Coupled fracture mode associated with anti-plane loading of cracks and notches

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Abstract The purpose of this paper is to investigate by means of the 3D Finite Element method a coupled fracture mode generated by anti-plane loading of a straight through-the-thickness crack in a linear elastic plate. This coupled fracture mode represents one of three-dimensional phenomena, which are currently largely ignored in numerical simulations and failure assessment of structural components weakened by cracks. It arises due to the boundary conditions on the plate free surfaces, which negate the transverse shear stress components corresponding to classical mode III. Instead, a new singular stress state in addition to the well-known 3D corner singularity is generated. This singular stress state (or coupled fracture mode) can affect or contribute significantly to the fracture initiation conditions. The coupled singular mode exists even if the applied anti-plane loading produces no singularities ($K_{III} = 0$). In this case there is a strong thickness effect on the intensity of the coupled fracture mode.

Keywords Crack, anti-plane loading, coupled fracture mode, 3D modelling

1. Introduction

The first systematic study on the three-dimensional stress states of a through-the-thickness crack subjected to mode I loading was conducted in Refs [1-4]. Accurate studies on three-dimensional stress distributions in front of cracks have been carried out in those references, extending Williams' two-dimensional eigenfunction expansions [5] to the three-dimensional case, and in [6] four distinct harmonic functions have been used to solve the problem according to Papkovich-Neuber's method.

Utilising a variational principle, a system of simplified governing equations has been derived [1-4] for the extension and bending deformations of an elastic plate with a through-the-thickness crack and investigated the three-dimensional stress states surrounding the crack tip. One important result from this work is that the area of the three-dimensional stress state around the crack tip spreads in the plane direction to the distance of approximately half of the plate thickness. Beyond this distance the stress state follows the classical plane stress solution. Many experimental studies conducted in the past including those carried out in [7] who applied an optical technique confirmed this fundamental result.

Another interesting three-dimensional effect, which was first presented in [8], is the disappearance of the in-plane singularity at a point when a corner front (crack front) intersects a free surface. At this point a new three-dimensional corner singularity develops instead. The problem of a vertex (corner) singularity is now well documented in a number of articles in the last thirty years (see, among the others, Refs [9-17]). This problem was recently re-examined in [18] with reference to fatigue crack growth. In this paper, the effect of a free surface on fatigue crack behaviour was investigated experimentally and numerically for relatively thick specimens, where the solutions provided in [8] for semi-infinite space can be applied. In [9] it was underlined that the 3D corner singularity in cracked plates, which usually can be neglected for mode I loading, should be taken into consideration for mixed mode loadings. However, despite all these studies the role of the corner singularities in fatigue and fracture phenomena remains largely unknown.

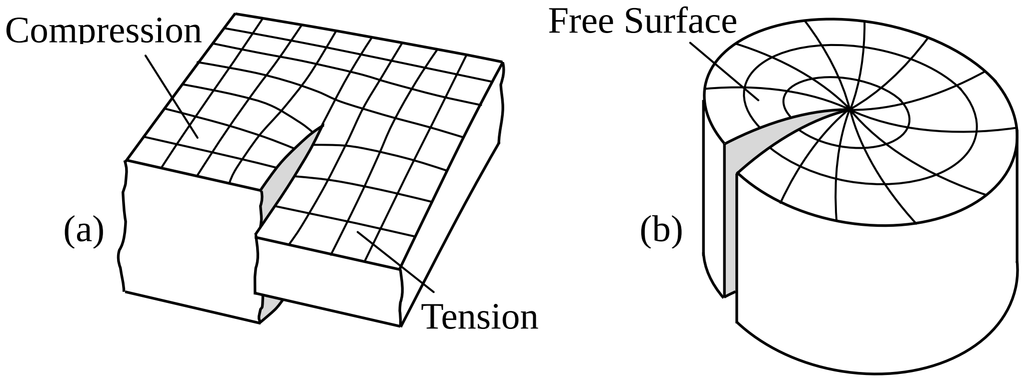

Fig.1 Generation of coupled fracture modes due to Poisson's effect and redistribution of stresses on free surfaces when a crack is subjected to shear (a) and anti-plane loading (b)

One more three-dimensional effect, which the two-dimensional numerical and analytical solutions are incapable to predict or analyse, is the existence of the coupled fracture modes. To illustrate these coupled fracture modes let us consider a through-the-thickness crack loaded in shear or anti-plane loading. In these cases additional local fracture modes are generated due to Poisson's effect and/or the redistribution of stresses on the free surfaces as illustrated: in Fig. 1 (a) – mode II loading and (b) – mode III loading in the case of a zero notch angle [16]. In many papers (see among others Refs [19, 20]) it was highlighted that modes II and III cannot exist independently and the presence of one of these modes always generates a coupled singular state.

The coupled modes induced by the primary modes II and III for through-the-thickness cracks have been known for a rather long period of time but the first systematic studies for semi-infinite cracks were conducted numerically using a careful FE modelling in [10, 21]. Recently the first coupled fracture mode (see Fig. 1a), which was called the out-of-plane or K_O-mode, was investigated for finite geometries representing welded joints as well as for sharp and round notches of arbitrary opening angle. It was demonstrated that the K_O-mode has many interesting and previously unknown features, which can influence mixed-mode fracture, crack initiation and fatigue growth phenomena.

A recent experimental and numerical study has been performed to investigate the fatigue crack growth tests in mixed-mode 2 + 3 on maraging steel and Ti-6Al-4V. The 3D evolutions of the crack fronts -measured by SEM after interrupted tests- have been analyzed [22]. A 3D finite element model of tested specimens has been prepared, with a refined mesh around the crack front. It has been shown that the profiles of stress intensity factors and energy release rate vary along the crack front. In particular due to the coupled Mode 2 and Mode 3 loadings K_2, K_3 and G have been found with an asymmetrical profile. When the inclination angle of the crack plane relative to the horizontal axis surface is equal to 45° the energy release rate is ten times higher near one free surface than on the other inducing a strong asymmetry in crack growth and showing the non negligible effects of the coupled modes.

One of these interesting features is a coupling of this mode with the non-singular terms at shear loading. It means, if a crack is loaded in shear with K_{II}= 0, the intensity of the coupled K_0- mode can be different from zero. At such loading the intensity of the out-of-plane mode is finite and capable of initiating brittle fracture [23, 24]. The latter result and other three-dimensional features are in contradiction with the classical two-dimensional view on fracture, which states that brittle fracture can only be initiated with non-zero stress intensities of primary fracture modes (conventional mode I, II and III).

Important features of the out-of-plane singular mode were revealed in a careful three-dimensional numerical study of typical welded lap joint geometry [25] aimed to investigate the contribution of this mode to the overall stress state in the close vicinity of the slit/notch tip. The extension of the present study to sharp and blunt notches under mode II loading has been recently made by the present authors [26, 27].

In contrast to mode II loading, the coupled mode corresponding to mode III loading is much less investigated and there were no systematic studies focusing on the investigation of this mode except many remarks in literature on the coupled nature of modes II and III as well as the influence of this coupled mode on fracture appearance, crack path and crack initiation [19, 20]. The aim of this paper is to investigate this coupled mode using the Finite Element method and provide numerical estimates of the possible contribution of this mode to brittle fracture.

In the beginning of the present paper, a numerical technique based on FE method is developed and validated for the investigation of the coupled mode associated with the anti-plane loading. Because this mode is localised in the close vicinity of the crack tip, a careful meshing is required in order to avoid large numerical errors in this region. Further, a systematically study of this mode is carried out focusing on the effect of Poisson's ratio on the stress intensity of this mode, which also varies along the crack front. It is also demonstrated that similar to the K_0-mode, a non-singular mode III loading (K_{III}=0) is capable of generating the coupled singular stress state along the crack front. In this case, a strong scale effect exists and the intensity of the coupled mode strongly depends on the thickness of the plate.

2. Approach

2.1 Geometry

Because the coupled singular modes are local modes and spread to the distance of approximately half of the plate thickness as explained in the Introduction, the problem geometry is truncated to a disk with such dimensions which avoid the effect of the finite boundaries on the stress state of the coupled mode as well as the influence of bending stresses. The antisymmetric boundary conditions are utilised to further simplify the geometry. The final geometry is shown in Fig. 2 and appropriate displacement boundary conditions corresponding to anti-plane loading were applied on the cylindrical surface. The origin of the Cartesian coordinate system (x,y,z) is located at the crack tip, at the mid-surface where x direction was chosen to be the direction of the crack bisector.

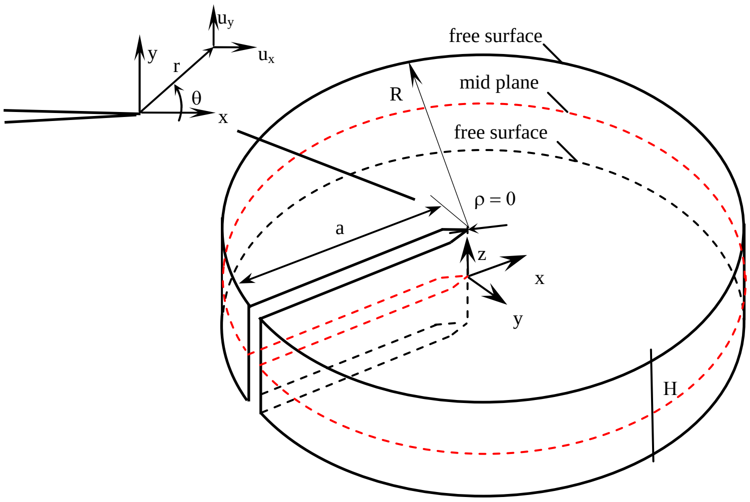

Figure 2. Geometry of the plate and coordinate system with z=0 in the mid-plane

2.2 Boundary Conditions

The displacement boundary conditions are applied to the outer cylindrical edge of the plate. In the beginning the out-of-plane displacement field corresponding to the first singular term in the asymptotic expansion of the stress field, which is valid far from the crack tip (model boundaries), was prescribed as follows:

$$w = u_z = \frac{2K_{III}}{\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\phi}{2} \qquad (1)$$

$$u_x = 0$$

$$u_y = 0$$

where μ is the shear modulus. It is linked to Young's modulus by a well-known relationship:

$$\mu = \frac{E}{2(1+\nu)} \qquad (2)$$

In further numerical examples Young's modulus was set at 200×10^9 Pa. However, the numerical results to be presented in the following sections can be easily rescaled for other values of mechanical properties, loading or plate thickness.

2.3 Validation

As highlighted in the Introduction section due to complexity of the numerical analysis and interpretation of the computational results there were almost no works investigating this singular mode. However, the three-dimensional effects, specifically the coupling of fracture modes II and III, were often recognised and acknowledged in many papers on stress analysis of cracked structural components, fracture and fatigue. A comparison of the present results obtained with ANSYS 11 and those published in [11] in terms of the ratio of the induced to applied stress intensity factors K_{II}^{C}/K_{III} as a function of the position along the crack front, z/H, are shown in Fig.3, demonstrating a good agreement. Description of all parameters of the modelling is given in [10] and the definition of the stress intensity factor of the coupled mode K_{II}^{C} will be provided in the next Section.

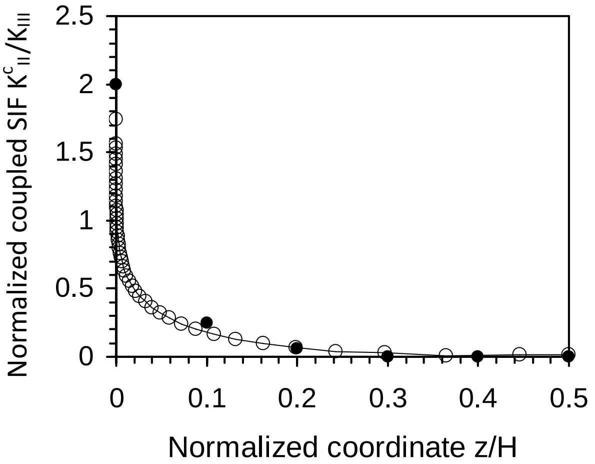

Fig.3 Comparison of the present FE calculations and published results: line is the current numerical results and filled circles are data from [11]

3. Numerical Results

3.1 Stress State

First, the results for the out-of-plane shear stress components along the bisector line for different distances from the crack tip are considered. The results can also be represented in non-dimensional form; however, the non-dimensional results are sometimes difficult to relate to practical situations. Therefore in the following computational examples, if it is not specified separately, the applied remote stress intensity factor, K_{III}= 1 MPa$\sqrt{m}$, Poisson's ratio ν=0.3 and the thickness of the plate, H= 40 mm.

The intensity of the computed stress component τ_{yz} defined similar to the classical definition of the stress intensity factor for mode II in plane problems of elasticity as

$$K_{III}(z) = \sqrt{2\pi}\lim_{x\to 0}\tau_{yz}(z)x^{1-\lambda_{III}} \quad (3)$$

can be obtained using a standard log-log regression analysis of the stress distribution. The singular power describes the asymptotic rate at which the stress components increase as the crack tip is approached. As expected, the stress field everywhere has been found to have a degree of singularity

λ_{III}= 0.5. This value remains constant from the crack tip until the external radius is reached.
It has been found that the in-plane shear stress components associated with the coupled mode change in a wider range from zero at the mid-plane as expected to values of the same order as the applied stress intensity factor. In contrast, the out-of-plane shear stresses associated with the applied K_{III} mode do not significantly change across the thickness and the maximum shear stresses take place at the mid-plane of the plate decreasing smoothly towards the free surfaces. As mentioned above, the results were obtained for the above specified conditions (applied stress intensity factor, material properties and plate thickness) and can be easily rescaled or represented in a dimensionless form if necessary.

3.2 Stress Intensity Factors

The stress intensity factor of the coupled mode can be defined similar to mode II, or as

$$K_{II}^{C}(z) = \sqrt{2\pi} \lim_{x\to 0} \tau_{xy}(z) x^{1-\lambda_{II}} \quad (4)$$

In general there are many similarities between the coupled mode and mode II corresponding to the shear loading of a crack. However there are some essential differences between these two singular modes. The coupled mode is a local mode, which is concentrated in the vicinity of the plate free surfaces. It is generated due to the boundary conditions, which negate the out-of-plane shear stress components corresponding to the applied mode. For this coupled fracture mode the effect of Poisson's ratio is rather small in comparison with the coupled mode generated by mode II loading [14]. The coupled modes rapidly decay with distance from the crack tip as it will be demonstrated later in this paper.
The distributions of the local intensities of mode III, $K_{III}(z)$ associated with the applied antisymmetric loading and $K_{II}^{C}(z)$, associated with the coupled mode are shown in Fig. 4. As it can be seen from this figure the coupled mode is localised in the close vicinity of the free surfaces and propagates into the thickness to the distance of approximately 0.2 H. There is a significant drop in the intensity of the applied mode in the precincts of the free surface. In the very close vicinity of the free surface the intensity of both modes is approaching to zero. As it was mentioned in the Introduction, at a point where a corner front (crack front) intersects a free surface the applied and coupled modes disappear and a new type of singularity, 3D corner singularity, develops instead. Consequently, the numerical results could be not very accurate in the vicinity of this location [10].

4. Anti-plane Loading with K_{III} = 0 and Scale Effect

4.1 Coupled Mode due to Anti-plane Loading

For an elastic plate with a crack, the possible contribution of higher order terms of the classical Williams' solution [28] to the fracture initiation was discussed with reference to a plane elastic problem in [29-31]. As discussed in Ref. [30] in some cases of practical interest, the contribution of the higher order stress terms to the stress state in the vicinity of the crack tip (in addition to the stress intensity factors and the T-stress) is not negligible. A set of equations has been proposed for accurately describing the crack tip stress components particularly for those cases where the mode I and mode II stress intensity factors used in combination with the T-stress component, are unable to capture with satisfying precision the complete stress field ahead the crack tip. In this session an accurate analysis of mode III higher order (non-singular) terms on the induced mode II is carried.

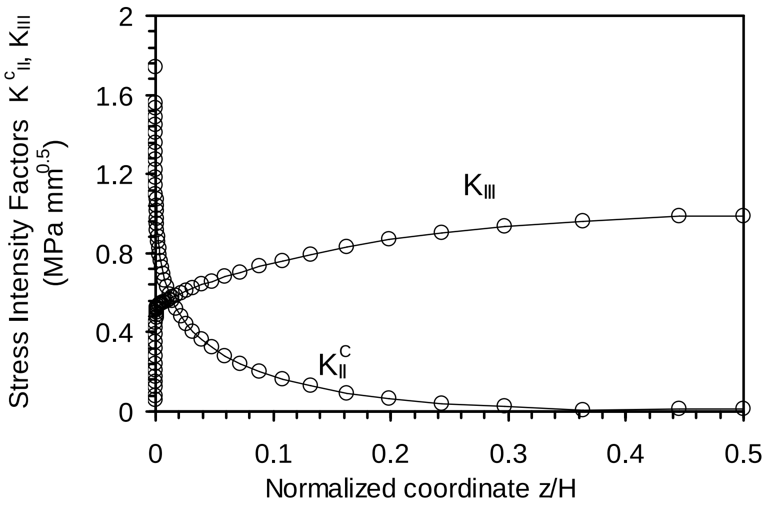

Fig. 4. Distribution of the stress intensity factors (mode III and the coupled mode) along the plate thickness, at a distance x = 0.05 mm from the crack tip.

The out-of-plane displacement, w, beyond the area of 3D effects (this 3D area is confined within a cylinder with the axis of symmetry along the crack front and radius equal to half of the plate thickness) can be expressed as [29]:

$$w = \sum_{n=0}^{\infty} \frac{r^{n+\frac{1}{2}}}{\mu} C_n \sin\left(\frac{1}{2} - n\right)\phi \qquad (5)$$

with

$$C_0 = K_{III}^{\infty}\sqrt{\frac{2}{\pi}} \qquad (6)$$

The previous results are related to the case when $C_0 \neq 0$ or $K_{III} \neq 0$ and all other terms in the asymptotic expansion (5) are zero ($C_n \equiv 0$ at $n = 1,2...\infty$). In Fe simulations the corresponding displacement boundary conditions far from the crack tip were applied to avoid effect of the finite boundaries of the FE model. In the following analysis the situation when a through-the-thickness crack loaded with $C_0 = K_{III} = 0$ is considered. It will be demonstrated that such a loading of a through-the-thickness crack is capable of inducing the singular coupled singular mode, the same as for the leading term in the asymptotic expansion of the two-dimensional displacement/stress field. It suggests that this coupled singular mode has a potential to cause fracture. In contrast, the classical two-dimensional theory of brittle fracture states that fracture by crack propagation is impossible due to the absence of the energy release rate when K_{III}=0.

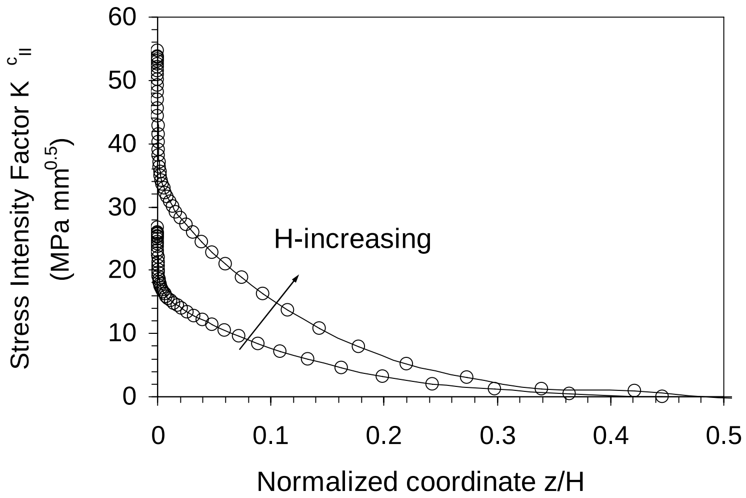

Fig.5. Effect of the thickness on the intensity of the coupled mode at distance x/H= 0.02 (a);

Consider the first non-singular term in the asymptotic expansion. Set its value C_1=1 MPa/$\sqrt{m}$ corresponding to n=1 in Eq. (5). Such loading produces no singularities corresponding to the applied antisymmetric loading, i.e. K_{III} = 0, which is also valid in the area of 3D effects (very close to the crack tip). The results of the FE calculations are presented in Fig.5, which reveals that a non-zero stress intensity of the coupled mode does exist at such loading. The latter effect has many implications for failure assessment. In particular, it indicates that brittle failure by crack propagation is possible even the intensity of the primary load (2D stress intensity factor) is close to zero $(K_{III} \approx 0)$.

It is interesting that the non-singular loading reveals a strong scale effect of deterministic nature, which can be also found from dimensionless considerations. Indeed, due to the localised nature of the coupled mode, its intensity has to be a linear function of the applied intensity, or $K_{II}^{C} \approx C_1$.

5. Conclusion

Below the most important findings of this work will be summarised:

a) Anti-plane loading of a through the thickness crack leads to generation of a singular stress state (or a coupled fracture mode), which has the similar singular behaviour as classical mode II. However there are some essential differences between these modes. The coupled mode is a local mode, which is concentrated in the vicinity of the plate free surfaces and is generated due to the Poisson's effect and boundary conditions, which negate the out-of-plane shear stress components corresponding to the applied mode. The coupled mode rapidly decays with distance from the crack tip. The local nature of the coupled modes means that the obtained results for the truncated geometry are applicable to other finite geometries provided that there is no interaction between the boundary conditions and the area of 3D effects.

b) The intensity of the primary (applied) anti-plane mode in the vicinity of the crack tip is moderately affected by Poisson's ratio. In contrast, the intensity of the coupled mode is largely unaffected by Poisson's ratio.

c) The singular coupled mode can be induced by, so called, non-singular anti-plane loading with K_{III} = 0. In this case a strong thickness effect is found and confirmed by FE calculations. The intensity of the coupled mode decays (increases) with the decrease (increase) of the plate thickness as a power function. When several anti-plane non-singular terms are applied to the crack in an elastic plate the scale effect can be rather complicated. The influence of the higher terms on the stress intensity of the coupled mode depends on the number of the asymptotic mode, n, and this influence is more significant for the higher mode numbers.

All these theoretical findings, specifically the effects of Poisson’s ratio and plate thickness on the stress intensities, have direct implications to the failure initiation conditions for cracks stressed in mode III. These findings demonstrate essential differences between classical two-dimensional considerations and 3D Fracture Mechanics. For example, the generation of the coupled singular mode at anti-plane loading with $K_{III} = 0$ indicates that contrary to the classical 2D theories, fracture under such loading conditions can be initiated due to the induced singular coupled modes. Such fracture is likely to take place close to free surfaces. It is also recognised that much work needs to be done to understand the contribution of the coupled modes to fracture initiation and fatigue.

Finally, all Finite Element results of the current study can be re-scaled to make these available for assessment of comparative studies. The only reason as to why it has been used particular values was to provide some physical feeling for the results of our finite element study.

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