral area corresponding to integral path is derived. Change integral path and calculate J integral of PVA-H colloid soft material in different integral path. The results can be summarized as follows:
(1) J integral of nonlinear fracture of PVA-H in a certain loading is independent on the integral path. Path-independent of the J integral obtained by Rice is proved once again by numerical method.
(2) The bigger the applied loading, the greater the J integral value of nonlinear fracture of PVA-H in the same integral path, approximately, for square relation.
(3) J integral value at the crack tip increases linearly along with the PVA-H crack length increasing. It is obvious that J integral or energy release rate of nonlinear fracture of PVA-H is severely affected by the crack length.
In the future, the above conclusions will provide certain theoretical guidance for PVA hydrogel as articular cartilage repair materials.

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References

[1] Gu ZQ, Xiao JM, Zhang XH. Biomed Mater Eng, 1998, 8(2): 75-81.
[2] S.M. Malmonge. Int J Artif Organs, 2001, 24(3): 119-122.
[3] Y.S. Pan, D.S. Xiong, X.L. Chen. Mechanical and Swelling Properties of Polyvinyl Alcohol Hydrogel. POLYMER MATERIALS SCIENCE AND ENGINEERING. 2007, 23(6): 228-231.
[4] J.R. Rice. A path independent integral and the approximate analysis of strain concentration by notches and cracks. Journal of Applied Mechanics, 1968, 35: 379-186.
[5] B. Moura, C.F. Shih. A treatment of crack tip contour integrals. International Journal of Fracture, 1987, 35: 295-310.
[6] K.N. Shivakumar, I.S. Raju. An equivalent domain integral method for three-dimensional mixed-mode fracture problem. Engineering Fracture Mechanics, 1992, 42: 935-959.
[7] F. Gao, Y. Jiang, B.F. Zeng. Advance in Polyvinyl alcohol hydrogel replacing articular cartilage. Foreign medical bone in science. 2004, 25(1): 41-43.